ON PARAMETERIZATIONS OF FIRST-ORDER UNIMODULAR FILTER BANKS

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ABSTRACT

In this paper, two parameterization methods are proposed for M-channel, first-order unimodular filter banks. For such a system with Mcmillan degree of ρ, the proposed parameterizations can reduce the number of free parameters by ρ^2, compared with existing methods. At the sanme time, the resulting structures are still complete and minimal in the Mcmillan sense. Besides, perfect reconstruction, unimodular and the filter length constraints can be *all* structurally imposed. Moreover, the proposed structures offer *unconstrained* optimizations in the design, which can be hardly achieved by existing methods.

1. INTRODUCTION

An $M \times M$ matrix polynomial matrix $\mathbf{E}(z)$ is said to be unimodular if $det\{\mathbf{E}(z)\} = c \neq 0$ [1–5]. When such an $\mathbf{E}(z)$ is the polyphase matrix of a filter bank (FB), the corresponding FB is called as the *unimodular* FB. This subclass of FBs is attractive as the perfect reconstruction (PR) property can be achieved with all the analysis and synthesis filters being FIR and causal [5]. In addition, an M-channel unimodular FB *always* has a system delay of $M-1$, which is the minimum among all M-channel FIR PR FBs. Moreover, it was shown in [4, 5] that unimodular FBs can yield high coding gain for highly correlated signals. Other interesting properties of unimodular matrices can be found in [6, 7].

In this paper, we study the parameterization of *first-order* unimodular PR FBs, where all the analysis and synthesis filters are of length $2M$ each. From the transform point of view, they correspond to *lapped unimodular transforms* [3]. The factorization of these systems through the degree-one building block was derived by Phoong *el al.* in [5]. Recently, their lifting-based implemetantions along with structural regularity were proposed in [6,7]. However, as we will show later, the structures presented in these works yield extra design parameters. Besides, when the design parameters are quantized, the first-order restriction may not be satisfied, i.e., the filter length may be longer than $2M$. Note that in some applications, especially in image coding, longer filter length will lead to the annoying ringing artifacts [2]. Taking these facts into account, we revisit the parameterization of first-order unimodular FBs in this paper. Two new parameterizations based on the SVD and the lifting structure were presented. Both of them are minimal and complete with fewer parameters than those required in the existing works. Besides, the new structures can lead to unconstrained optimization in the design. They can also structurally impose the first-order, or the filter length restriction in implementation.

Notations: For simplicity of presentation, we only consider real-coefficient FBs in this paper. Vectors and matrices are indicated in bold-faced letters. Subscripts will be provided only if their sizes are not clear from the context. Superscript T stands for transposition. Special matrices used extensively throughout this paper are the identity matrix $\mathbf{I}$ and the null matrix $\mathbf{0}$.

2. REVIEW OF DEGREE-ONE FACTORIZATION

Consider an M-channel unimodular FB with all analysis and synthesis filters of the same length $L = 2M$ each. Let $\mathbf{E}(z) = \mathbf{E}_0 + \mathbf{E}_1 z^{-1}$ and $\mathbf{R}(z) = \mathbf{R}_0 + \mathbf{R}_1 z^{-1}$ represent its analysis and synthesis polyphase matrices, respectively. The McMillan degree of $\mathbf{E}(z)$, denoted as ρ, was shown to be equal to the rank of $\mathbf{E}_1$ [3]. In [5], two-types of degree-one factorizations were proposed for first-order unimodular $\mathbf{E}(z)$, both of which are complete and minimal in the McMillan sense.

In Type-I factorization, $\mathbf{E}(z)$ is decomposed as a product of an invertible matrix $\mathbf{E}(1) = \mathbf{E}_0 + \mathbf{E}_1$ and ρ-many degree-one unimodular matrices $\hat{\mathbf{D}}_i(z)$ (for $i = 1, \cdots, \rho - 1$) as follows:

$$\mathbf{E}(z) = \mathbf{E}(1)\hat{\mathbf{D}}_1(z)\hat{\mathbf{D}}_2(z)\cdots\hat{\mathbf{D}}_\rho(z), \quad \text{Type I} \tag{1}$$

