ROBUST ADAPTIVE MATCHED SUBSPACE CFAR DETECTOR FOR GAUSSIAN SIGNALS

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ABSTRACT

An adaptive matched subspace CFAR detector of gaussian distributed signals is analyzed. The detector is an extension of the one developed in [2] to the case of unknown disturbance covariance matrix. This matrix is estimated from secondary data which can be non homogenous. In this latter case we use a robust estimate of the covariance matrix based on the sample spatial sign covariance matrix (SSCM). The performance of the adaptive scheme, specifically, the impact on the false alarm rate is studied by means of Monte Carlo simulations. The results show that the adaptive detector using the SSCM maintains the false alarm approximately constant in non homogenous situations.

1. INTRODUCTION

Detecting subspace signals in the presence of noise is a common problem in multidimensional signal processing. In [2], Gini *et al.*. studied a matched subspace CFAR detector of hovering helicopters, where the target was modeled as a subspace random signal in the presence of a compound Gaussian disturbance. Later, in [3] the same signal model was applied to detect and classify targets belonging to different classes in the presence of complex Gaussian noise with known covariance matrix. In real situations, this quantity must be adaptively estimated from signal free secondary data and the well known sample covariance matrix estimate leads to poor performances in the presence of non homogeneities in the secondary data. In this case we must resort to robust methods to estimate the covariance matrix. Many algorithms were proposed to deal with this situation, among them, the non homogeneity detector (NHD), recently proposed by Rangswamy *et al.* [6]. Using the NHD, one can select the cell containing the non homogeneity and eliminate it from the covariance estimation process using the sample covariance matrix. But in practical situations, the secondary data is not large enough to give a good estimate and using the NHD as a pre-processing step, reduces considerably the quantity of secondary data, causing a drastic degradation in the estimation process. In this paper, we use a robust method based on the spatial sign covariance matrix (SCM) [4], to estimate the unknown noise covariance matrix. Simulation results demonstrate that, under certain conditions, the sample spatial sign covariance matrix (SSCM), maintains the false alarm constant in the non homogenous secondary data.

2. PROBLEM STATEMENT AND DATA MODEL

Consider a radar which collects N pulses during the time on target. These pulses form the complex valued received vector $\mathbf{z} = [z(0), z(1), ..., z(N-1)]^T$. The detection problem can be seen as a binary hypotheses test :

$$\begin{cases} H_0 : \mathbf{z} = \mathbf{d} \\ H_1 : \mathbf{z} = \mathbf{d} + \mathbf{s} \end{cases} \tag{1}$$

The disturbance $\mathbf{d}$ is modeled as a complex Gaussian distributed random vector with zero mean and covariance matrix $\mathbf{M}_d = E\left\{\mathbf{z}\mathbf{z}^H\right\} = \sigma_d^2\mathbf{M}$, where $(.)^H$ is the conjugate transpose operator, σ_d^2 is the power of each disturbance component and $\mathbf{M}$ is the normalized covariance matrix, i.e.$[\mathbf{M}]_{i,i} = 1$ for $i = 1, 2, ..., N$. We assume that $\mathbf{M}$ is full-rank. In a shorthand notation $\mathbf{d} \sim CN(0, \sigma_d^2\mathbf{M})$. Also, the signal vector $\mathbf{s} \sim CN(0, \sigma_s^2\mathbf{M}_s)$, with covariance matrix $\mathbf{M}_s^{'} = E\left\{\mathbf{s}\mathbf{s}^H\right\} = \sigma_s^2\mathbf{M}_s$, where $\mathbf{M}_s$ is the normalized covariance matrix and σ_s^2 is the signal power. We assume that the rank of $\mathbf{M}_s$ is equal to r. This target signal model is equivalent to the linear Gaussian model [3] where the signal vector $\mathbf{s}$ is modeled as $\mathbf{s} = \mathbf{U}_s\boldsymbol{\beta}_s$, $\boldsymbol{\beta}_s$ is the $r \times 1$ mode weight random vector $\boldsymbol{\beta}_s \sim CN(0, \sigma_s^2\boldsymbol{\Lambda}_s)$, $\boldsymbol{\Lambda}_\mathbf{s}$ is the $r \times r$ diagonal matrix of non zero eigenvalues of the matrix $\mathbf{M}_s$ and $\mathbf{U}_s$ is the $N \times r$ unitary matrix of corresponding eigenvectors, called the mode matrix. We obtain $E\left\{\mathbf{s}\mathbf{s}^H\right\} = \mathbf{U}_\mathbf{s}E\left\{\boldsymbol{\beta}_s\boldsymbol{\beta}_s^H\right\}\mathbf{U}_\mathbf{s}^H = \sigma_s^2\mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_\mathbf{s}^H = \sigma_s^2\mathbf{M}_s$, ($\mathbf{M}_s$= $\mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H$ is the eigen decomposition of $\mathbf{M}_s$). The projection matrix on the signal subspace is then given by: $\mathbf{P}_s = \mathbf{U}_s(\mathbf{U}_s^H\mathbf{U}_s)^{-1}\mathbf{U}_s^H$. As in [2], we assume only knowledge of the mode matrix $\mathbf{U}_s$ but σ_s^2 and $\boldsymbol{\Lambda}_s$ are unknown. This means that we know the subspace were the target signal lies, but we do not know the corresponding power of each component.

