Noise Reduction Design of Perfect Reconstruction Oversampled Filter Banks

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Abstract— This paper studies the noise reduction design problem for oversampled filter banks (FBs) with perfect reconstruction (PR) constraint. Both the optimal design and worst case design are considered, where the former method caters for the noise with known power spectral density (PSD) and the latter one for the noise with unknown PSD. Explicit formulae involving only algebraic Riccati equation and matrix manipulations are provided for the general (IIR or FIR) oversampled PR FBs.

I. Introduction

Recently a great deal of research has been devoted to oversampled FBs with redundant signal expansions. The noise reduction properties, extra design freedom and improved capacity for signal and information representation are the main advantages of oversampled FBs [1], [3], [6], [7]. An elegant frame-theoretic approach is presented for the analysis and design of general oversampled FBs in [1], [2], [3]. Most of the research focuses on finite impulse response (FIR) oversampled FBs or infinite impulse response (IIR) FBs with some special structures, and there have been no unified and computationally effective tools available for systematic analysis and design of both IIR and FIR oversampled FBs. In our companion paper [5], explicit and numerically efficient formulae to compute the tightest frame bounds, to obtain the dual FB frame and to construct a tight (paraunitary) FB frame from an arbitray FB (FIR or IIR) satisfying perfect reconstruction (PR) condition are provided.

In this paper, we will consider the noise reduction problem for PR oversampled FBs based on the results of [4], [5]. Specifically, we will consider the following problem: given an analysis FB satisfying PR condition, how to pick up one synthesis FB from the set of all PR synthesis FBs such that it is “optimal” with respect to the noise assumption. We will present direct computational methods for two classes of noises: colored noises with known power spectral density and general noise with unknown power spectral density. For colored noises with known PSD, it is well-known that the optimal synthesis FB is the one whose range is orthogonal to the noise component [2]. However, as shown in [1], there is a lack of efficient method to get the synthesis FB corresponding to the dual frame which is optimal for white noises, let alone the general colored noise. A predictive quantization method is proposed in [2] to reduce the quantization noise. For noise with unknown PSD, the effective worst-case design or H_∞ filtering method is studied for critically-sampled FBs [11], [13], [12], In such case the PR synthesis FB (if exists) is unique for a given analysis FB, and there is no extra design freedom for optimal noise reduction subject to PR. In this paper, we study the noise reduction of oversampled FBs with PR constraint. It is worthwhile to point out that our method is frame theory based and the synthesis FB (dual frame) is not necessarily causal (however stability is guaranteed). Therefore simpler formulae are expected compared to the standard H_∞ filtering where the filters are confined to the causal and stable set.

II. Preliminaries

This sections collects some important results on oversampled FBs from [1]. It also reviews the state-space computational method for analysis and design of frames with oversampled FBs proposed in [4], [5]. Please refer [1], [3], [9], [10], [5] for more details.

A. PR oversampled FBs

Consider the N-channel oversampled FB with decimation factor M. Let $H_k(z)$ and $F_k(z)$, $k = 0, \dots N-1$, be the transfer functions of the analysis and synthesis filters, respectively. Write $H_k(z)$ and $F_k(z)$ as

$$H_k(z) = \sum_{n=-\infty}^{\infty} h_k[n] z^{-n} \text{ and } F_k(z) = \sum_{n=-\infty}^{\infty} f_k[n] z^{-n},$$

where $h_k[n]$ and $f_k[n]$ are impulse response coefficients of $H_k(z)$ and $F_k(z)$, respectively. Denote $E(z)$ and $R(z)$ the polyphase matrix of the analysis filters $\{H_k(z)\}$ and the synthesis filters $\{F_k(z)\}$, respectively, where $E_{ij}(z) = \sum_{n=-\infty}^{\infty} h_i[nN-j] z^{-n}$, and $R_{ji}(z) = \sum_{n=-\infty}^{\infty} f_i[nN-j] z^{-n}$, for $i = 0, \dots, N-1$ and $j = 0, \dots, M-1$. The oversampled filter bank with noise after blocking is shown in Fig. 1.

