DIRECT DESIGN OF NEAR PERFECT RECONSTRUCTION LINEAR PHASE NONUNIFORM FILTER BANKS WITH RATIONAL SAMPLING FACTORS

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ABSTRACT

A direct method of designing linear phase (LP) near perfect reconstruction (NPR) nonuniform filter banks (NUFBs) with rational sampling factors is presented. Conditions on the possible sampling factors of NUFBs are given. The band position relations and phase relations of the significant aliasing components are also analyzed. Based on these two relations, we derive a necessary condition on the elimination of significant aliasing distortion (ALD). Meanwhile LP property is met. Moreover, a set of relations of the filters are presented to minimize the ALD further. In our proposed method, the LP NUFBs with high stopband attenuation and low system delay can be easily designed without nonlinear optimization procedure.

1. INTRODUCTION

Nonuniform filter banks (NUFBs) have been successfully employed in many signal processing applications due to their flexibility in partitioning subbands. In some applications, such as image coding, the linear phase (LP) property of individual filter in the filter bank is highly desired. Unfortunately, the NUFBs designed by the most existing methods [1]-[5] do not posses LP property. The tree-structure method proposed in [6] is an easy way to design LP NUFBs via cascading uniform filter banks. However, the limitation of decimation factors and the long system delay are two major drawbacks using this method. In our previous work [7], we proposed a method, which is based on indirect structure, to design LP NUFBs, but this structure still have a long system delay.

In this paper, we propose a simple direct method for designing near perfect reconstruction (NPR) LP NUFBs. First of all, we discuss the condition on the feasibility of NUFBs, and give two conditions on the suitable position of each individual filter. Providing the above conditions fulfilled, we further analyze the band positions and phase relations of the significant aliasing distortion (ALD). Based on two sets of above relations, we derive a theorem on the elimination of significant ALD. With these conditions, the filter bank design problem is simplified as how to design each individual filter, leading to a low complexity of design. This can be done by using one of the available filter design tools. In this paper, we employ the Parks-McClellan algorithm. The desired magnitude response of the filter is specified as an optimization objective.

The outline of the paper is as follow. In Section 2, the basic theory of LP NUFBs is introduced. Section 3 will deduce a theorem for the cancellation of significant ALD. Then, some examples are given in Section 4. Finally, we summarize our results in the conclusion.

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2. THEORY OF NUFBS WITH RATIONAL SAMPLING FACTORS

The NUFBs with rational sampling factors is depicted in Fig. 1. We assume that the sampling factors p_i and q_i are coprime and the NUFB is critically sampled, that is

$$\sum_{i=0}^{M-1} \frac{p_i}{q_i} = 1 \, . \tag{1}$$

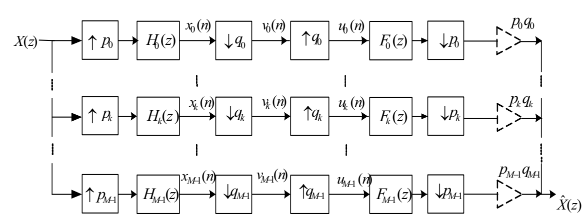


Fig. 1 *M*-channel nonuniform filter banks.

2.1. Basic Principle of LP NUFBs

Let $H_k(z)$ and $F_k(z)$ be the analysis and synthesis filters. Considering the affect of gain $p_k q_k$ in Fig. 1, the input signal $X(z)$ and output signal $\hat{X}(z)$ are related by

$$\begin{aligned} \hat{X}(z) &= \sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1}\sum_{l_k=0}^{q_k-1} X(zW_{q_k}^{l_k p_k}) H_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k} W_{q_k}^{l_k}) F_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k}) \\ &= X(z)T(z) + \sum_{k=0}^{M-1}\sum_{l_k=1}^{q_k-1} X(zW_{q_k}^{l_k p_k}) A_{k,\,l_k}(z) \, , \end{aligned} \tag{2}$$

where $W_i = e^{-j2\pi/i}$,

$$T(z) = \sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1} H_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k}) F_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k}) \tag{3}$$

is the transfer function of the filter bank, and

$$A_{k,l_k}(z)=\sum_{i_k=0}^{p_k-1} H_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k} W_{q_k}^{l_k}) F_k(z^{\frac{1}{p_k}} W_{p_k}^{i_k}) \qquad (4)$$

is the aliasing components.

