MAXIMUM-LIKELIHOOD PARAMETER ESTIMATION FOR CURRENT-BASED MECHANICAL FAULT DETECTION IN INDUCTION MOTORS

Martin Blödt[1], *Marie Chabert*[2], *Jérémi Regnier*[1], *Jean Faucher*[1]

[1]Laboratoire d'Electrotechnique et d'Electronique Industrielle
UMR INP Toulouse / CNRS No. 5828
2, Rue C. Camichel, 31071 Toulouse, France
blodt,regnier,faucher@leei.enseeiht.fr

[2]IRIT / ENSEEIHT
2, Rue C. Camichel
31071 Toulouse, France
marie.chabert@enseeiht.fr

ABSTRACT

This paper proposes a new method for mechanical fault detection in induction motors. The detection strategy is based on the estimation of a particular stator current parameter. The considered mechanical faults cause periodic load torque oscillations leading to a sinusoidal phase modulation of the stator current. The modulation index is related to the fault severity and can be used as a fault indicator. First, a simplified stator current model is proposed. The problem is then equivalent to the parameter estimation of a sinusoidal phase mono-component signal. Second, the maximum likelihood estimator is implemented using evolution strategies for optimization. The Cramer-Rao lower bounds are calculated and compared to the estimator performance through simulations. The estimation procedure is studied on experimental stator current signals from faulty and healthy motors.

1. INTRODUCTION

Induction motors can be found in an increasing number of applications from small motors in home and business applications to drives operating in high speed trains. The reliability, productivity and safety of an installation containing induction motors can be increased by an automatic and permanent monitoring system. Consequently, induction motor fault detection and diagnosis is of great concern. Mechanical faults are traditionally detected by monitoring and analyzing vibration data. However, vibration measurement is cost intensive and cannot always be realized. Alternatively, the diagnosis can be based on the available electrical quantities e.g. the stator current. The stator current is often already measured for control and protection purposes. This study proposes a new detection method based on the maximum likelihood estimation of an appropriate stator current parameter. This parametric approach first requires an appropriate fault model.

In this study, the influence of mechanical faults on the induction machine is modeled by an additional sinusoidal torque [1]. The total load torque Γ_{load} expresses as:

$$\Gamma_{load}[n] = \Gamma_{av} + \Gamma_c \cos(\omega_c n) \tag{1}$$

where Γ_{av} is the average load torque, Γ_c the additional torque amplitude and ω_c the normalized characteristic angular fault frequency, $n = 0, \ldots, N-1$. The magnetomotive force (MMF) model and the permeance wave approach provide the following expression of the stator current $i[n]$ [1]:

$$\begin{aligned} i[n] &= i_{st}[n] + i_{rt}[n] \\ &= I_{st}\cos(\omega_s n + \varphi_s) + I_{rt}\cos\big(\omega_s n + \beta\cos(\omega_c n)\big) \end{aligned} \tag{2}$$

β is the so-called modulation index of the sinusoidal phase modulation and it can be shown that $\beta \propto \frac{\Gamma_c}{\omega_c^2}$. ω_s is the normalized angular supply frequency and φ_s the initial phase angle between rotor and stator MMF depending on the motor load. $i_{st}[n]$ and $i_{rt}[n]$ denote the stator current components resulting from the stator and rotor MMF. The healthy case is obtained for $\beta = 0$.

Hence, the proposed fault detection strategy is based on the estimation of β from a discrete noisy observation of the stator current. The estimate is used as a fault indicator because β is directly related to the amplitude Γ_c of the torque oscillation and therefore to the fault severity. The estimate of the modulation index can later be used for detection in a binary hypothesis test. β is in general relatively small and takes typically values in $[0, 0.1]$. The other signal parameters are unknown and have also to be estimated in order to obtain a correct estimate of β. Conventional fault detection techniques compute a periodogram of the stator current and use the amplitude of sideband peaks for fault detection [2]. Drawbacks are the need for a high number of samples and the impossibility to distinguish phase modulation from amplitude modulation. Amplitude modulation can be the result of other faults. Thus, the knowledge of the modulation type is important for diagnosis purposes.

