SUBBAND AFFINE PROJECTION ALGORITHM USING VARIABLE STEP SIZE

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ABSTRACT

In applications with highly correlated inputs or long length of filter, adaptive filters suffer from slow convergence and large steady-state error. Affine projection algorithms based on the subband structure and step size controlling are good solutions for these problems. In this paper, we propose a new subband affine projection algorithm with variable step size. Experimental results on highly correlated inputs produce faster convergence, lower misadjustment error, and smaller complexity than conventional methods based on the fullband structure.

1. INTRODUCTION

The LMS (Least Mean Square)-type algorithms are most popular and widely used because of its simplicity and robustness. However, LMS adaptive filter suffers from slow convergence when the input signal is highly correlated [1]. To overcome this problem, the affine projection (AP) algorithm has been proposed. The improved performance of the AP algorithm is characterized by an updating-projection scheme of an adaptive filter on a P-dimensional data-related subspace [2]. By increasing the projection order P of the AP algorithm, the convergence rate accelerates. However, the increased projection order requires more computational complexity for updating the weights of the adaptive filter [3]. This complexity depends on the matrix inversion, and the matrix size increases with projection order of the AP adaptive filter. An alternative technique for improving the convergence rate of adaptive filter is subband adaptive filtering (SAF). In the SAF, the convergence rate and steady-state error are improved by reducing the dynamic spectral range of input signal. In [4], Pradhan suggested an innovative subband structure using the polyphase decomposition, noble identity, and maximal decimation. It achieves more rapid convergence rate without the aliasing problems and additional computations. The convergence analysis of Pradhan's subband structure [4] was performed in frequency domain [5]. The analysis of [5] showed that that Pradhan's subband adaptive filter is always stable and that its steady-state error is reduced. Recently, for fast convergence and efficient implementation, there has been increasing interest in combining advantages of the AP and SAF [6][7]. In those algorithms, fast versions of the conventional AP are used for reducing computational complexity, which is due to the matrix inversion. These variants have used the approximation schemes such as the iterative methods or the sparse weight-updating. Owing to the approximations, however, the deterioration in convergence is unavoidable. In the LMS-type algorithms and AP algorithm, the convergence rate and steady-state error are governed by the step size μ. To find the moderate trade-off between the fast convergence and low steady-state error, the step size needs to be controlled. For controlling of the step size in LMS-type algorithms, various schemes based on scalar have been proposed [8][9]. Recently, Shin [10] proposed a variable step size AP (VS-AP) with vector quantity. However, the maximum convergence rate of the VS-AP is limited in that of the conventional AP.

In this paper, we suggest a new subband affine projection (SAP) algorithm based on subband structure of [4], and develop an appropriate variable step size technique for the proposed SAP. In the proposed variable step size SAP (VS-SAP), the convergence rate and the steady-state error performance are improved by combining SAP with step size controlling. Moreover, the complexity of VS-SAP is reduced by applying the polyphase decomposition and noble identity to the maximally decimated adaptive filter. The convergence properties and complexity of the VS-SAP are superior to those of fullband VS-AP.

2. SUBBAND AFFINE PROJECTION ALGORITHM

Subband adaptive system identification model with the polyphase decomposition and noble identity is shown in Fig. 1 [4]. $\mathbf{s}^*$ is an unknown system that we wish to estimate. $x_i(n)$ and $d_i(n)$ denote an input and a desired signal that partitioned by the analysis filter $\mathbf{h}_i$. $\mathbf{s}_i(n)$ is $N_s \times 1$ vector as a polyphase component of an adaptive filter, $\mathbf{x}_{ij}(n)$ is an input signal vector of $\mathbf{s}_i(n)$. It is well known that AP algorithm is the undetermined optimization problem. Generally, Lagrangian theory is used for solving this optimization problem with equality constraints[1][11]. Based on the principle of minimum disturbance [1], we formulate the criterion for the M-subband AP filters as one of optimization subject to multiple constraints, as follows:

