THEORY AND DESIGN OF TWO-CHANNEL COMPLEX LINEAR-PHASE PSEUDO-ORTHOGONAL FILTERBANKS

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ABSTRACT

In recent years, two-channel complex-valued filterbanks have been studied and found theirs several important applications by many reseachers, such as complex signal and image processing. One of those important results is that there is no two-channel complex-valued linear-phase paraunitary filterbank (CLPPUFB), except for its filter lengths of 2. In this paper, we introduce a class of special complex-valued filterbanks which is a subclass of biorthogonal filterbanks: Those filterbanks are called complex pseudo-orthogonal filterbanks (CPOFB), based on the concept of pseudo-orthogonality (PO). This kind of orthogonality, we propose, is different from the conventional one essentially. This paper also shows possibility to design a two-channel complex linear-phase filterbank (CLPFB) with its filter lengths are more than 2, based on PO. Finally, we show a design example of a complex linear-phase pseudo-orthogonal filterbank (CLPPOFB). Moreover, such CPOFB can satisfy the linear-phase condition simultaneously with filter lengths are more than 2. This paper shows a theory, a design method and an example of CLPPOFB.

1. INTRODUCTION

In various fields of signal processing, Filterbanks (FBs) have been used. FBs decompose a signal, e.g. speech and image signals, into frequency subbands. Then, the signals are coded and transmitted, and finally composed on the side of a receiver. In this paper, we consider a kind of the two-channel complex-valued filterbanks (CFBs). Up to date, there are several important results, properties and applications of CFBs, e.g. motion estimation, texture analysis, image fusion, etc [1] – [9].

Paraunitary (PU) and the linear-phase (LP) property are important properties for FBs. One advantage of paraunitariness is that FBs can be designed more easier than any other implementation, because synthesis filters are the complex conjugate and the time-reverse version of analysis filters. On the other hand, the advantage of the LP property is very useful for image compression because LP filters can be used for the symmetric extension at boundaries of signals [10]. Hence, FBs meeting both PU and the LP properties are desired to design . However, it is impossible to construct two-channel CLPPUFB with filter lengths are more than 2 [9].

Thus, we propose the CPOFB based on the concept of PO in this paper. PO is different from conventional orthogonality, but it can preserve the advantage of orthogonality. That is, the simplicity of FB design. Moreover, such CPOFB can satisfy the LP condition simultaneously even if filter lengths are more than 2. This paper shows a theory, a design method and an example of CLPPOFB.

$Notations$: Bold faced letters indicates vectors and matrices. $\mathbf{I}$ is the identity matrix. $\overline{z}$ denotes the complex conjugate of z. The superscript H denotes complex conjugated transpose and T denotes transpose. $\mathbb{R}$ and $\mathbb{C}$ denote the set of real numbers and complex numbers, respectively. $\mathbb{C}^N$ represents the N-dimmensional complex vector space. $M_2(\mathbb{C})$ denotes the set of 2 × 2 matrices which entries are in $\mathbb{C}$. $\delta(k)$ is kronecker's delta.

2. REVIEW

2.1. Two-Channel Paraunitary Filterbank

Paraunitariness of the two-channel CFB is equivalent to the following equation.

$$\mathbf{E}(z)\mathbf{E}^H(z^{-1}) = c\mathbf{I} \ (c \in \mathbb{C}), \tag{1}$$

where $\mathbf{E}(z)$ is the analysis polyphase matrix. From this equation, we obtain next two equations.

$$h_1(n) = -(-1)^n \overline{h_0(N-n)} \tag{2}$$

$$\sum_n h_0(n)\overline{h_0(n-k)} = \delta(k), \tag{3}$$

where h_0 and h_1 are the lowpass and highpass filter of the CFB, respectively. If filter coefficients are real, then (1), (2) and (3) do not need complex conjugate. In addition to these conditions, the length of the lowpass filter must be even.

2.2. Lattice Structure

Designing a FB by using a lattice structure is more useful and easier than a direct implementation. The reason is that a LP

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or PU perfect reconstruction FB with long filter lengths can be constructed by cascading building blocks. Complete and minimal lattice structure of a two-channel PUFB and a LPFB have been proposed [11].