Section 2 describes an approximation of the faulty stator current by a monocomponent signal with a sinusoidal phase modulation. This simplified signal model is used in section 3 for the derivation of the maximum likelihood estimator. The estimator algorithm is implemented using evolution strategies described in 3.2. The theoretical Cramer-Rao bounds are calculated and compared to the estimator performance in numerical simulations. Section 4 describes the application of the estimator to mechanical load fault detection. An experimental setup provides the necessary test data and proves a good performance of the estimator for current-based fault detection.

2. STATOR CURRENT MODEL

First, the current must be expressed in its analytical form for an univocal phase definition [3]. Recall that if $x[n]$ is a real signal, the associated analytical signal is $x_a[n] = x[n] + jH\{x[n]\}$ where $H\{.\}$ denotes the Hilbert Transform. For further processing, the complex stator current will be demodulated. An estimate of the demodulation angular frequency ω_s is obtained from a spectral estimate of (2). The estimation is then performed on the complex envelope $i_{d,a}[n]$ of the real measured current:

$$i_{d,a}[n] = I_{st} \exp j\varphi_s + I_{rt} \exp j\big(\beta \cos(\omega_c n)\big) \quad (3)$$

The estimation problem complexity results from the presence of two components with the same average frequency but with different amplitudes and phases. However, under appropriate assumptions on the signal parameters, the demodulated stator current $i_{d,a}(t)$ can be approached by a mono-component phase-modulated signal as follows:

$$i_{d,a}[n] = \theta_0 \exp j\left[\theta_1 + \theta_2 \cos(2\pi\theta_3 n + \theta_4)\right] \quad (4)$$

where $\underline{\theta} = [\theta_0, \theta_1, \theta_2, \theta_3]$ are the unknown parameters. By identification, parameters $[\theta_0, \theta_1, \theta_2, \theta_3]$ can be related to the physical parameters as follows:

$$\theta_0 = I_{st}\sqrt{1+\gamma^2+2\gamma\cos\varphi_s}, \quad \theta_1 = \frac{\gamma \sin\varphi_s}{\gamma\cos\varphi_s + 1}$$
$$\theta_2 = \frac{\beta}{\gamma\cos\varphi_s+1}, \quad \theta_3 = f_c, \quad \gamma = I_{st}/I_{rt}$$

The estimated parameter θ_2 is proportional to the modulation index. However, it also depends on the motor load level through the ratio γ of the stator and rotor current amplitude and the phase angle φ_s. θ_4 is the initial phase of the modulation.

3. PARAMETER ESTIMATION

3.1. Maximum Likelihood Estimation

This section estimates the modulation index β from demodulated noisy stator current observations $\{z[n]\}_{n=0,\dots,N-1}$. Let $\{z_a[n] = z[n] + jH\{z[n]\}_{n=0,\dots,N-1}$ denote the stator current observations in their analytical form. The additive noise $\{g_a[n]\}_{n=0,\dots,N-1}$ is supposed complex white, zero-mean and Gaussian with $g_a[n] = g[n] + jH\{g[n]\}$. The noise variance σ^2 is supposed unknown.

Under this assumption, the maximum likelihood estimator (MLE) can be derived. Moreover, this estimator is asymptotically (i.e. for large data records) unbiased and its variance approaches the Cramer-Rao Lower Bound (CRLB) [4]. The MLE is obtained maximizing the noise probability density function with respect to the parameter vector $\underline{\theta}$. This problem can be seen as an extension of the parameter estimation of a non-modulated sinusoidal signal [5].