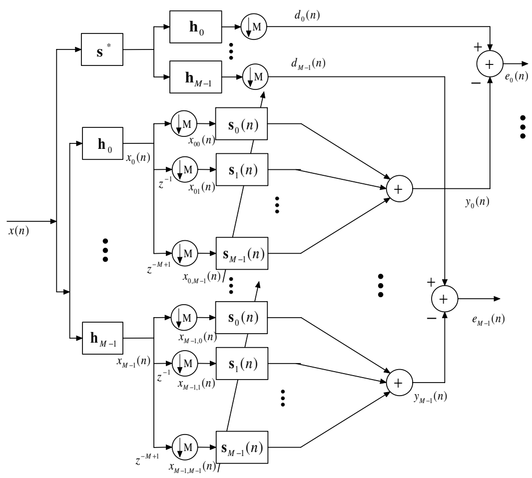

Fig. 1. System identification model for the subband adaptive filter for the M-subband case [4]

$$\mathit{Minimize} \quad \sum_{i=0}^{M-1} f[\hat{\mathbf{s}}_i(n)] = \|\mathbf{s}_0(n+1) - \mathbf{s}_0(n)\|^2 + \cdots + \|\mathbf{s}_{M-1}(n+1) - \mathbf{s}_{M-1}(n)\|^2 \tag{1}$$

subject to the constraints

$$\mathbf{d}_i(n) = \sum_{j=0}^{M-1} \mathbf{X}_{ij}^T(n)\mathbf{s}_j(n+1) \quad for\ i = 0, 1, \cdots, M-1 \tag{2}$$

where,

$$\mathbf{d}_i(n) = [d_i(n)\ \ d_i(n-1)\ \ \ldots\ \ d_i(n-P_s+1)]^T, \tag{3}$$

$$\mathbf{X}_{ij}(n) = [\mathbf{x}_{ij}(n)\ \mathbf{x}_{ij}(n-1)\ \ \cdots\ \ \mathbf{x}_{ij}(n-P_s+1)], \tag{4}$$

$$\mathbf{x}_{ij}(n) = [x_{ij}(n)\ \ x_{ij}(n-1)\ \ldots\ x_{ij}(n-N_s+1)]^T. \tag{5}$$

The parameter P_s is smaller than the dimension N_s of the input data space or, equivalently, the polyphase decomposed weight space in maximally decimated subband adaptive filter structure. That is, N_s and P_s are the length of the adaptive filter and the projection order in each subband, respectively. Applying the method of Lagrange multipliers with multiple constraints, we combine (1) and (2) to form the following cost function for the AP algorithm in the M-subband structure of Fig. 1 as

$$J(n) = \sum_{i=0}^{M-1}\left(f[\hat{\mathbf{s}}_i(n)] + [\mathbf{d}_i(n) - \sum_{j=0}^{M-1} \mathbf{X}_{ij}^T(n)\mathbf{s}_j(n+1)]^T \boldsymbol{\lambda}_i \right) \tag{6}$$

where, $\boldsymbol{\lambda}_i$ is the Lagrange multiplier vector. In (6), the cost function is quadratic, and also, it is convex since its Hassian matix is positive definite [1][11]. Therefore, the proposed cost function has a global minimum solution. Solving (6) for $\boldsymbol{\lambda}_i$ that minimizes the quadratic cost function with respect to $\mathbf{s}_i(n+1)$, this solution is obtained as

$$[\boldsymbol{\lambda}_0^T\ \cdots\ \boldsymbol{\lambda}_{M-1}^T]^T = 2[\mathbf{A}^T(n)\mathbf{A}(n)]^{-1}[\mathbf{e}_0^T(n)\ \cdots\ \mathbf{e}_{M-1}^T(n)]^T \tag{7}$$

$$\mathbf{A}(n) = \begin{bmatrix} \mathbf{X}_{00}(n) & \mathbf{X}_{10}(n) & \ldots & \mathbf{X}_{(M-1)0}(n) \\ \mathbf{X}_{01}(n) & \mathbf{X}_{11}(n) & \ldots & \mathbf{X}_{(M-1)1}(n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{0(M-1)}(n) & \mathbf{X}_{1(M-1)}(n) & \ldots & \mathbf{X}_{(M-1)(M-1)}(n) \end{bmatrix} \tag{8}$$

$$\mathbf{e}_i(n) = \mathbf{d}_i(n) - \sum_{j=0}^{M-1} \mathbf{X}_{ij}^T(n)\mathbf{s}_j(n+1) \tag{9}$$