For an analysis filter bank $\{H_k(z)\}$ with polyphase matrix $E(z)$, recall the following results from [1], [3]

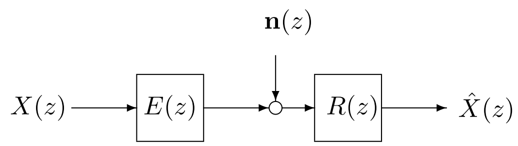

Fig. 1. Oversampled filter bank with noise after blocking

Lemma 1: $\{H_k(z)\}$ implements PR if and only if its polyphase matrix $E(z)$ is of full column rank on the unit circle.

Lemma 2: Assume that $E(z)$ has full rank on the unit circle. Then all synthesis polyphase matrices $R(z)$ can be characterized by

$$R(z) = R_0(z) + U(z)(I_N - E(z)R_0(z)) \tag{1}$$

where $R_0(z) = (E^{\sim}(z)E(z))^{-1}E^{\sim}(z)$ is the synthesis FB corresponding to the dual frame and $E^{\sim}(z) = E^*(z^{-1})$.

B. State space representations and factorization

A transfer matrix $E(z) = \sum_{i=-\infty}^{\infty} E_i z^{-i} \in \mathbb{C}^{N\times M}$ is called causal if $E_i = 0$ for all $i < 0$, is called anti-causal if $E_i = 0$ for all $i > 0$, and is called strictly anti-causal if $E_i = 0$ for all $i \geq 0$. Note that the anti-causal $E(z)$ include strictly anti-causal $E(z)$ as a special case with $E_0 = 0$.

Definition 1: For any rational causal transfer matrix $E(z) = \sum_{i=0}^{\infty} E_i z^{-i} \in \mathbb{C}^{N\times M}$, if the matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times M}, C \in \mathbb{R}^{N\times n}$ and $D \in \mathbb{R}^{N\times M}$ are such that $E(z) = D + C(zI - A)^{-1}B$, then (A, B, C, D) is called a state space realization of $E(z)$ and denoted as $E(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. In such case, $E(z)$ is described by the following causal state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k). \end{aligned}$$

The realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ is minimal if the dimension of A is minimal.

Definition 2: For any rational anti-causal transfer matrix $E(z) = \sum_{i=-\infty}^{0} E_i z^{-i} \in \mathbb{C}^{N\times M}$, if the matrices $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times M}, C \in \mathbb{R}^{N\times n}$ and $D \in \mathbb{R}^{N\times M}$ are such that $E(z) = C(z^{-1}I - A)^{-1}B + D$, then (A, B, C, D) is called an anti-causal state space realization of $E(z)$ and denoted as $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]_{ac}$. In such case, $E(z)$ is described by the following anti-causal state space model

$$\begin{aligned} x(k) &= Ax(k+1) + Bu(k+1) \\ y(k) &= Cx(k) + Du(k). \end{aligned}$$

For any transfer matrix $E(z)$, if $E(z)$ has a causal state-space realization $E(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$, then $E^{\sim}(z)$ has an anti-causal state-space realization $E^{\sim}(z) = \left[\begin{array}{c|c} A^* & C^* \\ \hline B^* & D^* \end{array}\right]$.

A rational transfer matrix $N(z)$ is called inner if $N(z)$ is causal stable satisfying $N^{\sim}(z)N(z) = I$ for all $z = e^{j\theta}, \theta \in [0, 2\pi)$. Note that in such case $N^{\sim}(z)$ is anti-causal and stable. Note also that the inner condition is equivalent to the paraunitary condition [1], [3]. Thus, inner implies paraunitary.