The joint probability density function $p(Z, \underline{\theta})$ expresses as [5]:

$$p(Z,\underline{\theta}) = \frac{1}{(2\pi\sigma^2)^N} \exp\left[-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1} |z_a[n] - i_{d,a}[n]|^2\right] \quad (5)$$

Maximizing $p(Z,\underline{\theta})$ is equivalent to maximizing the following expression $L_0(\underline{\theta})$ with respect to $\underline{\theta}$ [5]:

$$L_0(\underline{\theta}) = -\frac{1}{N}\sum_{n=0}^{N-1} |z_a[n] - i_{d,a}[n]|^2 \quad (6)$$

and finally, the estimation problem is equal to maximizing $L(\underline{\theta})$ with:

$$\begin{aligned} L(\underline{\theta}) &= \frac{1}{N}\sum_{n=0}^{N-1} 2\left(z[n]i_d[n] + H\{z[n]\}H\{i_d[n]\}\right) - |i_{d,a}[n]|^2 \\ &= -\theta_0^2 + \frac{2\theta_0}{N}\sum_{n=0}^{N-1} z[n]\cos(\theta_1 + \theta_2\cos(2\pi\theta_3 n + \theta_4)) \\ &\quad + H\{z[n]\}\sin(\theta_1 + \theta_2\cos(2\pi\theta_3 n+\theta_4)) \end{aligned} \quad (7)$$

Straightforward derivations lead to:

$$L(\underline{\theta}) = -\theta_0^2 + 2\theta_0 \Re\left\{ e^{-j\theta_1}\frac{1}{N}\sum_{n=0}^{N-1} z_a[n] e^{-j\theta_2\cos(2\pi\theta_3 n + \theta_4)}\right\} \quad (8)$$

where $\Re\{.\}$ denotes the real part. Let define the complex function

$$B(z,\theta_2,\theta_3,\theta_4) = \frac{1}{N}\sum_{n=0}^{N-1} z_a[n]e^{-j\theta_2\cos(2\pi\theta_3 n+\theta_4)} \quad (9)$$

$L(\underline{\theta})$ is maximized with respect to θ_1 if the term in braces $\{.\}$ in equation (8) is real. This is the case if

$$\theta_1 = \arg\{\, B(z,\theta_2,\theta_3,\theta_4)\,\} \quad (10)$$

so that the function $L(\underline{\theta})$ becomes:

$$L(\underline{\theta}) = -\theta_0^2 + 2\theta_0\, |B(z,\theta_2,\theta_3,\theta_4)| \quad (11)$$

The value of θ_0 that maximizes (11) is

$$\theta_0 = |B(z,\theta_2,\theta_3,\theta_4)| \quad (12)$$

so that the function to maximize becomes

$$L(\theta_2,\theta_3,\theta_4) = |B(z,\theta_2,\theta_3,\theta_4)|^2 \quad (13)$$

Hence, the maximum likelihood estimation procedure can be summarized as follows: first, the function $|B(z,\theta_2,\theta_3,\theta_4)|$ is maximized using numerical optimization and the parameters $(\theta_2,\theta_3,\theta_4)$ are obtained. The other parameters θ_0 and θ_1 can be derived from the analytical expressions (10) and (12).

3.2. Optimization Method

The maximum of the function $|B(z, \theta_2, \theta_3, \theta_4)|$ cannot be found analytically. Therefore, numerical methods must be used. The search space is relatively limited in this typical application because the characteristic fault frequency θ_3 and typical values of the modulation index θ_2 are well known ($\theta_2 \in [0, 0.1]$, $\theta_3 \in [0.11, 0.125]$, $\theta_4 \in [0, 2\pi]$). First, a grid search has been implemented with a fixed step size in the search space. However, the computation was still time-expensive, so that an evolutionary algorithm is finally used.