The time-reverse property of the analysis filter $h_k(n)$ and synthesis filter $f_k(n)$ is possessed, that is:

$$f_k(n)=h_k(N_k-n) \qquad (5a)$$

or $$F_k(z)=z^{-N_k}H_k(z^{-1}),\quad 0\le k\le M-1. \qquad (5b)$$

where N_k is the order of analysis filter $h_k(n)$.

For the LP filter banks, both the analysis and synthesis filters posses the LP property, that is:

for symmetric $h_k(n)$, $\quad h_k(n)=h_k(N_k-n)$; $\qquad$ (6a)

for anti-symmetric $h_k(n)$, $h_k(n)=-h_k(N_k-n)$. $\qquad$ (6b)

Combining (5a), (6a) and (6b), we have:

$$f_k(n)=(-1)^{u_k}h_k(n), \qquad (7)$$

where $u_k=\begin{cases}0, & \text{when } h_k(n) \text{ is symmetric}\\ 1, & \text{when } h_k(n) \text{ is anti-symmetric}\end{cases}$, which means that the synthesis filters are also LP filters.

Substituting (5b) into (3), we get

$$\begin{aligned}T(z)&=\sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1}(z^{\frac{1}{p_k}}W_{p_k}^{i_k})^{-N_k}H_k(z^{\frac{1}{p_k}}W_{p_k}^{i_k})H_k((z^{\frac{1}{p_k}}W_{p_k}^{i_k})^{-1})\\ &\underline{\underline{z=e^{j\omega}}}\sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1}e^{j\omega(-\frac{N_k}{p_k})}W_{p_k}^{-i_kN_k}\,|H_k(e^{j\omega\frac{1}{p_k}}W_{p_k}^{i_k})|^2 .\end{aligned} \qquad (8)$$

To eliminate the PHD, Eq. (9) should be fulfilled.

$$\frac{N_0}{p_0}=\frac{N_1}{p_1}=\frac{N_2}{p_2}=\cdots=\frac{N_{M-1}}{p_{M-1}}=C, \qquad (9)$$

where C is a positive integer. Then

$$T(e^{j\omega})=\sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1}e^{-j\omega C}|H_k(e^{j\omega\frac{1}{p_k}}W_{p_k}^{i_k})|^2=e^{-j\omega C}\sum_{k=0}^{M-1}\sum_{i_k=0}^{p_k-1}|H_k(e^{j\omega\frac{1}{p_k}}W_{p_k}^{i_k})|^2, \qquad (10)$$

which has LP property, and the PHD is eliminated.

To make the AMD (amplitude distortion) be eliminated, we should make the plot of $|T(e^{j\omega})|$ to be 'flat' as soon as possible. For analysis conveniently, we rewrite (8) as: $T(e^{j\omega})=e^{-j\omega C}\sum_{k=0}^{M-1}p_k(|H_k(e^{j\omega})|^2\downarrow p_k)$, where $\downarrow p_k$ denotes the decimation with rate p_k. Clearly, the AMD can be canceled by using the following constraint:

$$\sum_{k=0}^{M-1}p_k\left|H_k(e^{j\omega})\right|^2\downarrow p_k=1,\quad 0\le\omega\le\pi. \qquad (11)$$

From (11), we can find out that the main factor influencing the AMD is the transition band of each individual filter. Based on this observation, we propose to adjust the transition band shape so as to minimize the AMD. This can be easily and efficiently done by using the Parks-McClellan algorithm.

2.2. Feasibility of Rational Sampling Rate

First, we introduce two concepts: image region of input signal and un-image region of input signal $X(\omega)$.