where $\hat{\mathbf{D}}_i(z) = \mathbf{I} - \hat{\mathbf{u}}_i\hat{\mathbf{v}}_i^\dagger + \hat{\mathbf{u}}_i\hat{\mathbf{v}}_i^\dagger z^{-1}$(for $i = 1, \cdots, \rho - 1$), while $\hat{\mathbf{u}}_i$ and $\hat{\mathbf{v}}_i$ are $M \times 1$ vectors satisfying $\hat{\mathbf{v}}_i^\dagger\hat{\mathbf{u}}_i = 0$. As $\hat{\mathbf{v}}_i^\dagger\hat{\mathbf{u}}_i = 0$, it can be calculated that the inverse of $\hat{\mathbf{D}}_i(z)$ takes the form of $\hat{\mathbf{D}}_i^{-1}(z) = \mathbf{I} + \mathbf{u}_i\mathbf{v}_i^\dagger - z^{-1}\mathbf{u}_i\mathbf{v}_i^\dagger$. Since $\hat{\mathbf{D}}_i^{-1}(z)$ is a degree-one causal matrix, it is clear that $\mathbf{R}(z) = \mathbf{E}^{-1}(z)$ is also causal with degree of ρ. It should be pointed here that *all* order-one unimodular matrices can be factorizated into (1). However, $\mathbf{E}(z)$ and its inverse $\mathbf{R}(z)$ generated by (1) may *not* necessarily have order one. Instead, their orders can vary from 1 to ρ, i.e., the filter length can vary from $2M$ to $(\rho+1)M$. To meet the first-order condition, $\hat{\mathbf{u}}_i$ and $\hat{\mathbf{v}}_i$ in (1) need to further satisfy [5–7]

$$\hat{\mathbf{v}}_i^\dagger\hat{\mathbf{u}}_j = 0, \text{ i.e., } \hat{\mathbf{u}}_j \perp \hat{\mathbf{v}}_i, \; 1 \leq i, j \leq \rho. \tag{2}$$

It was shown in [5] that through some simple mathematical manipulations, $\mathbf{E}(z)$ and $\mathbf{R}(z)$ can be explicitly expressed into the *order-one* form. Let us define two matrices $\hat{\mathcal{U}}$ and $\hat{\mathcal{V}}$ from $\hat{\mathbf{u}}_i$ and $\hat{\mathbf{v}}_i$ as follows: $\hat{\mathcal{U}} = \begin{bmatrix}\hat{\mathbf{u}}_1 & \hat{\mathbf{u}}_2\cdots\hat{\mathbf{u}}_\rho\end{bmatrix}$ and $\hat{\mathcal{V}} = \begin{bmatrix}\hat{\mathbf{v}}_1 & \hat{\mathbf{v}}_2\cdots\hat{\mathbf{v}}_\rho\end{bmatrix}$. Then, denote the $M \times M$ matrix $\mathbf{Q}$ as $\mathbf{Q} = \hat{\mathcal{U}}\hat{\mathcal{V}}^\dagger$. Under (2), the order-one form for $\mathbf{E}(z)$ in (1) is [5–7]

$$\mathbf{E}(z) = \mathbf{E}(1)(\mathbf{I} - \mathbf{Q} + \mathbf{Q}z^{-1}). \tag{3}$$

Accordingly, its inverse $\mathbf{R}(z)$ can be shown to be [5–7]

$$\mathbf{R}(z) = \mathbf{E}^{-1}(1)(\mathbf{I} + \mathbf{Q} - \mathbf{Q}z^{-1}). \tag{4}$$

An equivalent, but alternative degree-one factorization of $\mathbf{E}(z)$ is as follows [5]

$$\mathbf{E}(z) = \mathbf{E}_0\mathbf{D}_0(z)\mathbf{D}_1(z)\cdots\mathbf{D}_{\rho-1}(z), \quad \text{Type II,} \tag{5}$$

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where $\mathbf{E}_0$ is an invertible matrix and each $\mathbf{D}_i(z)$ is a degree-one unimodular matrix taking the form of $\mathbf{D}_i(z) = \mathbf{I} + z^{-1}\mathbf{u}_i\mathbf{v}_i^\dagger$, with the $M \times 1$ vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ verifying

$$\mathbf{v}_i^\dagger \mathbf{u}_j = 0, \text{ i.e., } \mathbf{u}_j \perp \mathbf{v}_i,\ 1 \leq i, j \leq \rho. \tag{6}$$