The implemented algorithm is a $(\mu + \lambda)$-evolution strategy. A comprehensive introduction can be found in [6]. The algorithm is initialized by randomly placing a population of μ parents in the search space. The λ children are created using self-adaptive mutation with the advantage that the standard deviation in the mutation process is automatically adapted. In the following selection process, all the individuals are evaluated and the μ best individuals out of the parents and children generation form the next parent generation. The algorithm continues with a new mutation cycle. The algorithm stopping rule is a maximum number of 100 iterations. The number of parents and children is μ=10 and λ=100. The evolution strategy shows a good performance and computes about 10-20 times faster than the grid search.

3.3. Cramer-Rao Lower Bounds

The estimation mean square error results from estimation bias and variance. The MLE is an unbiased estimator for large data records i.e. the remaining error is the variance in the case of independent and identically distributed observations. The Cramer-Rao lower bounds (CRLB) provide an inferior bound for any unbiased estimator variance. The MLE variance approaches asymptotically these bounds. The CRLB are given by the diagonal elements of the inverse fisher information matrix $[F(\underline{\theta})]$:

$$\mathrm{var}(\hat{\theta}_i) \geq [F^{-1}(\underline{\theta})]_{ii} \tag{14}$$

In the case of the probability density function given in (5), the elements of the Fisher information matrix can be calculated using the following formula [5]:

$$[F(\underline{\theta})]_{ij} = \frac{1}{\sigma^2} \sum_{n=0}^{N-1} \left[\frac{\partial i_d[n]}{\partial \theta_i} \frac{\partial i_d[n]}{\partial \theta_j} + \frac{\partial H\{i_d[n]\}}{\partial \theta_i} \frac{\partial H\{i_d[n]\}}{\partial \theta_j} \right] \tag{15}$$

For the considered estimation problem, $[F(\underline{\theta})]$ is symmetric and takes the following form:

$$[F(\underline{\theta})] = \begin{bmatrix} f_{00} & 0 & 0 & 0 & 0 \\ 0 & f_{11} & f_{12} & f_{13} & f_{14} \\ 0 & f_{12} & f_{22} & f_{23} & f_{24} \\ 0 & f_{13} & f_{23} & f_{33} & f_{34} \\ 0 & f_{14} & f_{24} & f_{34} & f_{44} \end{bmatrix} \tag{16}$$

The expressions of all matrix elements f_{ii} are given in appendix A. They match the more general results in [7].

In order to test the estimator performance, simulations have been carried out using a monocomponent PM signal with additive white gaussian noise. The chosen signal parameters are: θ_0=1, θ_1=0.5, θ_2=0.005, θ_3=0.125, θ_4=0.3, f_s=0.25 and SNR=50 dB. The SNR was chosen lower for the simulation as in reality in order to obtain a higher MSE. The mean square error (MSE) was estimated using 1000 simulation runs for each data record length. The MSE together with the theoretical CRLB are displayed in Fig. 1 as functions of the data record length. Obviously, the increase in the data record length leads to a decreasing estimation error which approaches the CRLB.

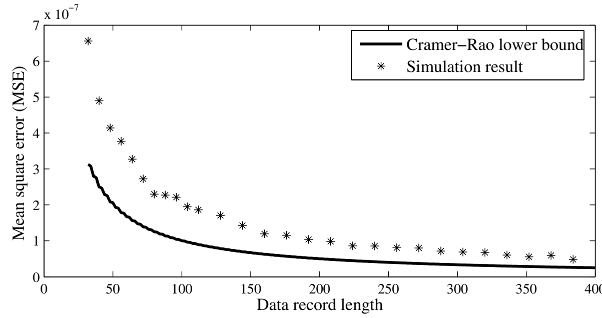


Fig. 1. Mean square estimation error of parameter θ_2 and theoretical Cramer-Rao lower bound versus data record length

4. EXPERIMENTAL RESULTS

The experimental setup consists of an induction motor coupled to a DC machine. The DC machine produces load torque oscillations at rotational frequency through an armature current control. These torque oscillations are similar to typical effects of mechanical load faults. The stator current measurement is performed at a sampling rate of 25 kHz by a 24-bit data acquisition board. The signal is further low-pass filtered, downsampled to 200 Hz and processed using Matlab.