2.2.1. *Paraunitariness*

Two-channel real-valued PUFBs can be designed by the following lattice structure [11].

$$\mathbf{E}_{poly}(z) = \mathbf{R}_N \mathbf{\Lambda}(z) \mathbf{R}_{N-1} \cdots \mathbf{R}_1 \mathbf{\Lambda}(z) \mathbf{R}_0, \tag{4}$$

where $\mathbf{E}_{poly}$ is the analysis polyphase matrix, $\mathbf{R}_k$ and $\mathbf{\Lambda}(z)$ are

$$\mathbf{R}_k = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ -\sin\theta_k & \cos\theta_k \end{bmatrix}, \quad \mathbf{\Lambda}(z) = \begin{bmatrix} 1 & 0 \\ 0 & z^{-1} \end{bmatrix}, \tag{5}$$

respectively ($\theta_k \in \mathbb{R}$).

2.2.2. *Linear-phase property*

The LP property of two-channel real-valued FBs imply that filter coefficients of each lowpass and highpass filter have to be symmetry and antisymmetry (in the case of complex coefficients, filters have to be hermitian symmetry and antisymmetry). The LPFBs can be represented as follows [11] :

$$\mathbf{E}_{poly}(z) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{S}_N \mathbf{\Lambda}(z) \mathbf{S}_{N-1} \cdots \mathbf{S}_1 \mathbf{\Lambda}(z) \mathbf{S}_0, \tag{6}$$

where $\mathbf{E}_{poly}$ is the analysis polyphase matrix and $\mathbf{S}_k$ is

$$\mathbf{S}_k = \begin{cases} \begin{bmatrix} a & b \\ b & a \end{bmatrix} & (\text{for real} : a, b \in \mathbb{R}) \\ \begin{bmatrix} a & b \\ \overline{b} & \overline{a} \end{bmatrix} & (\text{for complex} : a, b \in \mathbb{C}) \end{cases}$$

and $\mathbf{\Lambda}(z)$ is same as (5).

3. COMPLEX PSEUDO-ORTHOGONAL FILTERBANKS

3.1. Pseudo-Orthogonality

In this section, we introduce the new concept, PO. First, we consider an operator $\langle \cdot, \cdot \rangle$ denoted as follows:

For $\mathbf{x} = (x_1 \cdots x_N)^T, \mathbf{y} = (y_1 \cdots y_N)^T \in \mathbb{C}^N$, we define $\langle \cdot, \cdot \rangle : \mathbb{C}^N \times \mathbb{C}^N \to \mathbb{C}$

$$\langle \mathbf{x}, \mathbf{y} \rangle := \sum_{k=1}^{N} x_k \overline{y_k}. \tag{7}$$

It is well-known that (7) satisfies the inner product definition. If $\langle \mathbf{x}, \mathbf{y} \rangle = 0$, then we call $\mathbf{x}$ and $\mathbf{y}$ are orthogonal.

Next, we consider the above $\langle \cdot, \cdot \rangle$ without complex conjugate

$$\langle \mathbf{x}, \mathbf{y} \rangle_p := \sum_{k=1}^{N} x_k y_k. \tag{8}$$

For real numbers, $\langle \cdot, \cdot \rangle_p$ satisfies the inner product definition, whereas it does not for complex numbers since for some $\mathbf{x} \in \mathbb{C}^N$, $\langle \mathbf{x}, \mathbf{x} \rangle_p \leq 0$.

Now, we define the pseudo-orthogonality with $\langle \cdot, \cdot \rangle_p$.

Definition 1 $\mathbf{x}$ *and* $\mathbf{y}$ $(\in \mathbb{C}^N)$ *are pseudo-orthogonal if and only if*

$$\langle \mathbf{x}, \mathbf{y} \rangle_p = 0.$$

Then, we extend this pseudo-orthogonality to the polyphase matrix version.

Definition 2 *A CFB is pseudo-orthogonal if and only if its analysis polyphase matrix* $\mathbf{E}(z)$ *satisfy*

$$\mathbf{E}(z)\mathbf{E}^T(z^{-1}) = c\mathbf{I}. \quad (c \in \mathbb{C}) \tag{9}$$

3.2. The properties and the lattice structure of CPOFB

In this section, we discuss properties and an implementation of CPOFB.

First, properties of CPOFB are almost same as real-valued PUFBs. From (9), the following equations can be verified

$$h_1(n) = -(-1)^n h_0(N-n) \tag{10}$$

$$\sum_n h_0(n) h_0(n-k) = \delta(k), \tag{11}$$

where h_0 and h_1 were mentioned previously. The highpass filter is the time-reverse and sign alternated version of the lowpass filter, without complex conjugate. However if the lowpass filter satisfies the LP property, that means its coefficients are hermitian-symmetry, the resulting highpass filter related in (10) also satisfies the LP property. As well as the PUFB, the lowpass filter of CPOFB must be even.

Next, one of the easiest design method of CPOFB is using a lattice structure. Here, we consider building blocks of the lattice for CPOFB.