Remark 1: For a given transfer matrix, it might have both causal and anti-causal realizations, and it might have only causal (anti-causal) realization, depending on the location of its poles and zeros. For example, $E_1(z) = \frac{1}{z-0.5}$ has the causal realization $E_1(z) = \left[\begin{array}{c|c} 0.5 & 1 \\ \hline 1 & 0 \end{array}\right]$, and also has the anti-causal realization $E_1(z) = -2 + \frac{-4}{z^{-1}-2} = \left[\begin{array}{c|c} 2 & -4 \\ \hline 1 & -2 \end{array}\right]_{ac}$. However, $E_2(z) = z$ has only anti-causal realization $E_2(z) = \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]_{ac}$, and $E_3(z) = z^{-1}$ has only causal realization $E_3(z) = \left[\begin{array}{c|c} 0 & 1 \\ \hline 1 & 0 \end{array}\right]$.

For $E(z) \in \mathbb{C}^{N\times M}$ with a minimal causal realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$. Assume that

A1) $\left[\begin{array}{cc} A - e^{j\theta}I & B \\ C & D \end{array}\right]$ *has full column rank for all* $\theta \in [0, 2\pi)$, *and* D *has full column rank.*

Then $E(z)$ can be factorized in the following form

$$E(z) = N(z)M(z)^{-1} \tag{2}$$

where $M(z)$ and $N(z)$ are rational transfer matrices with $N(z)$ inner, and

$$\left[\begin{array}{c} M(z) \\ N(z) \end{array}\right] := \left[\begin{array}{c|c} A + BF & BW^{-\frac{1}{2}} \\ \hline F & W^{-\frac{1}{2}} \\ C + DF & DW^{-\frac{1}{2}} \end{array}\right] \tag{3}$$

$$W = W^* = D^*D + B^*XB \tag{4}$$

$$F = -W^{-1}(B^*XA + D^*C), \tag{5}$$

and X is the unique stabilizing solution of the following Riccati equation

$$A^*XA - X + C^*C - (A^*XB + C^*D)W^{-1}(B^*XA + D^*C) = 0. \tag{6}$$

Theorem 1: Suppose that $E(z)$ with a minimal realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ satisfies Assumption A1). Let $M(z)$, $N(z)$, F, W and X be defined as in (2)-(6), respectively. Then the state space realization for the synthesis filter $R_0(z) = [E^{\sim}(z)E(z)]^{-1}E^{\sim}(z)$ corresponding to the dual frame is given by $R_0(z) = R_{0c}(z) + R_{0ac}(z)$, where $R_{0c}(z)$ and $R_{0ac}(z)$ are respectively the causal stable and anti-causal stable part of $R_0(z)$ given below

$$R_{0c}(z) = \left[\begin{array}{c|c} A + BF & (A+BF)Y(C+DF)^* + BW^{-1}D^* \\ \hline F & W^{-1}D^* + FY(C+DF)^* \end{array}\right] \tag{7}$$

$$R_{0ac}(z) = \left[\begin{array}{c|c} (A+BF)^* & (C+DF)^* \\ \hline W^{-1}B^* + FY(A+BF)^* & 0 \end{array}\right]_{ac}, \tag{8}$$

and Y is the solution of the following Lyapunov equation

$$Y-(A+BF)Y(A+BF)^*=BW^{-1}B^*. \tag{9}$$

III. Optimal design to known noises

The blocking noise $\mathbf{n}(z)$ in Fig. 1 is assumed to be a wide sense stationary (WSS) signal with the PSD $S_{nn}(z)$. Recall that the power norm of $\mathbf{n}(z)$ is defined as $\|\mathbf{n}\|_{\mathcal{P}}=\sqrt{\frac{1}{2\pi}\int_0^{2\pi}Tr\{S_{nn}(e^{j\omega})\}d\omega}$. Then the PSD of $\mathbf{e}(z)=\hat{X}(z)-X(z)$ is given by $S_{ee}(z)=R(z)S_{nn}(z)R^{\sim}(z)$. The optimization objective function is defined as the power norm of $\mathbf{e}(z)$

$$J=\|\mathbf{e}\|_{\mathcal{P}}=\sqrt{\frac{1}{2\pi}\int_0^{2\pi}Tr\{R(e^{j\omega})S_{nn}(e^{j\omega})R^*(e^{j\omega})\}d\omega}.$$