For the k-th channel ($0\le k\le M-1$) of NUFBs, the regions $[2d_k\frac{\pi}{p_k},\ (2d_k+1)\frac{\pi}{p_k}]$ or $[-(2d_k+1)\frac{\pi}{p_k},\ -2d_k\frac{\pi}{p_k}]$, $d_k=0,\ 1,\ 2,\ \cdots\left\lfloor\frac{p_k-1}{2}\right\rfloor$ in frequency domain are defined as the un-image regions of input signal (real line in Fig.2), where $\lfloor i\rfloor$ denotes the maximum integer less than or equal to i, and the regions $[(2r_k-1)\frac{\pi}{p_k},\ 2r_k\frac{\pi}{p_k}]$, or $[-2r_k\frac{\pi}{p_k},\ -(2r_k-1)\frac{\pi}{p_k}]$, $r_k=1,2,\cdots\left\lfloor\frac{p_k}{2}\right\rfloor$ are the image region of input signal (dashed in Fig. 2).

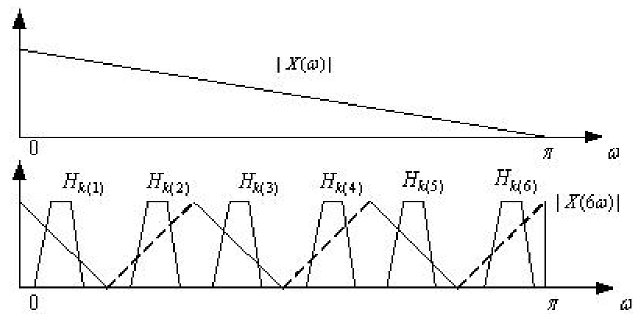


Fig. 2 The analysis filters in image and un-image regions.

Then we derive the relation about feasible sampling rates without giving the detailed derivation procedures.

For the un-image case (the passband position of $H_k(z)$, $0\le k\le M-1$ is in the un-image region), we have

$$\frac{q_k}{p_k}\left(\sum_{i=0}^{k-1}\frac{p_i}{q_i}+2d_k\right)=C_k,\quad 1\le k\le M-1, \qquad (12)$$

and $$d_k=\begin{cases}0, & k=0\\ 0,\ 1,\ 2\cdots\left\lfloor\frac{p_k-1}{2}\right\rfloor, & 1\le k\le M-2\\ (p_{M-1}-1)/2, & k=M-1\end{cases}$$

where C_k is a positive integer.

For the image case (the passband position of $H_k(z)$, $0\le k\le M-2$ is in the image region), we have

$$\frac{q_k}{p_k}\left(2r_k-\sum_{i=0}^{k}\frac{p_i}{q_i}\right)=B_k,\quad 0\le k\le M-2, \qquad (13)$$

and $$r_k=\begin{cases}p_k/2 & k=0\\ 1,\ 2\cdots\left\lfloor\frac{p_k}{2}\right\rfloor, & 1\le k\le M-2\\ \text{not exist} & k=M-1\end{cases}$$

where B_k is a positive integer.

We say that a sampling rate is feasible, if (12) is satisfied for some d_k or (13) is satisfied for some r_k for each *k*, $1 \le k \le M-1$.

3. CANCELLATION OF SIGNIFICANT ALD

As for the significant ALD, we mean the ALD caused by the adjacent filters. First, we give a necessary condition.

Theorem 1: Under the preconditions Eq. (5a) and (9), the necessary condition on the cancellation of the significant ALD of LP NUFBs with rational sampling factors is depicted as follows:

The analysis filters $h_k(n)$ should satisfy the alternate symmetric property described as: $h_0(n)$ is symmetric, $h_1(n)$ is anti-symmetric, $h_2(n)$ is symmetric, $\cdots$.

In the following part (Sec. 3.1 and 3.2), we will focus on the deduction of the theorem 1.

3.1. The Band Position Relations of the Significant Aliasing Components

Clearly, $A_{k,l_k}(z)$ shown in (4) can be considered as the result of $H_k(zW_{q_k}^{l_k})F_k(z)$ decimated by p_k. That is,

$$A_{k,l_k}(z) = p_k H_k(zW_{q_k}^{l_k})F_k(z) \downarrow p_k . \qquad (14)$$

Without lose of generality, we only consider the center point of each significant aliasing region.

Due to page limitation, we only consider the un-image case. Fig. 3 shows one example of this case.