The parameter estimation has been carried out on 183 data records, each composed of 64 samples. The introduction of a small torque oscillation with amplitude Γ_c=0.14 Nm allows to reproduce the effects of a mechanical fault. The amplitude of the torque oscillation is only 0.8% of the mean load torque. The histogram of the estimated modulation index θ_2 is shown in Fig. 2 for a set of healthy and faulty data. The probability density functions are very different. A decision between the healthy and faulty case can simply be made by considering an adequate threshold for the modulation index θ_2. The detection performances are studied through the receiver operating characteristic (ROC) curves. The ROC curves display the probability of detection P_D with respect to the probability of false alarm P_F [4]. The ROC curves (Fig. 3(a)) have been experimentally obtained for several levels of load torque oscillation Γ_c with 18 Nm average torque. An increase in Γ_c, equivalent to an increase in fault severity, leads to a higher P_D for the same value of P_F. The ROC approaches the ideal case where P_D always

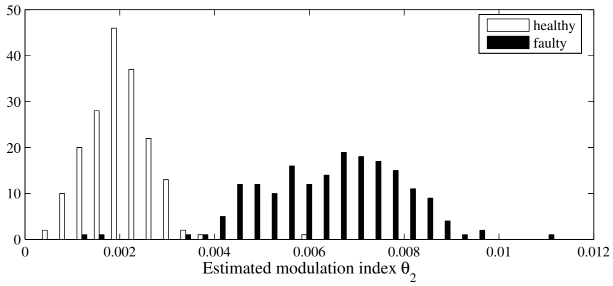
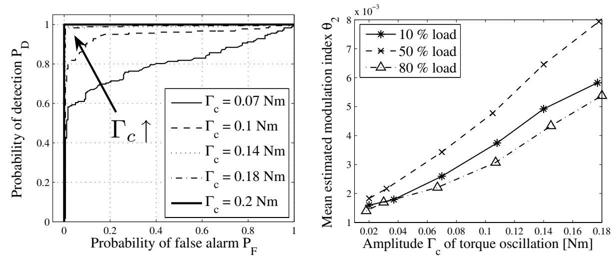


Fig. 2. Histogram of estimated modulation index θ_2 for a set of healthy and faulty data



(a) ROC for 50% load

(b) $E[\theta_2]$ versus Γ_c

Fig. 3. Experimental ROC and mean estimated modulation index $E[\theta_2]$ versus Γ_c for different average loads

equals 1, except for $P_D = 0$.

The link between the estimated modulation index θ_2 and the fault severity has been experimentally verified by processing data with different load torque oscillation amplitudes and different average load torque levels. Fig. 3(b) shows the mean estimated modulation index versus the amplitude of the load torque oscillation. An increase in the torque oscillation amplitude, equivalent to an increase in fault severity, leads to a considerably higher modulation index for all different load levels. The increase is approximately linear. Hence, a critical level of torque oscillation can be fixed and a decision about the motor condition can be taken based on the estimation of θ_2.

5. CONCLUSION

This paper presented a new approach to mechanical fault detection in induction motors based on stator current monitoring. The fault-related load torque oscillations produce characteristic sinusoidal phase modulations of the stator current. The modulation index depends on the torque oscillation amplitude and is therefore an indicator for the fault severity. A maximum likelihood estimation procedure was proposed in order to estimate the modulation index. This presents an advantage over methods based on classical spectral analysis because the fault indicator is directly obtained. Furthermore, shorter data records can be used and the type of modulation can be identified.