3. CFAR DETECTION

Gini *et al.* [2], developed a CFAR detection algorithm based on the GLRT principle for known $\mathbf{M}$. The obtained results are reported here without details. The detection statistic based on the GLRT is given by :

$$L(\mathbf{z}) = \frac{\max\limits_{\boldsymbol{\beta}_s,\sigma_d^2} f_{\mathbf{Z}|H_1}(\mathbf{z} \mid H_1, \boldsymbol{\beta}_s, \sigma_d^2)}{\max\limits_{\sigma_d^2} f_{\mathbf{Z}|H_0}(\mathbf{z} \mid H_0, \sigma_d^2)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \tag{2}$$

the conditional pdf under hypothesis H_1 is given by:

$$f_{\mathbf{Z}|H_1}(\mathbf{z} \mid H_1, \boldsymbol{\beta}_s, \sigma_d^2) = \frac{1}{(\pi\sigma_d^2)^{\frac{N}{2}}\,|\mathbf{M}|} \exp\left(-\frac{(\mathbf{z} - \mathbf{U}_s\boldsymbol{\beta}_s)^H\mathbf{M}^{-1}(\mathbf{z} - \mathbf{U}_s\boldsymbol{\beta}_s)}{\sigma_d^2}\right) \tag{3}$$

The pdf under H_0 is obtained from (3) by setting $\boldsymbol{\beta}_s = \mathbf{0}$.

Replacing the ML estimates of the unknown parameters in (2) (see [1] for more details), the test statistic can be written as:

$$\frac{\mathbf{z}^H \mathbf{Q}_s \mathbf{z}}{\mathbf{z}^H \mathbf{M}^{-1} \mathbf{z}} \underset{H_0}{\overset{H_1}{\gtrless}} \lambda \tag{4}$$

where $\mathbf{Q}_s = \mathbf{M}^{-1}\mathbf{U}_s(\mathbf{U}_s^H\mathbf{M}^{-1}\mathbf{U}_s)^{-1}\mathbf{U}_s^H\mathbf{M}^{-1}$ and the threshold λ is selected to provide the desired probability of false alarm $P_F = \alpha_0$.

Using the Cholesky factorization $\mathbf{M} = \mathbf{L}\mathbf{L}^H$, were $\mathbf{L}$ is an $N \times N$ lower triangular matrix, we obtain the whitened vector $\mathbf{x} = \mathbf{L}^{-1}\mathbf{z}$ and the whitened target signal vector $\mathbf{q}_s = \mathbf{L}^{-1}\mathbf{s}$. The projection matrix onto the new signal subspace is $\mathbf{P}_s = \mathbf{L}^{-1}\mathbf{U}_s(\mathbf{U}_s^H\mathbf{M}^{-1}\mathbf{U}_s)^{-1}\mathbf{U}_s^H\mathbf{L}^{-H}$. After some easy algebra, we can express the detector (4) as:

$$\Gamma(\mathbf{x}) = \frac{\mathbf{x}^H \mathbf{P}_s \mathbf{x}}{\mathbf{x}^H (I - \mathbf{P}_s) \mathbf{x}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{5}$$

We note that the quantity $\mathbf{I} - \mathbf{P}_s$ represents the projection matrix on the subspace orthogonal to the signal subspace spanned by the projection matrix $\mathbf{P}_s$.