Likewise, denote $\mathcal{U} = \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 \cdots \mathbf{u}_\rho\end{bmatrix}$, $\mathcal{V} = \begin{bmatrix}\mathbf{v}_1 & \mathbf{v}_2 \cdots \mathbf{v}_\rho\end{bmatrix}$ and $\mathbf{P} = \hat{\mathcal{U}}\hat{\mathcal{V}}^\dagger$. Then, under (6), $\mathbf{E}(z)$ and $\mathbf{R}(z)$ generated by Type II factorization can be re-written into [5–7]

$$\mathbf{E}(z) = \mathbf{E}_0(\mathbf{I} + \mathbf{P}z^{-1}), \tag{7}$$

$$\mathbf{R}(z) = (\mathbf{I} - \mathbf{P}z^{-1})\mathbf{E}_0^{-1} \tag{8}$$

Remarks: To design a first-order unimodular FB through (1) or (5), one needs to parameterize an $M \times M$ invertible matrix $\mathbf{E}(1)$ or $\mathbf{E}_0$, 2ρ-many $M \times 1$ vectors $\{\hat{\mathbf{u}}_i, \hat{\mathbf{v}}_i\}$ or $\{\mathbf{u}_i, \mathbf{v}_i\}$ ($1 \leq i \leq \rho$) under the constraint of (2) or (6). Altogether, this requires $M^2 + 2M\rho - \rho^2$ parameters for real-coefficient systems [6, 7].

3. PROPOSED STRUCTURES

In this paper, we propose more efficient parameterizations with fewer parameters, while still retaining the completeness and the minimality of the structure. Note that for any first-order unimodular FB, its polyphase matrices $\mathbf{E}(z)$ and $\mathbf{R}(z)$ can be *always* represented through the order-one forms in (3)-(4) or (7)-(8). Hence, instead of using the $M \times 1$ vectors $\{\hat{\mathbf{u}}_i, \hat{\mathbf{v}}_i\}$ or $\{\mathbf{u}_i, \mathbf{v}_i\}$ ($1 \leq i \leq \rho$) as free parameters, we will characterize the $M \times M$ matrices $\mathbf{Q}$ in (3)-(4) and $\mathbf{P}$ in (7)-(8). The following Lemma presents the necessary and sufficient conditions to yield an PR unimodular FB with degree of ρ.

Lemma 1. *$\mathbf{E}(z)$ and $\mathbf{R}(z)$ given by (3)-(4) or (7)-(8) correspond to a PR unimodular FB with degree of ρ if and only if $\mathbf{P}$ and $\mathbf{Q}$ satisfy (1) rank($\mathbf{P}$) = rank($\mathbf{Q}$) = ρ; (2) $\mathbf{P}^2 = \mathbf{Q}^2 = \mathbf{0}$, i.e., they are nil-potent matrices with nil-potency index of 2.*

The proof of Lemma 1 can be easily obtained by substituting (3)-(4) or (7)-(8) into the PR restriction $\mathbf{R}(z)\mathbf{E}(z) = \mathbf{I}_M$. Details are omitted here. Lemma 1 suggests that the design of a first-order unimodular FB can be converted into the parameterization of an $M \times M$ matrix $\mathbf{X}$ verifying

$$\text{rank}(\mathbf{X}) = \rho \quad \text{and} \quad \mathbf{X}^2 = \mathbf{0} \tag{9}$$

In what follows, we will investigate the characterizations of such a matrix through the SVD and the lifting structure in Section 3.1 and Section 3.2, respectively.

3.1. The SVD-based Parameterization

Note that using the SVD, any $M \times M$ matrix $\mathbf{X}$ with rank of ρ can be always represented as $\mathbf{X} = \mathbf{U}\Delta\mathbf{V}^T$, where Δ is a diagonal matrix with non-zero diagonal elements, while $\mathbf{U}$ and $\mathbf{V}$ are $M \times \rho$ orthonormal matrices. Taking advantage of the orthonormal property of $\mathbf{U}$ and $\mathbf{V}$, one can derive that $\mathbf{X}^2 = \mathbf{0}$ *if and only if*

$$\mathbf{V}^T\mathbf{U} = \mathbf{0}. \tag{10}$$

Eq. (10) implies that $\mathbf{X}^2 = \mathbf{0}$ holds if and only if the $M \times \rho$ orthonormal matrix $\mathbf{V}$ is in the null space of $\mathbf{U}$. Let the $M \times (M - \rho)$ matrix $\mathbf{U}^\perp$ denote the orthogonal compliment of $\mathbf{U}$. Thus, $\mathbf{V}$ can be completely represented by