The theoretical Cramer-Rao lower bounds of the estimation variance were calculated. The estimator was implemented using evolution strategies for numerical optimization. The estimator performance was tested by simulations and approaches the Cramer-Rao bound. The estimation procedure was successfully tested on experimental stator current data from healthy and faulty motors. A simple detector based on a threshold for the modulation index was proposed. ROC curves and mean estimated modulation indices demonstrate the effectiveness for different levels of torque oscillation.

6. REFERENCES

[1] Martin Blödt, Marie Chabert, Jean Faucher, and Bruno Dagues, "Mechanical load fault detection in induction motors by stator current time-frequency analysis," in *Proc. IEEE International Electric Machines and Drives Conference (IEMDC '05)*, San Antonio, Texas, May 2005.

[2] R. R. Obaid, T. G. Habetler, and D. J. Gritter, "A simplified technique for detecting mechanical faults using stator current in small induction motors," in *Conference Record of the IEEE Industry Applications Conference, 2000*, Rome, Italy, Oct. 2000, pp. 479–483.

[3] Patrick Flandrin, *Time-Frequency/Time-Scale Analysis*, Academic Press, San Diego, 1999.

[4] Harry L. van Trees, *Detection, Estimation, and Modulation Theory: Part I*, John Wiley & Sons, New York, 2001.

[5] David C. Rife and Robert R. Boorstyn, "Single-tone parameter estimation from discrete-time observations," *IEEE Trans. Inform. Theory*, vol. 20, no. 5, pp. 591–598, Sept. 1974.

[6] Hans-Georg Beyer and Hans-Paul Schwefel, "Evolution strategies: A comprehensive introduction," *Natural Computing*, vol. 1, no. 1, pp. 3–52, Mar. 2002.

[7] Mounir Ghogho, Asoke K. Nandi, and Ananthram Swami, "Cramer-Rao bounds and maximum likelihood estimation for random amplitude phase-modulated signals," *IEEE Trans. Signal Processing*, vol. 47, no. 11, pp. 2905–2916, Nov. 1999.

A. ELEMENTS OF THE FISHER INFORMATION MATRIX

$$f_{00} = \frac{N}{\sigma^2}, \quad f_{11} = \frac{\theta_0^2}{\sigma^2} N$$
$$f_{12} = \frac{\theta_0^2}{\sigma^2} \sum_{n=0}^{N-1} \cos(2\pi\theta_3 n + \theta_4)$$
$$f_{13} = \frac{\theta_0^2}{\sigma^2} (-2\pi\theta_2) \sum_{n=0}^{N-1} n \sin(2\pi\theta_3 n + \theta_4)$$
$$f_{14} = \frac{\theta_0^2}{\sigma^2} (-\theta_2) \sum_{n=0}^{N-1} \sin(2\pi\theta_3 n + \theta_4)$$
$$f_{22} = \frac{\theta_0^2}{\sigma^2} \sum_{n=0}^{N-1} \cos^2(2\pi\theta_3 n + \theta_4)$$
$$f_{23} = \frac{\theta_0^2}{\sigma^2} (-2\pi\theta_2) \sum_{n=0}^{N-1} \frac{1}{2} n \sin(4\pi\theta_3 n + 2\theta_4)$$
$$f_{24} = \frac{\theta_0^2}{\sigma^2} (-\theta_2) \sum_{n=0}^{N-1} \frac{1}{2} \sin(4\pi\theta_3 n + 2\theta_4)$$
$$f_{33} = \frac{\theta_0^2}{\sigma^2} (2\pi\theta_2)^2 \sum_{n=0}^{N-1} n^2 \sin^2(2\pi\theta_3 n + \theta_4)$$
$$f_{34} = \frac{\theta_0^2}{\sigma^2} 2\pi\theta_2^2 \sum_{n=0}^{N-1} n \sin^2(2\pi\theta_3 n + \theta_4)$$
$$f_{44} = \frac{\theta_0^2}{\sigma^2} \theta_2^2 \sum_{n=0}^{N-1} \sin^2(2\pi\theta_3 n + \theta_4)$$