False alarm probability

The false alarm probability is given by $P_F = \Pr\{\Gamma(\mathbf{x}) > \gamma \mid H_0\}$. In [5], it is shown that the quadratic form $\mathbf{x}^H\mathbf{P}_s\mathbf{x} \sim \chi^2_{2r}$ and the denominator $\mathbf{x}^H(\mathbf{I} - \mathbf{P}_s)\mathbf{x} \sim \chi^2_{2(N-r)}$. Therefore, since the numerator and denominator are mutually independent, the statistic $F = \frac{\mathbf{x}^H\mathbf{P}_s\mathbf{x}/(2r)}{\mathbf{x}^H(\mathbf{I}-\mathbf{P}_s)\mathbf{x}/(2N-2r)}$ is distributed according to an F distribution with $2r$ and $2(N-r)$ degrees of freedom, were $r = rank(\mathbf{M}_s)$.The change of variable of the form $W = F/(1+F)$ transforms the F distribution into a Beta one given by:

$$\begin{aligned} f_W(w) &= \frac{(N-1)!}{(r-1)!(N-r-1)!} w^{r-1}(1-w)^{N-r-1}, \\ & \quad 0 \leq w \leq 1 \end{aligned} \tag{6}$$

We have then: $P_{FA} = \Pr\{\Gamma(\mathbf{x}) > \gamma \mid H_0\} = \Pr\{F > \eta\} = \Pr\{W > \xi\}$, where $\eta = \gamma(N-r)/r$ and $\xi = \eta/(1+\eta)$, we have then:

$$P_{FA} = 1 - \frac{(N-1)!}{(r-1)!(N-r-1)!} \int_0^{\xi} w^{r-1}(1-w)^{N-r-1} dw \tag{7}$$

It is important to highlight that the false alarm depends only on the number of pulses N and on the rank of the signal covariance matrix r. Thus the detector has the desired CFAR property with respect to $\sigma_d^2, \beta_s, \mathbf{M}_s$ and $\mathbf{M}$. The threshold is derived from (7).

4. ADAPTIVE DETECTION

In the previous sections, $\mathbf{M}$ is supposed *a priori* known. But, in practice, it must be adaptively estimated from secondary data, and the performance of the resulting detector is highly dependent on the accuracy of the estimation procedure. The secondary data must be representative of the samples in the test cell. Generally, we take L cells that are just in the neighborhood of the test cell and the simplest way to estimate the covariance matrix, is to use the sample covariance matrix $\mathbf{S}_{av} = \frac{1}{L}\sum_{i=1}^{L} \mathbf{x}_i\mathbf{x}_i^H$, which is the well known maximum likelihood (ML) estimator. When the secondary data are homogenous, the performance of this method is optimal. If the assumption of the homogeneity of the data is violated, for example, in the presence of interfering targets or/and clutter edge in the secondary data, the result can become dramatic. This is the dominant situation in a radar environment. One solution for this case is to use a pre-processing technique in order to select a representative set of data from the contaminated data. In [6] a method was proposed, based on a test of non homogeneity to determine the cells containing the non homogeneity. An other way to proceed is to use robust estimation of the covariance matrix. In this paper we consider a robust method based on a non parametric technique using the sample spatial sign covariance matrix (SSCM) [4].