$\mathbf{A}(n)$ is $MN_s \times MP_s$ matrix. Let $\boldsymbol{\Phi}(n)$ be $\mathbf{A}^T(n)\mathbf{A}(n)$. It can be represented as

$$\boldsymbol{\Phi}(n) = \begin{bmatrix} \mathbf{G}_0(n) & & \mathbf{C}(n) \\ & \ddots & \\ \mathbf{C}^T(n) & & \mathbf{G}_{(M-1)}(n) \end{bmatrix} \tag{10}$$

In (10), $\boldsymbol{\Phi}(n)$ is $MP_s \times MP_s$ matrix, and the elements of off-diagonal component $\mathbf{C}(n)$ consist of sample cross-correlations between the signals for the ith and jth subband ($i \neq j$), whereas, the elements of diagonal component $\mathbf{G}_i(n)$ consist of sample auto-correlations. Assuming that the input signal is wide-sense stationary and ergodic, the cross-correlation at zero lag, $\gamma_{\mathbf{x}_{00}\mathbf{x}_{10}+\mathbf{x}_{01}\mathbf{x}_{11}}(k,l)$, can be expressed as

$$\gamma_{\mathbf{x}_{00}\mathbf{x}_{10}+\mathbf{x}_{01}\mathbf{x}_{11}}(0) = \left[\mathbf{x}_{00}^T(k)\mathbf{x}_{10}(k) + \mathbf{x}_{01}^T(k)\mathbf{x}_{11}(k)\right]/N_s. \tag{11}$$

For analytical simplicity, we further assume that the input signal is white giving us a flat spectrum. From these assumptions, $E\{\mathbf{x}_{00}^T\mathbf{x}_{00} + \mathbf{x}_{01}^T\mathbf{x}_{01}\} = \sigma_{\mathbf{x}_0}^2$ ($\sigma_{\mathbf{x}_0}^2$ is the variance of subband signal, $\mathbf{h}_0^T\mathbf{x}$) and $E\{\mathbf{x}_{00}^T\mathbf{x}_{10} + \mathbf{x}_{01}^T\mathbf{x}_{11}\} = 0$. For colored inputs, $E\{\mathbf{x}_{00}^T\mathbf{x}_{10} + \mathbf{x}_{01}^T\mathbf{x}_{11}\} \neq 0$. However, if the frequency responses of the analysis filters do not overlap significantly, it is always true that $E\{\mathbf{x}_{00}^T\mathbf{x}_{10} + \mathbf{x}_{01}^T\mathbf{x}_{11}\} \ll E\{\mathbf{x}_{00}^T\mathbf{x}_{00} + \mathbf{x}_{01}^T\mathbf{x}_{01}\}$. That is, the elements of off-diagonal component $\mathbf{C}(n)$ are remarkably small compared with the elements of diagonal component $\mathbf{G}_i(n)$. Therefore, we can consider $\mathbf{C}(n) \approx \mathbf{0}$. From (10), we can easily get $\boldsymbol{\Phi}^{-1}(n)$ following as

$$\boldsymbol{\Phi}^{-1}(n) = \begin{bmatrix} \mathbf{G}_0^{-1}(n) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_1^{-1}(n) & & \vdots \\ \vdots & & \ddots & \mathbf{0} \\ \mathbf{0} & \ldots & \mathbf{0} & \mathbf{G}_{(M-1)}^{-1}(n) \end{bmatrix} \tag{12}$$

With the above approximations, the recursive relation for updating the coefficients of the subband adaptive filters can be obtained as

$$\mathbf{S}(n+1) = \mathbf{S}(n) + \mu\mathbf{A}(n)\boldsymbol{\Phi}^{-1}(n)\mathbf{E}(n), \tag{13}$$

$$\mathbf{S}(n) = \left[\mathbf{s}_0^T(n)\ \ \mathbf{s}_1^T(n)\ \ \cdots\ \ \mathbf{s}_{M-1}^T(n)\right]^T,$$

$$\mathbf{E}(n) = \left[\mathbf{e}_0^T(n)\ \ \mathbf{e}_1^T(n)\ \ \cdots\ \ \mathbf{e}_{M-1}^T(n)\right]^T,$$

where μ is step size. The proposed SAP shown in (13) has smaller complexity than that of the fullband AP. In (4), the size of matrix depends on a spectral magnitude range of input signal and it can be decreased by the reduced bandwidth in each subband. When the size of the data matrix in the fullband is $N \times P$, in the subband, it becomes $N_s \times P_s = (N/M) \times (P/M)$. Therefore, the computational complexity for weight-updating is reduced in the proposed SAP.