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_2(\mathbb{C})$. In order to A to be a pseudo-orthogonal matrix, we should replace (c, d) by the flipped and sign alternated version of (a, b)

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{c=-b, d=a} A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}.$$

If both $\mathbf{A}$ and $\mathbf{B}$ are pseudo-orthogonal matrices, the product $\mathbf{AB}$ is also pseudo-orthogonal matrix. Since $\mathbf{\Lambda}(z)$ in (5) is also pseudo-orthogonal matrix, we can obtain the analysis polyphase matrix $\mathbf{E}_{poly}(z)$ of CPOFB by using the following lattice structure

$$\mathbf{E}_{poly}(z) = \mathbf{P}_N \mathbf{\Lambda}(z) \mathbf{P}_{N-1} \cdots \mathbf{P}_1 \mathbf{\Lambda}(z) \mathbf{P}_0 \tag{12}$$

where $\mathbf{P}_k \in M_2(\mathbb{C})$ is

$$\mathbf{P}_k = \begin{bmatrix} a_k & b_k \\ -b_k & a_k \end{bmatrix}. \tag{13}$$

Conversely, any CPOFB can be expressed (12).
proof:
Order-N polyphase matrix $\mathbf{E}_{poly}$ can be expressed as

$$\mathbf{E}_{poly} = \sum_{k=0}^{N} \mathbf{E}_k z^{-k}, \tag{14}$$

where $\mathbf{E}_k \in M_2(\mathbb{C})$. The condition of pseudo-orthogonality is

$$\mathbf{E}_{poly}^T(z^{-1})\mathbf{E}_{poly}(z) = c\mathbf{I}. \quad (c \in \mathbb{C}) \tag{15}$$

Let $\mathbf{E}_0$ and $\mathbf{E}_N$ be

$$\mathbf{E}_0 = \begin{bmatrix} e_{11}^{(0)} & e_{12}^{(0)} \\ e_{21}^{(0)} & e_{22}^{(0)} \end{bmatrix}, \quad \mathbf{E}_N = \begin{bmatrix} e_{11}^{(N)} & e_{12}^{(N)} \\ e_{21}^{(N)} & e_{22}^{(N)} \end{bmatrix}, \tag{16}$$

and then it can be verified that

$$\mathbf{E}_N^T\mathbf{E}_0 = \begin{bmatrix} e_{11}^{(N)} & e_{21}^{(N)} \\ e_{12}^{(N)} & e_{22}^{(N)} \end{bmatrix} \begin{bmatrix} e_{11}^{(0)} & e_{12}^{(0)} \\ e_{21}^{(0)} & e_{22}^{(0)} \end{bmatrix} = 0. \tag{17}$$

Let $\mathbf{U}$ be

$$\mathbf{U} = \begin{bmatrix} e_{22}^{(N)} & -e_{12}^{(N)} \\ e_{12}^{(N)} & e_{22}^{(N)} \end{bmatrix}, \tag{18}$$

and then multiply $\mathbf{U}$ to $\mathbf{E}_{poly}$ from the left side.

$$\mathbf{U}\mathbf{E}_{poly} = \begin{bmatrix} \times & \times \\ 0 & 0 \end{bmatrix} + \cdots + \begin{bmatrix} 0 & 0 \\ \times & \times \end{bmatrix} z^{-N}, \tag{19}$$

where $\times$ denotes nonzero entries. With some manipulation, we can obtain the following the order-reduction representation.

$$\mathbf{E}_{poly} = c\,\mathbf{U}^T\mathbf{\Lambda}(z)\hat{\mathbf{E}}(z) \quad (\exists c \in \mathbb{C}), \tag{20}$$

where $\hat{\mathbf{E}}(z)$ is the order-$(N-1)$ pseudo-orthogonal matrix. Thus we can obtain (12) by repeating manipulation like from (16) to (20) until the order of polyphase matrix reaching to zero.