The Robust estimation of the covariance matrix

Robust estimation of covariance is required in many applications such as directions of arrival (DOA), radar, sonar etc.. When the data are Gaussian, the sample covariance estimate is the best estimate in the maximum likelihood sense. Unfortunately, this is extremely sensitive to deviations from the model assumption. These deviations may be caused by the existence of non homogeneities in the data. In such situations, robust estimation techniques should be considered. There are many methods for robust covariance estimation based on M estimators, S estimators, minimum volume ellipsoid estimator, minimum covariance determinant, estimates based on projection, etc.. In this paper, we will borrow a method from DOA estimation, proposed by Visuri *et al.* in [4]. This method is non parametric and based on the spatial sign covariance matrix estimate.

We begin by the multivariate spatial sign concept. The spatial sign function for an n variate complex vector $\mathbf{x}$ is defined as:

$$\mathrm{S}(\mathbf{x}) = \begin{cases} \frac{\mathbf{x}}{||\mathbf{x}||}, & \mathbf{x} \neq 0 \\ 0, & \mathbf{x} = 0 \end{cases} \tag{8}$$

Where $||\mathbf{x}|| = (\mathbf{x}^H\mathbf{x})^{1/2}$.
For an n variate complex data set, $\mathbf{x}_1, .., \mathbf{x}_N$, the sample SCM (SSCM) is

$$\mathbf{R}_S = \frac{1}{N}\sum_{i=1}^{N} \mathrm{S}(\mathbf{x}_i)\mathrm{S}^H(\mathbf{x}_i) \tag{9}$$

We now present the procedure used. Consider an n variate complex valued random variable $\mathbf{x}$ with the covariance matrix $\mathbf{\Sigma}_0$ and theoretical spatial sign covariance matrix (SCM) $\mathbf{\Sigma}_1$. Define $\tilde{\mathbf{x}} = \begin{pmatrix} \Re(\mathbf{x}) \\ \Im(\mathbf{x}) \end{pmatrix}$ and let the $2n \times 2n$ covariance matrix $\tilde{\mathbf{\Sigma}}_0$ and SCM $\tilde{\mathbf{\Sigma}}_1$ be:

$$\tilde{\mathbf{\Sigma}}_i = \begin{pmatrix} \tilde{\Sigma}_i^{11} & \tilde{\Sigma}_i^{12} \\ \tilde{\Sigma}_i^{21} & \tilde{\Sigma}_i^{22} \end{pmatrix}, \quad i = 0, 1. \tag{10}$$

From Lemma 1 in [4], we get

$$\mathbf{\Sigma}_i = \tilde{\Sigma}_i^{11} - j\tilde{\Sigma}_i^{12} + j\tilde{\Sigma}_i^{21} + \tilde{\Sigma}_i^{22}, i = 0, 1 \tag{11}$$

It is shown that the SSCM is a good estimate of the theoretical SCM.

In this work we use the SSCM in the detection statistic (4) and (5) instead of the unknown disturbance covariance matrix $\mathbf{M}$ and evaluate the performance of the obtained detector. Since for any CFAR detector, the most important parameter is the false alarm rate which must be constant, we will investigate the behavior of the P_{FA}.

5. NUMERICAL PERFORMANCE ANALYSIS

To evaluate numerically the performance of the modified algorithm, we assume a PSD of a target signal given by (see [3] for details) $S(f) = \frac{\sigma_S^2}{B}(1 - \frac{|f-f_d|}{B})rect(\frac{f-f_d}{2B})$ where $f \in [-0.5 \;\; 0.5]$ is the frequency normalized to the pulse repetition frequency (PRF), f_d is the normalized mean target doppler frequency, B the normalized bandwidth and $rect(.)$ is the standard rectangular function. This PSD represent an approximation of the spectrum of the signal backscattered by a hovering helicopter. The elements of the matrix $\mathbf{M}_s$ are obtained by inverse Fourier transform

$$\begin{aligned} [\mathbf{M}_s]_{i,k} &= \frac{1}{2B_1}\int_{f_d-B}^{f_d+B} e^{j2\pi(i-k)}df \\ &= sinc[2B(i-k)]e^{-j2\pi f_d(i-k)}, i,k = 1,\ldots,N \quad (12) \end{aligned}$$

As in [3], we consider $[\mathbf{M}]_{m,l} = \rho^{|m-l|}$, with $\rho = 0.9$ is the one lag correlation coefficient of the disturbance.