3. VARIABLE STEP SIZE TECHNIQUE FOR SAP

To control step size in the SAP, we have modified the results in [10]. We examine the mean square deviation (MSD) with an weight error vector that defined as $\tilde{\mathbf{S}}(n) = \mathbf{S}^* - \mathbf{S}(n)$. From the update recursion (13), the MSD satisfies

$$\begin{aligned} E\{\|\tilde{\mathbf{S}}(n+1)\|^2\} &- E\{\|\tilde{\mathbf{S}}(n)\|^2\} \\ &= \mu^2 E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)\} \\ &- 2\mu E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}(n)\} \\ &\equiv -\triangle(\mu) \end{aligned} \tag{14}$$

$$\begin{aligned} \triangle(\mu) &= -\mu^2 E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)\} \\ &+ 2\mu E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}(n)\} \end{aligned} \tag{15}$$

Since (15) is a quadratic concave function, the MSD can be minimized by the maximum value of μ. From this result, the optimum step size at iteration n is given by

$$\mu^* = \frac{E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}(n)\}}{E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)\}}. \tag{16}$$

The error vector in (13) that includes measurement noise can be rewritten as

$$\begin{aligned} \mathbf{E}(n) &= \mathbf{A}^T(n)\mathbf{S}^* - \mathbf{A}^T(n)\mathbf{S}(n) + \mathbf{z}(n) \\ &= \mathbf{A}^T(n)\tilde{\mathbf{S}}(n) + \mathbf{z}(n) \end{aligned} \tag{17}$$

where $\mathbf{z}(n)$ is white Gaussian measurement noise with a variance σ_z^2. Assuming the input data matrix $\mathbf{A}(n)$ is statistically independent upon $\mathbf{z}(n)$ and neglecting the dependency of $\tilde{\mathbf{S}}(n)$ on past noise, $E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)\}$ is rewritten by using the result of [10]

$$\begin{aligned} &E\{\mathbf{E}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)\} \\ &= E\{[\tilde{\mathbf{S}}^T(n)\mathbf{A}(n) + \mathbf{z}^T(n)]\mathbf{\Phi}^{-1}(n)[\mathbf{A}^T(n)\tilde{\mathbf{S}}(n) + \mathbf{z}(n)]\} \\ &= E\{\tilde{\mathbf{S}}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}^T(n) + \mathbf{z}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{z}(n)\} \\ &= E\{\tilde{\mathbf{S}}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}^T(n)\} \\ &\quad + E\{\mathbf{z}^T(n)\mathbf{\Phi}^{-1}(n)\mathbf{z}(n)\} \\ &= E\{\|\tilde{\mathbf{S}}^T(n)\|_\Sigma^2\} + \sigma_z^2 Tr[E\{\mathbf{\Phi}^{-1}(n)\}] \end{aligned} \tag{18}$$

where $E\{\|\tilde{\mathbf{S}}(n)\|_\Sigma^2\} = E\{\tilde{\mathbf{S}}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}^T(n)\}$. From (18), the optimum step size is approximated as [10]

$$\mu^*(n) = \frac{E\{\|\tilde{\mathbf{S}}(n)\|_\Sigma^2\}}{E\{\|\tilde{\mathbf{S}}(n)\|_\Sigma^2\} + \sigma_z^2 Tr[E\{\mathbf{\Phi}^{-1}(n)\}]}, \tag{19}$$

Let $\mathbf{p}(n) \equiv \mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}^T(n)$.

$$\|\mathbf{p}(n)\|^2 = \mathbf{S}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}(n) \tag{20}$$