$$\mathbf{V} = \mathbf{U}^\perp\mathbf{W}, \tag{11}$$

where $\mathbf{W}$ is an $(M-\rho) \times \rho$ matrix. The orthonormal property of $\mathbf{V}$ further requires that $\mathbf{W}$ should be orthonormal as well. Note that by applying Sylverter's rank inequality [8] to the equation $\mathbf{X}^2 = \mathbf{0}$, we have rank($\mathbf{X}$) + rank($\mathbf{X}$) $- M < 0$; hence, $M - \rho > \rho$. This indicates that $\mathbf{W}$ is a "tall" matrix with its row dimension greater than its column dimension. Hence, $\mathbf{W}$ can be chosen to be orthonormal.

From the above analysis, we know that $\mathbf{X}$ satisfying (9) can be always characterized as

$$\mathbf{X} = \mathbf{U}\Delta\mathbf{W}^T(\mathbf{U}^\perp)^T \tag{12}$$

Note that once $\mathbf{U}$ is found, its orthogonal compliment $\mathbf{U}^\perp$ can be obtained through the Gram-schmidt orthogonalization process [8]. Hence, $\mathbf{X}$ can be fully characterized through an $M \times \rho$ orthonormal matrix $\mathbf{U}$, a $\rho \times \rho$ non-singular diagonal matrix Δ and an $(M - \rho) \times \rho$ orthonormal matrix. It is obvious that Δ contains ρ free parameters. Also, recall that using the Givens decomposition, an $i \times j$ orthogonal matrix can be completely parameterized by $ij - \frac{j(j+1)}{2}$ parameters. Hence, the degrees of design freedom held by $\mathbf{X}$ are

$$M\rho - \frac{\rho(\rho+1)}{2} + (M - \rho)\rho - \frac{\rho(\rho+1)}{2} + \rho = 2\rho(M - \rho).$$

Accordingly, if $\mathbf{P}$ or $\mathbf{Q}$ are parameterized as in (12), the total number of free parameters in (3) or (7) is $M^2 + 2\rho(M - \rho)$ (The extra M^2 comes from the $M \times M$ invertible matrix $\mathbf{E}(1)$ or $\mathbf{E}_0$). As we have mentioned before, if $\{\hat{\mathbf{u}}_i, \hat{\mathbf{v}}_i\}$ or $\{\mathbf{u}_i, \mathbf{v}_i\}$ are used directly as free parameters, $M^2 + 2M\rho - \rho^2$ parameters are required in (1) or (5). Therefore, using the proposed parameterization through the SVD, we can gain a reduction of ρ^2 parameters. Note that as $\rho \leq M/2$, the maximum number of reduced parameters is $\lfloor M/2 \rfloor^2$.

Not only does the proposed SVD-based parameterization reduce the number of free parameters, it also facilitates the *unconstrained* optimization in the design. Note that in (12), the Givens rotation angles in $\mathbf{U}$ and $\mathbf{W}$ as well as the diagonal elements in Δ can be arbitrarily varied under a mild condition $\Delta(i, i) \neq 0$ ($1 \leq i \leq \rho$). On the other hand, the parameterizations of $\{\hat{\mathbf{u}}_i, \hat{\mathbf{v}}_i\}$ or $\{\mathbf{u}_i, \mathbf{v}_i\}$ should satisfy the constraints of (2) or (6), which make the optimization more complicated. Moreover, just like the degree-one factorization in (1) and (5), the proposed structure is also *complete* and *minimal* in the Mcmillan sense. Proof of both the completeness and the minimality is straightforward. Details are hence omitted here.

3.2. The Lifting-based Parameterization

Although the SVD-based parameterization can reduce the number of free parameters, it will incur high computation cost due to the presence of Givens rotation matrices. For both (1) and (5), it was shown in [6, 7] that the degree-one building block $\hat{\mathbf{D}}_i(z)$ and $\mathbf{D}_i(z)$ can be further decomposed into a series of *lifting steps*. The lifting factorization provides fast, reversible and multiplierless implementations of. Besides, one can also structurally impose the regularity condition. Motivated by the work in [6, 7], we propose a new lifting-based implementation for $\mathbf{E}(z)$ and $\mathbf{R}(z)$. The main difference here is that our lifting structure is based on the order-one form in (3)-(4) or (7)-(8), while the one in [6, 7] stems from the degree-one factorization in (1) and (5). To this end, we will consider a new parameterization of $\mathbf{X}$ satisfying (9).