4. COMPLEX LINEAR-PHASE PSEUDO-ORTHOGONAL FILTERBANK

We discussed the lattice structure of CPOFB and CLPFB in the previous sections. The analysis polyphase matrix of CPOFB ($\mathbf{E}_{CPO}(z)$) and CLPFB ($\mathbf{E}_{CLP}(z)$) can be rewritten as follows:

$$\mathbf{E}_{CPO}(z) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{P}_N\mathbf{\Lambda}(z)\mathbf{P}_{N-1}\cdots\mathbf{P}_1\mathbf{\Lambda}(z)\mathbf{P}_0 \tag{21}$$

$$\mathbf{E}_{CLP}(z) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{H}_N\mathbf{\Lambda}(z)\mathbf{H}_{N-1}\cdots\mathbf{H}_1\mathbf{\Lambda}(z)\mathbf{H}_0, \tag{22}$$

where $\mathbf{P}_k$ and $\mathbf{H}_k$ are

$$\mathbf{P}_k = \begin{bmatrix} a_k & b_k \\ -b_k & a_k \end{bmatrix}, \ \mathbf{H}_k = \begin{bmatrix} a_k & b_k \\ \overline{b_k} & \overline{a_k} \end{bmatrix} \ (a_k, b_k \in \mathbb{C}).$$

If some building blocks satisfy both (21) and (22), we can obtain a CLPPOFB. Comparing with (21) and (22), we can easily obtain the following necessary and sufficient condition

$$a_k = \overline{a_k}, \quad \overline{b_k} = -b_k. \tag{23}$$

Therefore, a_k has to be real number and b_k has to be purely imaginary number. Finally, we can obtain a lattice structure for CLPPOFB. The analysis polyphase matrix of CLPPOFB ($\mathbf{E}_{CLPPO}$) can be represented as follows:

$$\mathbf{E}_{CLPPO}(z) = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \mathbf{V}_N\mathbf{\Lambda}(z)\mathbf{V}_{N-1}\cdots\mathbf{V}_1\mathbf{\Lambda}(z)\mathbf{V}_0 \tag{24}$$

where $\mathbf{V}_k$ is

$$\mathbf{V}_k = \begin{bmatrix} s_k & jt_k \\ -jt_k & s_k \end{bmatrix}, \tag{25}$$

where s_k and t_k are real number.

5. DESIGN EXAMPLE

In this section, we present a design example of CLPPOFB based on lattice structure discribed the previous section.

We implement and optimize filterbank based on (24). In different applications, various objective functions can be used in optimization, for example stopband attenuation and coding gain. In this paper, we minimize the Φ which is calculated by summing error of the passband energy and stopband energy as follows:

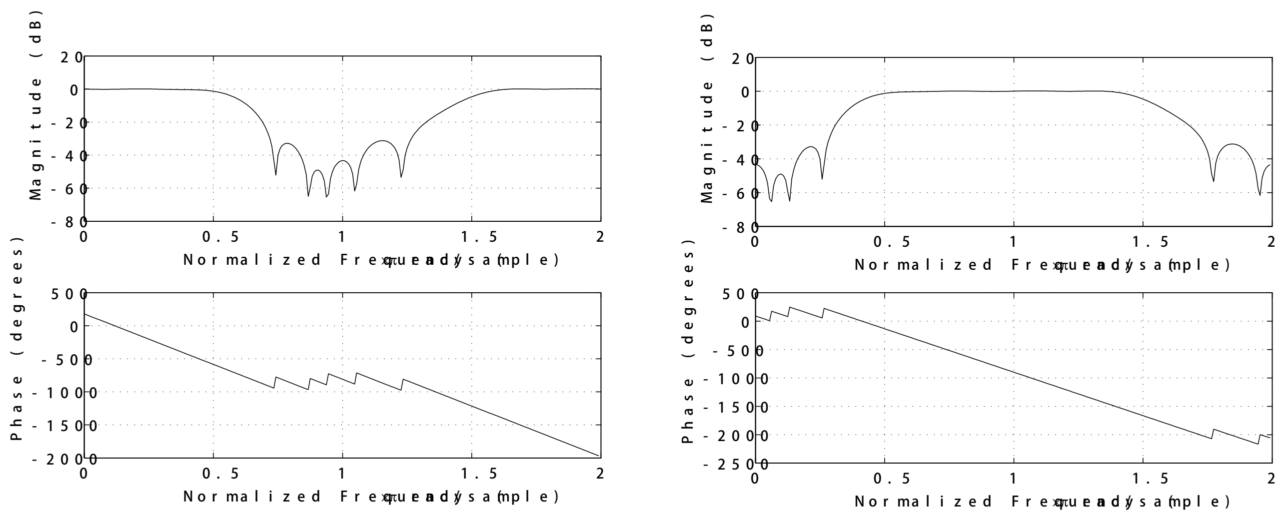
$$\Phi = \sum_{i=0}^{1}(E_{pass}^{(i)} + E_{stop}^{(i)}),$$