For a given analysis FB with polyphase representation $E(z)$ satisfying PR condition, it is shown in [2] that the optimal synthesis FB can be written as

$$R(z)=[E^{\sim}(z)S_{nn}^{-1}(z)E(z)]^{-1}E^{\sim}(z)S_{nn}^{-1}(z).$$

However, even for the simplest case of white noise where $S_{nn}(z)=I$, no numerically efficient algorithms exist for the general $E(z)$ [1]. In this section, we provide the direct state space computational formulae for the design of optimal synthesis FB $R(z)$.

Assume the state-space realization of $E(z)$ and $S_{nn}^{-\frac{1}{2}}(z)$ are given as follows

$$E(z)=\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right] \text{ and } S_{nn}^{-\frac{1}{2}}(z)=\left[\begin{array}{c|c}A_2 & B_2\\ \hline C_2 & D_2\end{array}\right].$$

Then the state-space realization of $E(z)S_{nn}^{-\frac{1}{2}}(z)$ is given by

$$E(z)S_{nn}^{-\frac{1}{2}}(z)=:\left[\begin{array}{c|c}\bar{A} & \bar{B}\\ \hline \bar{C} & \bar{D}\end{array}\right]=\left[\begin{array}{cc|c}A & BC_2 & BD_2\\ 0 & A_2 & D_2\\ \hline \multicolumn{2}{c|}{C_2} & DD_2\end{array}\right]. \tag{10}$$

Define $\bar{F}=-\bar{W}^{-1}(\bar{B}^*\bar{X}\bar{A}+\bar{D}^*\bar{C})$, where $\bar{X}$ is the solution of the following Riccati equation

$$\bar{A}^*\bar{X}\bar{A}-\bar{X}+\bar{C}^*\bar{C}-(\bar{A}^*\bar{X}\bar{B}+\bar{C}^*\bar{D})\bar{W}^{-1}(\bar{B}^*\bar{X}\bar{A}+\bar{D}^*\bar{C})=0,$$

and $\bar{W}=\bar{D}^*\bar{D}+\bar{B}^*\bar{X}\bar{B}$. Let $\bar{Y}$ be the solution of the following Lyapunov equation

$$\bar{Y}-(\bar{A}+\bar{B}\bar{F})\bar{Y}(\bar{A}+\bar{B}\bar{F})^*=\bar{B}\bar{W}^{-1}\bar{B}^*.$$

Theorem 2: Suppose that the state space model $E(z)S_{nn}^{-\frac{1}{2}}(z)$ is given by (10), and the Assumption A1) is satisfied. Let $\bar{W},\bar{F},\bar{Y}$ be defined as above. Then the optimal synthesis FB minimizing J_1 is given by $R(z)=R_c(z)+R_{ac}(z)$, where

$$R_{0c}(z)=\left[\begin{array}{c|c}\bar{A}+\bar{B}\bar{F} & (\bar{A}+\bar{B}\bar{F})\bar{Y}(\bar{C}+\bar{D}\bar{F})^*+\bar{B}\bar{W}^{-1}\bar{D}^*\\ \hline \bar{F} & \bar{W}^{-1}\bar{D}^*+\bar{F}\bar{Y}(\bar{C}+\bar{D}\bar{F})^*\end{array}\right]$$

$$R_{0ac}(z)=\left[\begin{array}{c|c}(\bar{A}+\bar{B}\bar{F})^* & (\bar{C}+\bar{D}\bar{F})^*\\ \hline \bar{W}^{-1}\bar{B}^*+\bar{F}\bar{Y}(\bar{A}+\bar{B}\bar{F})^* & 0\end{array}\right]_{ac}.$$