Note that since rank$(\mathbf{X}) = \rho$, $\mathbf{X}$ can be decomposed into

$$\mathbf{X} = \mathcal{U}\mathcal{V}^T, \tag{13}$$

where both $\mathcal{U}$ and $\mathcal{V}$ are $M \times \rho$ matrices with rank of ρ. The condition $\mathbf{X}^2 = 0$ implies that

$$\mathcal{U}\mathcal{V}^T\mathcal{U}\mathcal{V}^T = 0 \Leftrightarrow \mathcal{V}^T\mathcal{U} = 0 \tag{14}$$

Eq. (14) suggests that the chacterization of $\mathbf{X}$ in (9) can be converted into two full rank matrices $\mathcal{U}$ and $\mathcal{V}$ satisfying (14)

As $\mathcal{U}$ is of full rank, after proper row permutation, its $\rho \times \rho$ lower matrix can be made to be invertible. In other words, there exists an $M \times M$ permutation matrix $\mathbf{T}$ so that $\mathcal{U}$ can be represented as

$$\mathcal{U} = \mathbf{T}\begin{bmatrix}\mathcal{U}_u \\ \mathcal{U}_d\end{bmatrix} = \mathbf{T}\begin{bmatrix}\mathbf{A} \\ \mathbf{I}_\rho\end{bmatrix}\mathcal{U}_d, \tag{15}$$

where $\mathcal{U}_d$ is an $\rho \times \rho$ matrix, $\mathcal{U}_u$ is an $(M-\rho)\times\rho$ arbitrary matrix and $\mathbf{A} = \mathcal{U}_u\mathcal{U}_d^{-1}$. Next, denote the $\rho \times M$ matrix $\mathcal{V}^T$ as

$$\mathcal{V}^T = \begin{bmatrix}\mathcal{V}_l & \mathcal{V}_r\end{bmatrix}\mathbf{T}^T, \tag{16}$$

where the submatrices $\mathcal{V}_l$ and $\mathcal{V}_r$ are of sizes $\rho\times(M-\rho)$ and $\rho\times\rho$, respectively. Then, substituting (15) and (16) into (14) yields

$$\mathcal{V}_r = -\mathcal{V}_l\mathbf{A} \tag{17}$$

Therefore, under the constraint of (14), $\mathcal{V}^T$ in (16) can be rewritten into

$$\mathcal{V}^T = \mathcal{V}_l\begin{bmatrix}\mathbf{I}_{M-\rho} & -\mathbf{A}\end{bmatrix}. \tag{18}$$

Accordingly, $\mathbf{X} = \mathcal{U}\mathcal{V}^T$ can be completely parameterized by

$$\mathbf{X} = \mathcal{U}\mathcal{V}^T = \mathbf{T}\begin{bmatrix}\mathbf{A} \\ \mathbf{I}_\rho\end{bmatrix}\mathbf{B}\begin{bmatrix}\mathbf{I}_{M-\rho} & \mathbf{A}\end{bmatrix}\mathbf{T}^T, \tag{19}$$

in which $\mathbf{T}$ is an $M \times M$ permutation matrix, $\mathbf{A}$ is an arbitrary $(M-\rho)\times\rho$ matrix and $\mathbf{B} = \mathcal{U}_d\mathcal{V}_l$ is a $\rho \times (M-\rho)$ full rank matrix.

Now, let us get back to the order-one form in (3). If $\mathbf{Q}$ is parameterized through (19), one can prove that $\mathbf{E}(z)$ and $\mathbf{R}(z)$ can be expressed as

$$\mathbf{E}(z) = \mathbf{E}(1)\mathbf{T}\begin{bmatrix}\mathbf{I} & \mathbf{A} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ -\mathbf{B}z^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & -\mathbf{A} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{T}^T \tag{20}$$