From (10),

$$E\{\|\mathbf{p}(n)\|^2\} \approx E\{\mathbf{S}^T(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{A}^T(n)\tilde{\mathbf{S}}(n)\} \tag{21}$$

Instead of expectation, using $\|\hat{\mathbf{p}}(n)\|^2$ estimated with the time averaging, (19) is represented by

$$\mu^*(n) = \frac{\|\hat{\mathbf{p}}(n)\|^2}{\|\hat{\mathbf{p}}(n)\|^2 + \sigma_z^2 Tr[E\{\mathbf{\Phi}^{-1}(n)\}]} \tag{22}$$

where $\hat{\mathbf{p}}(n) = \alpha\hat{\mathbf{p}}(n-1) + (1-\alpha)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n)$ [10]. Combining (13) with (22), we obtain the variable step size subband affine projection (VS-SAP) algorithm as

$$\mathbf{S}(n+1) = \mathbf{S}(n) + \mu(n)\mathbf{A}(n)\mathbf{\Phi}^{-1}(n)\mathbf{E}(n) \tag{23}$$

$$\mu(n) = \mu_{max}\frac{\|\hat{\mathbf{p}}(n)\|^2}{\|\hat{\mathbf{p}}(n)\|^2 + C} \tag{24}$$

where μ_{max} is set to the value that provides fastest convergence speed for the initial convergence stage. C is a positive constant related to $\sigma_z^2 Tr[E\{\mathbf{\Phi}^{-1}(n)\}]$ and it can be approximated as $(MP_s)/SNR$ [10]. For the stability of VS-SAP,

$$0 < \mu(n) \le \mu_{max} < 2 \tag{25}$$

In (22)–(24), the computational complexity for finding the optimum step size of VS-SAP is reduced by applying the polyphase decomposition and noble identity to adaptive filter. And also, the complexity for its weight-updating is reduced. Therefore, total complexity of the proposed method is considerably smaller than that of the conventional method.

4. SIMULATIONS

In order to evaluate the convergence rate and steady-state error performance of the VS-SAP, we perform computer simulations in system identification scenario. The lengths of analysis filters are increased with M so that the ratio of the transition band to the passband is maintained nearly the same for all values of M. In particular, we use the cosine modulated filter banks [12] for analysis and synthesis filters. The prototype filters' lengths are 32 and 64 for M = 2 and 4, respectively. The input signals are obtained by filtering a zero-mean white Gaussian random sequence through the IIR (infinite impulse response) filter, $\mathbf{H}_{IIR} = 1/(1 + 0.999z^{-1} + 0.99z^{-2} + 0.995z^{-3} + 0.99z^{-4})$. Fig. 2 shows the misalignment curves of the fullband AP [2], variable step size AP(VS-AP) [10],