Corollary 1: Given an analysis filter bank with polyphase representation $E(z)$ satisfying A1), assume the PSD matrix of the noise $\mathbf{n}(z)$ is $S_{nn}(e^{j\omega})=S$ with S being a positive definite constant matrix. Then the synthesis FB minimizing J_1 is given by $R(z)=R_c(z)+R_{ac}(z)$, where

$$R_{0c}(z)=\left[\begin{array}{c|c}A+BF & (A+BF)Y(C+DF)^*+BW^{-1}D^*\\ \hline F & W^{-1}D^*+FY(C+DF)^*\end{array}\right]$$

$$R_{0ac}(z)=\left[\begin{array}{c|c}(A+BF)^* & (C+DF)^*\\ \hline W^{-1}B^*+FY(A+BF)^* & 0\end{array}\right]_{ac}.$$

IV. Worst-case design to unknown noises

In some applications, it is not realistic to obtain the PSD matrix of the noises. The method in previous section is therefore not applicable. In this section, we present the worst-case design method for oversampled PR FBs using frame theory. It will be shown that the solution is simpler than the well-known H_∞ filtering design since causality is not required. Surely causality is an important requirement for implementation. But there are plenty of applications where causality is not necessary, for instance, image processing or block data processing [3], [8]. Besides, our results on non-causal (but stable) FBs present a systematic interpretation of the noise reduction from frame theory.

Assume that no information on the PSD of $\mathbf{n}(z)$ is available except that its power norm is bounded. In such case, we try to minimize the power induced norm

$$J_2=\sup_{\|n\|_{\mathcal{P}}}\frac{\|\mathbf{e}\|_{\mathcal{P}}}{\|\mathbf{n}\|_{\mathcal{P}}}=\|R(z)\|_\infty:=\sup_{w\in[0,2\pi)}\bar{\sigma}\left(R(e^{j\omega})\right)$$

where $\bar{\sigma}(\cdot)$ denotes the largest singular value. Therefore the optimization problem is to find $R(z)$ from the set of all synthesis filter banks (causal and/or noncausal) providing PR such that $\|R(z)\|_\infty$ is minimized. For more details about the norms of random signals, please refer to [14]

Theorem 3: Suppose that $E(z)$ with a minimal realization $\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]$ satisfies assumption A1). Let $M(z)$, $N(z)$, F, W and X be defined as in (2)-(6), respectively. Then the synthesis FB that minimizes J_2 is given by $R_0(z)=R_{0c}(z)+R_{0ac}(z)$, where $R_{0c}(z)$ and $R_{0ac}(z)$ are given by (7) and (8) respectively.

Remarks on the proof. Due to space limitation, we can not give the proof here. The main tool is the two block matrix completion. Theorem 3 shows that the dual synthesis FB in Theorem 1 is also optimal in the sense of worst case design (or induced norm of power norm of signals). The result is simpler than the well-known $\mathcal{H}_\infty$ filtering [11], [12], [13] because of non-causality is allowed in our setup.

V. Numerical examples

This section presents an example to illustrate our design methods.

Consider an oversampled FB with IIR analysis filters with $N=3$ and $M=2$, where $H_0(z),H_1(z),H_2(z)$

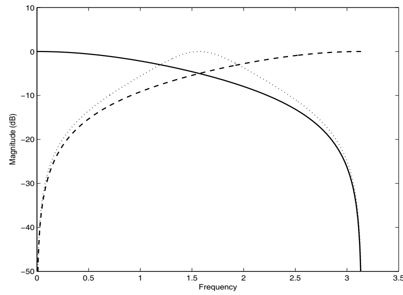

Fig. 2. Frequency responses of the analysis filters. $H_0(z)$: solid; $H_1(z)$: dotted; $H_2(z)$: dashed