$$E_{pass}^{(i)} = \int_{\Omega_{i,p}} (1 - |H_i(\omega)|)^2 d\omega,$$

$$E_{stop}^{(i)} = \int_{\Omega_{i,s}} |H_i(\omega)|^2 d\omega,$$

where $i \in \{0, 1\}$, $H_0(\omega)$ and $H_1(\omega)$ are transfer functions of the lowpass filter and the highpass filter, $E_{pass}^{(0)}$, $E_{stop}^{(0)}$, $E_{pass}^{(1)}$, and $E_{stop}^{(1)}$ are the passband and stopband energy error of the lowpass filter and highpass filter respectively. We set $\Omega_{0,p} = [0, \frac{30\pi}{256}] \cup [\frac{227\pi}{256}, 1]$, $\Omega_{0,s} = [\frac{90\pi}{256}, \frac{166\pi}{256}]$, $\Omega_{1,p} = [0, \frac{38\pi}{256}] \cup [\frac{218\pi}{256}, 1]$ and $\Omega_{1,s} = [\frac{90\pi}{256}, \frac{166\pi}{256}]$, where $\Omega_{0,p}$, $\Omega_{0,s}$, $\Omega_{1,p}$, and $\Omega_{1,s}$ are the passband and stopband of the lowpass and highpass filter, respectively. For simplicity to optimize the FB, we adopted the following building block as (25):

$$\mathbf{V}_k = \begin{bmatrix} \cos\theta_k & -j\sin\theta_k \\ j\sin\theta_k & \cos\theta_k \end{bmatrix} (\theta_k \in \mathbb{R}). \tag{26}$$



(a) Result of lowpass filter (b) Result of highpass filter

Fig. 1. Design example based on the structure (24) with filter length of 18.

The frequency responses and phases are shown in Fig. 1 when theirs filter lengths are 18 in Fig. 1. These frequency responses indicate both the lowpass and highpass filters have good stopband attenuation. Of course, both filters have the LP propety.

6. CONCLUSION

In this paper, the CPOFB based on the concept of PO has been proposed. This kind of orthogonality is not same as the conventional one essentially. However, we can design CPOFB as useful and easy as conventional one. Addtionally, it is possible that a two-channel complex-valued linear-phase *orthogonal-like* FB, CLPPOFB, its filter lengths are more than 2. Therefore, it is expected CLPPOFB is useful for image/video compression. Our future work is to investigate CLPPOFB's application for them.

7. REFERENCES

[1] X. Q. Gao, T. Q. Nguyen, and G. Strang, "A study of two-channel complex-valued filterbanks and wavelets with orthogonality and symmetry properties," *IEEE Trans. Signal Process.*, vol. 50, no. 4, pp. 824–833, Apr, 2002.

[2] X. Q. Gao, T. Q. Nguyen, and G. Strang, "Theory and lattice structure of complex paraunitary filterbanks with filters of (hermitian-) symmetry/antisymmetry properties," *IEEE Trans. Signal Process.*, vol. 49, no. 5, pp. 1028–1043, May, 2001.

[3] J. Magarey and N. Kingsbury, "Motion estimation using a complex-valued wavelet transform," *IEEE Trans. Signal Process.*, vol. 46, no. 4, pp. 1069–1084, Apr, 1998.

[4] N. Kingsbury, "Complex wavelets for shift invariant analysis ans filtering of signals," *Appl. Comput. Harmon. Anal.*, vol. 10, no. 3, pp. 234–253, May, 2001.

[5] N. Kingsbury, "Image processing with complex wavelets," *Phil. R. Soc. London A*, Sept, 1999.

[6] P. R. Hill, D. R. Bull, and C. N. Canagarajah, "Image fusion using a new framework for complex wavelet transforms," *IEEE Conf. on Image Processing ICIP 2005*, vol. 2, pp. 1338–1341, 11–14, Sept, 2005.

[7] S. Hatipoglu, S. K. Mitra, and N. Kingsbury, "Texture classification using dual-tree complex wavelet transform," *Proc. 7th International IEE Conf. on Image Processing and Its Applications*, Manchester, England, July 12-15, 1999, pp 344–347.

[8] P. Loo and N. Kingsbury, "Digital watermarking using complex wavelets," *Proc. IEEE Conf. on Image Processing*, Vancouver, pp. 11–13, Oct, 1995.

[9] X.-P. Zhang, M. D. Desai, and Y.-N. Peng, "Orthogonal complex filter banks and wavelets: some properties and design," *IEEE Trans. Signal Process.*, vol. 47, no. 4, pp. 1039–1048, Apr, 1999.

[10] G. Strang and T. Q. Nguyen, *Wavelets and Filter Banks*. Wellesley, MA: Wellesley-Cambridge, 1997.

[11] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Englewood Cliffs, NJ: Prentice-Hall, 1993.