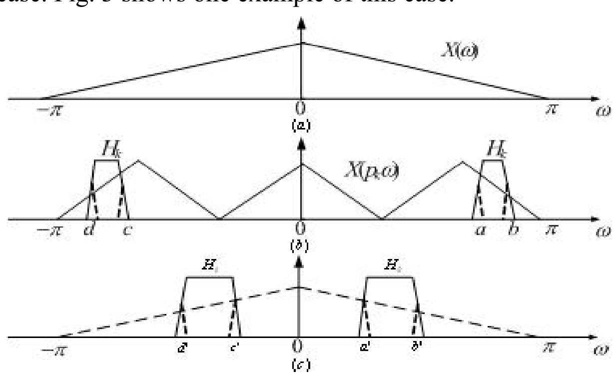


Fig. 3 Significant ALD for the un-image case. (a) Input signal. (b) H_k in the un-image regions and the significant ALD. (c) The corresponding aliasing after decimation of p_k.

The term $H_k(zW_{q_k}^{l_k})F_k(z)$ can cause significant ALD at the following positions: $(a).C_k\frac{\pi}{q_k}$, $(b).(C_k+1)\frac{\pi}{q_k}$, $(c).-C_k\frac{\pi}{q_k}$, $(d).-(C_k+1)\frac{\pi}{q_k}$, where C_k can be obtained in (12) (see Fig. 3 (b)), and they correspond to $l_k = C_k$, C_k+1, $q_k - C_k$, and $q_k-(C_k+1)$ respectively. According to (14) and (12), the corresponding four positions of $A_{k,l_k}(z)$ are: $(a').\sum_{i=0}^{k-1}\frac{\pi}{q_i}p_i$, $(b').\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$, $(c').-\sum_{i=0}^{k-1}\frac{\pi}{q_i}p_i$, and $(d').-\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$ (Fig. 3 (c)).

The image case also satisfies the similar relation, and this relation makes the following fact clearly.

For the *k*-th channel, $0 \le k \le M-2$, significant ALD can be generated in positions $\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$ and $-\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$ for some l_k in $A_{k,l_k}(z)$; for *k*+1-th channel, the significant ALD in the same positions can also be generated for some l_{k+1}. This relation implies that the significant ALD could be canceled if we select analysis and synthesis filters properly.

Then we will analyze whether we can cancel the significant ALD, and if this is the case, how we cancel it.

3.2. Phase Relations of Significant ALD and Elimination of Significant ALD

For the sake of easy analysis, we rewrite the term $A_{k,l_k}(z)$ into another form according to (7) and (14):

$$A_{k,l_k}(z) = (-1)^{u_k} D_{k,l_k}(z), \qquad (15)$$

where $D_{k,l_k}(z) = p_k H_k(zW_{q_k}^{l_k})H_k(z) \downarrow p_k$. (16)

We divide the significant ALD into two classes: *Class 1:* in the *image* region and *Class 2:* in the *un-image* region. The following analysis and results focus on *Class 1*. However, the similar analysis and results can be applied to *Class 2*.

Class 1: The significant ALD in the *un-image* region.

There are four types LP filters [6]. Due to space limitation, we only discuss *Case 1*.

Case 1: The analysis filters $H_k(z)$ is *symmetric* and *even order* in the *un-image* region in [*0*, π].

The term $H_k(zW_{q_k}^{l_k})H_k(z)$ can cause significant ALD in the positions $C_k\frac{\pi}{q_k}$ and $(C_k+1)\frac{\pi}{q_k}$ (points *a* and *b* in Fig. 3(b)). Then we define $Ph_{1(k,l_k)}$ as the phase of the term $H_k(zW_{q_k}^{l_k})H_k(z)$ in the position $l_k\frac{\pi}{q_k}$ and also define $Ph_{2(k,l_k)}$ as the phase of $D_{k,l_k}(z)$ in the position $\sum_{i=0}^{k-1}\frac{\pi}{q_i}p_i$ or $\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$ (points *a*' or *b*' in Fig. 3(c)). Then we give the inherent phase relation as follow

$$Ph_{2(k,C_k+1)} = Ph_{2(k+1,C_{k+1})}\,,\ 0 \le k \le M-2\,, \tag{17}$$

which means that $D_{k,C_k+1}(z)$ and $D_{k+1,C_{k+1}}(z)$ have the same phase expression in the same position $\sum_{i=0}^{k}\frac{\pi}{q_i}p_i$ in [0, π].