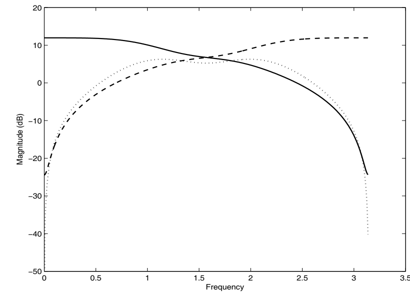

Fig. 3. Frequency responses of the optimal synthesis filters for correlated noise . $F_0(z)$: solid; $F_1(z)$: dotted; $F_2(z)$: dashed.

are low-pass, band-pass and high-pass butterworth filters given respectively by $H_0(z) = \frac{0.4046z+0.4046}{z-0.1908}$, $H_1(z) = \frac{0.2836z^2-0.2836}{z^2+0.4327}$, and $H_2(z) = H_0(-z)$. The frequency responses of $H_1(z)$, $H_2(z)$ and $H_i(z)$ are shown in Fig. 2. Let $S = \begin{bmatrix} 2 & 0.2 & 0.005 \\ 0.2 & 2 & 0.2 \\ 0.005 & 0.2 & 2 \end{bmatrix}$, the optimal synthesis filters are given by

$$F_0(z) = \frac{\begin{array}{c}(2.024z^{10} + 2.459z^9 + 10.55z^8 + 19.55z^7 \\ +44.96z^6 + 37.11z^5 + 8.977z^4 + 11.51z^3 \\ +1.637z^2 - 0.4691z - 0.0736)\end{array}}{z^{10} + 5.251z^8 + 22.41z^6 + 5.251z^4 + z^2}$$

$$F_1(z) = \frac{0.7599z^8 - 21.34z^6 + 12.29z^4 + 8.618z^2 - 0.329}{z^9 + 5.251z^7 + 22.41z^5 + 5.251z^3 + z}$$

$$F_2(z) = -F_0(-z),$$

and the frequency responses are shown in Fig. 3. The synthesis filters by worst case design in Theorem 3 are given by

$$F_0(z) = \frac{\begin{array}{c}(1.038z^{10} + 1.115z^9 + 4.625z^8 + 8.863z^7 \\ +21.44z^6 + 16.83z^5 + 3.876z^4 + 5.221z^3 \\ +0.8681z^2 - 0.2127z - 0.0378)\end{array}}{z^{10} + 4.493z^8 + 20.83z^6 + 4.493z^4 + z^2}$$

$$F_1(z) = \frac{0.3445z^8 - 9.676z^6 + 5.573z^4 + 3.907z^2 - 0.149}{z^9 + 4.493z^7 + 20.83z^5 + 4.493z^3 + z}$$

$$F_2(z) = -F_0(-z),$$

and the frequency responses are shown in Fig. 4.

As shown in the example, all the synthesis filters are easy to obtain. The computation can be carried out easily using MATLAB.

VI. Concluding remarks

The noise reduction for PR oversampled FBs is studied in this paper. For the noise with known PSD, the optimal solution minimizing the power norm of the error is provided. For the noise with unknown PSD, the worst case design minimizing the induced power norm is given. For both cases, explicit formulae involving only algebraic Riccati equation and matrix manipulations are presented for the general (IIR or FIR) oversampled PR FBs.

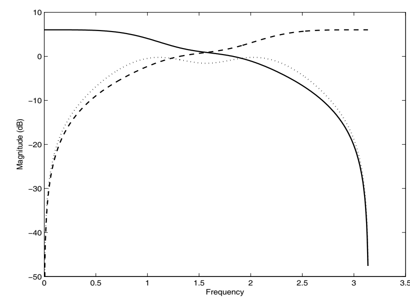

Fig. 4. Frequency responses of the synthesis filters by worst case method. $F_0(z)$: solid; $F_1(z)$: dotted; $F_2(z)$: dashed.

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