It is common to assume the rank r of the signal subspace equal to the minimum number of dominant eigenvalues whose sum exceeds $r = 0.99T_r(\mathbf{M}_s)$, where $Tr(.)$ is the trace operator. The corresponding eigenvectors represent a basis for the signal subspace.

The first step is to determine the detection threshold by solving (7) with $P_{FA} = \alpha_0$. The secondary data are obtained from L range bins surrounding the cell under test. We consider the case of homogenous and the case of non homogenous secondary data.

Homogenous case: we fix $\alpha_0 = 10^{-2}$, $L = 64, N = 8$, $f_d = 0$, $B = 0.2$ and by means of Monte Carlo simulations, we determine the P_{FA} for the detector using the sample covariance matrix estimate and the one using the SSCM. The first one gives $P_{FA} = 1.43.10^{-2}$ and the second $P_{FA} = 1.90.10^{-2}$. One notes that the sample covariance matrix gives the better performance since it is the ML estimate.

Non homogenous case: We consider that a number n_c of cells in the secondary data, contain noise plus clutter. We note by CNR the clutter to noise ratio. Figure 1 shows the false alarm probability for different CNR when the sample covariance matrix and SSCM are used. The results show that for high CNR and values of n_c approximately ≤ 25, the SSCM maintains the false alarm at a constant value less than that obtained with the sample covariance matrix (ML in the figure), but with an increase in the value of the P_{FA} with respect to the nominal value $\alpha_0 = 10^{-2}$. When the number n_c is large, this detector losses its CFAR property.

Figure 2 shows the results when n_i cells contain noise plus interfering targets. The interfering targets are supposed to follow a complex Gaussian pdf with the INR referring to interference-to-noise ratio. The SSCM (dashed lines) maintains the false alarm rate ($\simeq 2.10^{-2}$) independently of the INR value. For low values of INR the ML gives better performance, but for high values of INR, a drastic degradation of its P_{FA} is observed.

Figure 3 compares the detection probabilities of the detectors using SSCM, sample covariance matrix and the one with perfectly known covariance matrix, in homogenous situations. For high SNR (greater than 10dB), the plots are the same. For lower SNR, the detection loss is negligible.

As seen before, the detector using the SSCM is robust to both interfering targets and clutter edge, since it maintains the false alarm constant. We now investigate its probability of detection in non homogenous situation. Figure 4 shows the probability of detection when n_c cells in the secondary data contain a clutter edge, and Figure 5 shows the case of n_i cells containing interfering targets. In both cases the curves are practically the same.

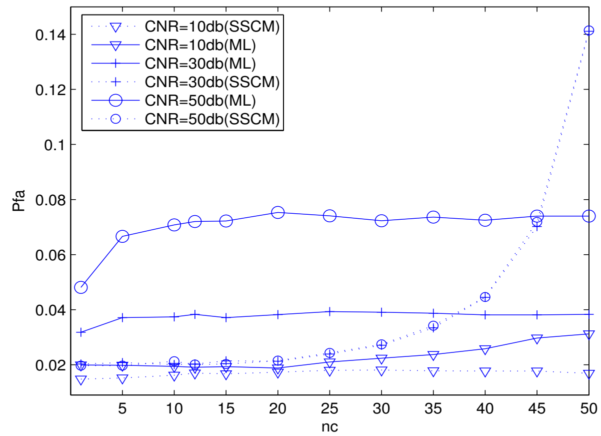

Fig. 1. Probability of false alarm using the ML and SSCM in the presence of clutter edge.

6. CONCLUSION

In this paper we have analyzed the CFAR matched subspace detector when the disturbance covariance matrix is unknown and adaptively estimated from secondary data. The practical case of non homogenous secondary data is assumed and a robust method of estimating the covariance matrix based on the sample spatial sign covariance matrix (SSCM) is used. The simulations show that this method can, under certain conditions, maintain the CFAR property of the detector in non homogenous background with a value of P_{FA} less than the one obtained using the sample covariance matrix.