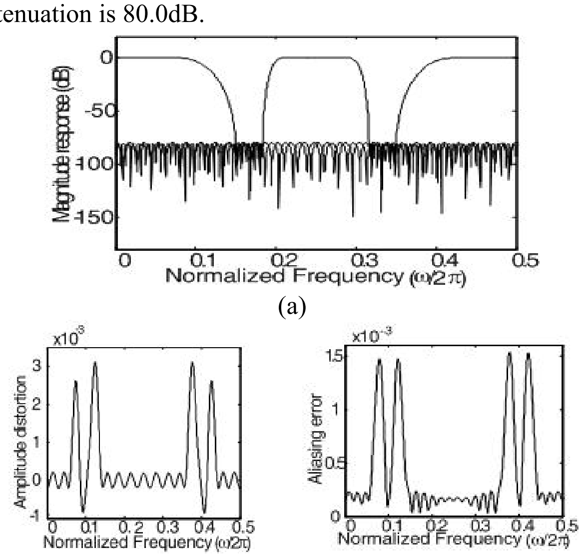
For the other cases in *Class 1* and the cases in *Class 2*, we can find the similar results for the proper l_k and l_{k+1} . Based on those results, we conclude that to eliminate significant ALD, we should ensure the theorem 1 to be satisfied.

In addition, it is necessary to give a lemma to minimize the ALD through analysis.

Lemma 1*:* Let $T_{l(k)}$ and $T_{r(k)}$ denote the width of $H_k(z)$'s transition band which will partition the lower and higher frequency components of input signal respectively. To minimize the ALD, we should choose $T_{r(k)}p_k = T_{l(k+1)}p_{k+1}$, $0 \le k \le M-2$.

4. DESIGN EXAMPLES

Using our proposed method, two examples are shown employing Parks-McClellan algorithm. The first one is a 3-channel NPR LP NUFBs with sampling rate (1/5, 3/5, 1/5), and the orders of analysis filters: $N_0 = N_2 = 56$, $N_1 = 168$. Fig. 4 shows the magnitude response of analysis filters, AMD and ALD. The magnitude error is $E_{pp} = 3.966\times10^{-3}$, the aliasing error is $E_a = 1.532\times10^{-3}$, and the stopband attenuation is 80.0dB.



(a)

(b) (c)

Fig. 4 3-channel NUFBs with sampling rate (1/5, 3/5, 1/5). (a) Magnitude response of each analysis filter; (b) Amplitude distortion. (c) Aliasing error.

The second example is a 4-channel case with sampling rate (1/2, 1/4, 3/16, 1/16), and orders $N_0 = N_1 = N_3 = 169$, $N_2 = 507$. Fig. 5 shows the magnitude response of analysis filters. The resulting $E_{pp} = 3.966\times10^{-3}$, $E_a = 1.532\times10^{-3}$, and stopband attenuation is 89.2dB.

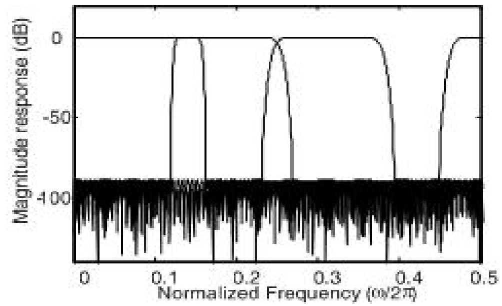


Fig. 5 Magnitude response of 4-channel NUFBs with sampling rate (1/2, 1/4, 3/16, 1/16).

5. CONCLUSIONS

We proposed a new simple method to design NPR LP NUFBs with rational sampling factors. The feasibility of desired system was analyzed. After some detailed analysis, a necessary condition on the elimination of significant ALD is derived. With the requirements fulfilled, the proposed FBs can be easily designed. In addition, our method has lower system delay compared with the LP NPR NUFBs by the indirect method, and can attain a higher stopband attenuation of each analysis and synthesis filter.

6. REFERENCES

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