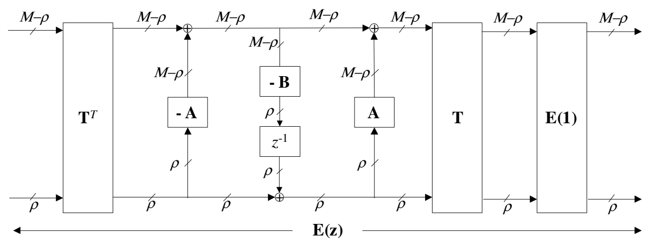
and

$$\mathbf{R}(z) = \mathbf{T}^T\begin{bmatrix}\mathbf{I} & -\mathbf{A} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{0} \\ \mathbf{B}z^{-1} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{I} & \mathbf{A} \\ \mathbf{0} & \mathbf{I}\end{bmatrix}\mathbf{T}\mathbf{E}^{-1}(1) \tag{21}$$

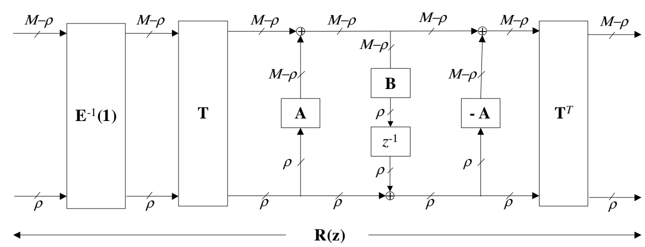
In (20) and (21), as $\mathbf{A}$ and $\mathbf{B}$ are of sizes $(M-\rho)\times\rho$ and $\rho\times(M-\rho)$, respectively, the total number of free parameters is also $M^2 + 2\rho(M-\rho)$, which is the same as required in the SVD-based parameterization. From the implementation perspective, (20) and (21) are more advantageous since they lead to lifting-based implementations, as shown in Figure 1. In fact, when $\rho = 1$, they boil down to the degree-one lifting structures proposed in [6, 7]. But when $\rho > 1$, the structure in [6, 7] will require extra ρ^2 parameters than that of (20). Also, our proposed structure is more robust to quantization. That is, even when $\mathbf{A}$ and $\mathbf{B}$ are quantized, the resulting $\mathbf{E}(z)$ and $\mathbf{R}(z)$ still satisfy the PR, unimodular and order-one constraints. Whereas, for the structure in [6, 7], if the lifting coefficients are quantized, the order (or filter length) of the resulting FB may be affected.

Remarks:

1. By parameterizing $\mathbf{Q}$ through (19), one can get a similar lifting structure for Type II order-one form in (7). In fact, it is not difficult to show that the resulting lifting structure of (7) can be obtained by replacing z^{-1} with $(z^{-1}-1)$ in (20) and (21).
2. In (20) and (21), the total number of possible permutation matrix $\mathbf{T}$ is $M!$ for an M-channel system. To simplify the optimization, the permutation matrix $\mathbf{T}$ is usually set to be, e.g., $\mathbf{T} = \mathbf{I}$. However, in this way, one cannot get global optimal solution. On the other hand, the SVD-based structure does not contain any permutation matrix, which is more attractive in unconstrained optimization.



(a)



(b)

Figure 1: The proposed lifting-based implementations for first-order unimodular filter banks. (a) The analysis bank $\mathbf{E}(z)$. (b)The synthesis bank $\mathbf{R}(z)$.

4. DESIGN EXAMPLES

This section presents two design examples using the proposed parameterizations. The optimization criteria is the *coding gain*, in which the input signal is modeled as an AR(1) process with a coefficient of 0.95.

Design Example I in Figure 2is a four-channel ($M = 4$), degree-two ($\rho = 2$) unimodular FB. It is obtained by applying the SVD-based parameterization to (3), which requires 28 parameters in total. Whereas, if the degree-one factorization in (1) is used, 32 parameters is required. Figure 2 shows the frequency response of the resulting design. The coding gain for this example is 8.47dB, which is greater than the 7.96dB of paraunitary first-order FB ??.