7. ACKNOWLEDGEMENT

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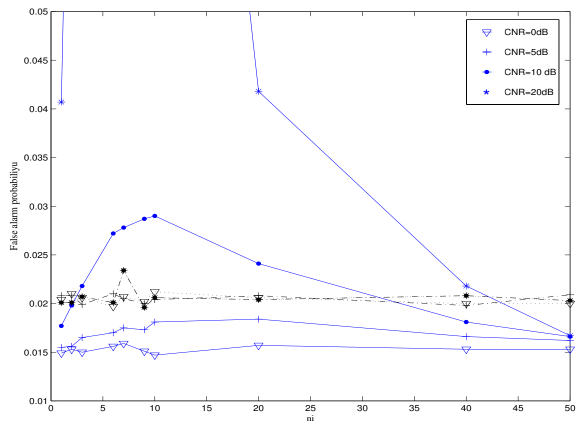

Fig. 2. false alarm Probability using SSCM and ML estimate in the presence of interfering targets.

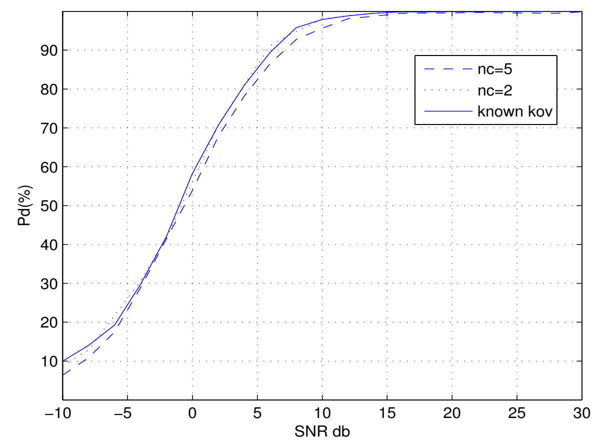

Fig. 4. detection Probability of the detector using SSCM, when nc cells in the secondary data contain clutter. CNR=10dB.

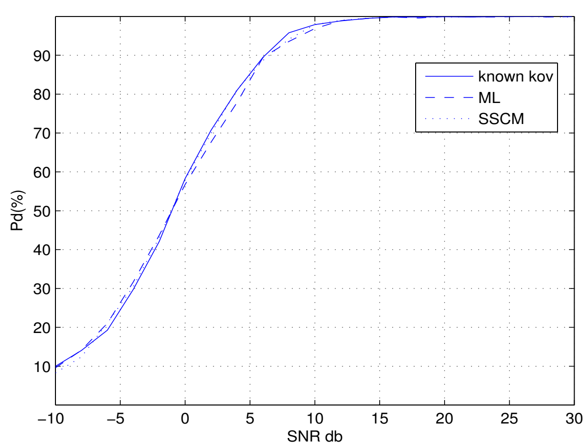

Fig. 3. detection Probability of detectors with SSCM and ML estimate and known covariance matrix in homogenous situation.

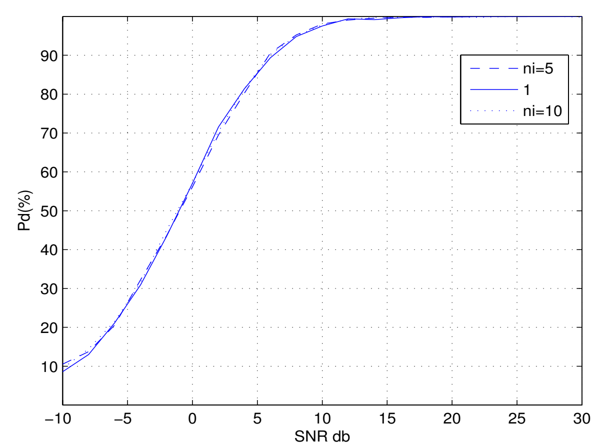

Fig. 5. detection Probability of the detector using SSCM,when ni cells in the secondary data contain interfering targets. INR=10 dB