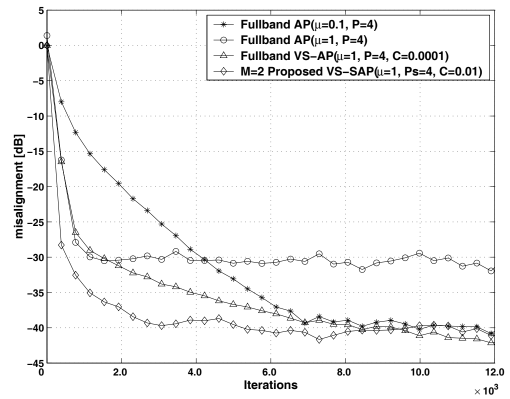

Fig. 2. Misalignment of the estimated system

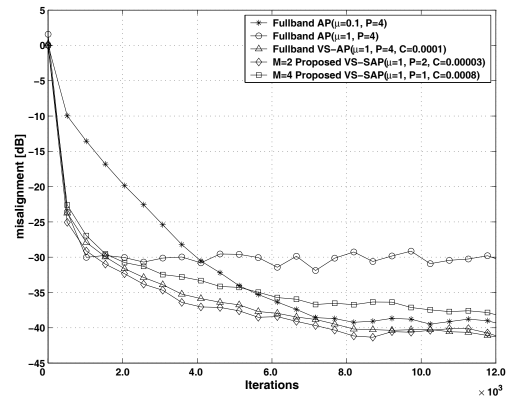

Fig. 3. Misalignment of the VS-SAP with the reduced projection order

and the proposed $M = 2$ VS-SAP. The unknown system to be identified is length $N = 32$ FIR (finite impulse response) filter with coefficients chosen randomly. The maximum step size is set to 1 ($\mu_{max} = 1$) for VS-AP and VS-SAP. And the fixed step size for the conventional AP is $\mu = 0.1$. The projection order in each algorithm is $P = P_s = 4$. We choose $\alpha = 0.99$, $C = 1.0 \times 10^{-4}$, and $\alpha = 0.99$, $C = 1.0 \times 10^{-2}$ for VS-AP and VS-SAP respectively. In Fig. 2, the performances of the VS-SAP are superior to those of other algorithms in convergence rate and misalignment. The misalignment curves obtained by using the reduced projection order $P_s = P/M$ are shown in Fig. 3. The projection orders of VS-AP and the fullband AP are all $P = 4$, whereas the proposed VS-SAP uses the reduced projection order, $P_s = P/M$. In Fig. 3, the convergence property of the proposed VS-SAP is similar to that of VS-AP. However, we can expect that the complexity of the VS-SAP is gradually reduced by increasing M.

5. CONCLUSIONS

We proposed the subband affine projection algorithm with variable step size. The proposed VS-SAP produces better performance when compared to conventional methods. Moreover, the proposed SAP can be simplified by partitioning into the same number of subbands as the projection order. These results lead us to the conclusion that the proposed VS-SAP is one of the best solutions in the adaptive signal processing application with highly correlated input signal. Several simulation results were included to verify the theoretical results and to show the improved performance of the proposed methods.

6. REFERENCES

[1] S. Haykin, *Adaptive filter theory*, 4th Ed., NJ:Prentice-Hall, 2002.

[2] K. Ozeki and T. Umeda, ”An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties,” *Electron, Comm. Jap.*, vol. 67-A, no. 5, pp. 19-27, 1984.

[3] S. G. Sakaran and A. A. Beex, ”Convergence behavior of the affine projection algorithm,” *IEEE Trans. Signal Proc.*, vol. 48, no. 4, pp. 1086-1097, April 2000.

[4] S. S. Pradhan and V. U. Reddy, ”A new approach to subband adaptive filtering,” *IEEE Trans. Signal Proc.*, vol. 45, no. 3, pp. 655-664, Mar. 1999.

[5] S. Miyagi and H. Sakai, ”Convergence analysis of alias-free subband adaptive filters based on a frequency domain technique,” *IEEE Trans. Signal Proc.*, vol. 52, no. 1, pp. 79-89, Jan. 2004

[6] Q. G. Liu, B. Champagne and K. C. Ho, ”On the use of a modified fast affine projection algorithm in subbands for acoustic echo cancelation,” *IEEE Proc, Digital Signal Proc. Workshop*, pp. 354-357, Sep. 1996.

[7] E. Chau, H. Sheikhzadeh, and R. L. Berennan, ”Complexity reduction and regularization of a fast affine projection algorithm for oversampled subband adaptive filters,” *IEEE Proc. ICASSP 2004*, vol. 5, pp. V 109-112, May 2004.

[8] R. H. Kwong and E. W. Johnstone, ”A variable step size LMS algorithm,” *IEEE Trans. Signal Proc.*, vol. 40, no. 7, pp. 1633-1642, Jul. 1992.

[9] T. Aboulnasr and K. Mayyas, ”A robust variable step-size LMS-type algorithm: Analysis and simulation,” *IEEE Trans. Signal Proc.*, vol. 45, no. 3, pp. 631-639, Mar. 1997.

[10] H. C. Shin and A. H. Sayed, ”Variable step-size NLMS and affine projection algorithm,” *IEEE Signal Proc. Lett.*, vol. 11, no. 2, pp. 132-135, Feb. 2004.

[11] T. K. Moon and W. C. Stirling, *Mathematical methods and algorithms*, Englewood Cliffs, NJ: Prentice-Hall, 2000.

[12] P. P. Vaidyanathan, *Multirate System and Filter Banks*, Englewood Cliffs, NJ: Prentice-Hall, 1993.