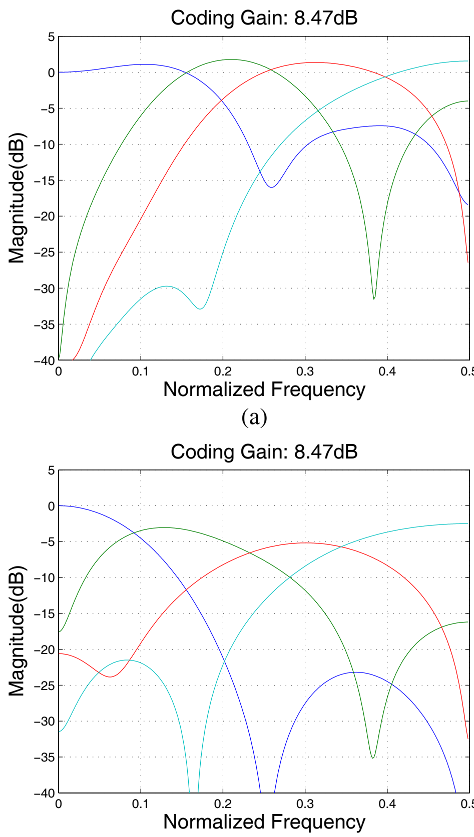
Design Example II in Fig. 3 is based on the lifting structure in (20)-(21) with $M = 8$ and $\rho = 2$. The invertible matrix $\mathbf{E}(1)$ in

(20) is chosen to be the integer-approximated DCT. Its coding gain is 9.12dB. The resulting free matrices $\mathbf{A}$ and $\mathbf{B}$ are listed below.

$$\mathbf{A} = \begin{bmatrix} \frac{103}{512} & -\frac{165}{256} & -\frac{11}{256} & \frac{137}{512} & \frac{11}{256} & \frac{21}{512} \\ -\frac{107}{512} & -\frac{125}{256} & \frac{23}{128} & \frac{23}{256} & \frac{169}{512} & \frac{53}{128} \end{bmatrix}^T$$

$$\mathbf{B} = \begin{bmatrix} -\frac{43}{256} & -\frac{71}{512} & -\frac{5}{64} & -\frac{67}{512} & -\frac{3}{512} & \frac{47}{512} \\ -\frac{137}{512} & -\frac{85}{512} & -\frac{47}{512} & -\frac{43}{256} & -\frac{5}{256} & \frac{27}{256} \end{bmatrix}$$

Since each element in $\mathbf{A}$ and $\mathbf{B}$ takes the form of $k/2^n$, this FB has a multiplierless implementation.

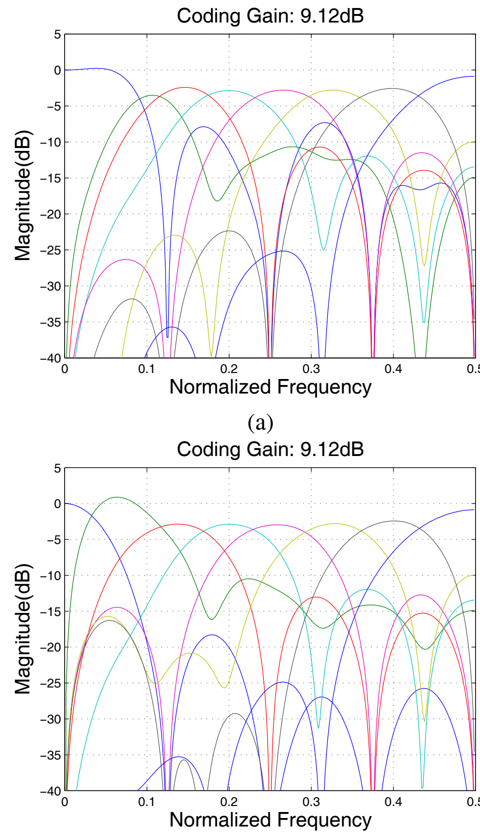


(b)

Figure 2: Frequency response of a 4-channel order-one unimodular FB with $\rho = 2$. (a)The analysis filters. (b)The synthesis filters.

5. CONCLUSION

This paper proposes two new parameterization methods for first-order unimodular filter banks. Both of them are complete and minimal in the Mcmillan sense. Compared with existing methods, the proposed factorizations can reduce the number of free parameters by ρ^2, where ρ is the degree of the FB. From the implemenation perspective, they are robust to quantization, which can structurally impose the PR, unimodular properties and first-order constraint. Two design examples are presented to verify the vailidity of the theory. It should be pointed that although our derivations in this paper focus on the real-coefficient system, they can be easily extended to complex systems as well.



(b)

Figure 3: Frequency response of an 8-channel order-one unimodular FB with $\rho = 2$. (a)The analysis filters. (b)The synthesis filters.

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