SAR IMAGES IMPROVEMENTS BY USING THE S-METHOD

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ABSTRACT

Synthetic aperture radar processors are generally made with the stationary targets in mind, hence commonly used technique for the SAR signal analysis is a two-dimensional Fourier transform. Moving targets induce Doppler-shift and Doppler spread in the returned signal, producing blurred or smeared images. Standard techniques for these kinds of the problems are motion compensation and time-frequency analysis application. Both of them are computationally intensive. Here, we will present a numerically simple S-method based approach, already applied in the ISAR imaging. This approach improves readability of the SAR images what will be analytically proved and demonstrated on the simulated SAR setup.

1. INTRODUCTION

Synthetic aperture radar (SAR) involves the coherent summation of the signals received from a sequence of locations. The radar is carried on an aircraft or spacecraft platform moving at uniform speed V in a straight line and constant altitude H. The forward motion provides scanning in the along track direction called azimuth or cross range. The scanning configuration for side looking radar is given in the Fig.1. The radar beam is directed to the side (most commonly perpendicular to the flight path of the aircraft, as in the Fig.1) and down toward the surface. The beam is wide in the vertical direction and so intersects the surface in an oval with the long axis extended in the across track, called range direction. The radar transmits the pulses of electromagnetic energy and the return echoes are sampled in order to provide scanning in the (ground) range direction. The coordinate along the line of sight of the radar is termed slant range. Fig.1 shows the relation between slant range and ground range, where θ is the look angle. Slant range indicates the distance between the radar and the target.

When the target moves and the radar is static the process is known as inverse synthetic aperture radar (ISAR). Both, the SAR and the ISAR principle are well understood and simple when the conditions are ideal. They are usually considered separately in the literature. The oscillating and the rotating targets in the SAR cause a time-dependent Doppler frequencies or micro-Doppler shift. Thus, the two-dimensional (2D) Fourier transform of the radar signal will be blurred, hence the 2D Fourier transform is not the proper tool for observing the SAR signals in the case of the moving targets [1], [2], [3]. In this case motion compensation [1] or time-frequency representations [2] are usually used in order to obtain clear images. Both of them are computationally intensive. Here, we will propose a numerically simple S-method, already used in the case of the ISAR images [4], for obtaining clear SAR images of the moving targets. By using the S-method radar image can be produced starting with the already obtained radar image, by using the 2D Fourier transform, and then making an additional simple matrix calculations.

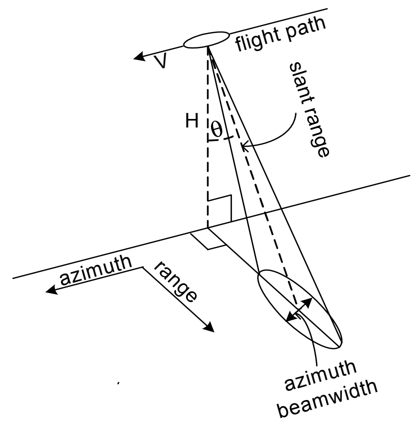

Fig. 1. Scanning configuration for SAR

2. SIGNAL MODEL

Continuous wave (CW) radar transmits signal in a form of coherent series of chirps [1].

$$v_p(t) = \begin{cases} \exp(j\pi B f_r t^2) & \text{for} \quad 0 \le t \le T_r \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where T_r is the repetition time and $f_r = 1/T_r$ is the repetition frequency. Although the signal has duration T_r, after pulse compression, it can behave like a pulse with duration equivalent to $1/B$, where B is frequency bandwidth of the transmitted radar pulse [1].

In one revisit the transmitted signal consists of M such chirps:

$$v(t) = \exp(-j\omega_0 t) \sum_{m=0}^{M-1} v_p(t - mT_r) \qquad (2)$$

where ω_0 is the radar operating frequency. Total signal duration is $T_c = MT_r$. It is called coherent integration time (CIT).

Signal of the form (2) is transmitted toward a target. If the target distance from the radar is d, then the received signal is delayed with the respect to the transmitted signal for $t_d = 2d/c$, where c is the propagation rate, equal to the speed of light. Thus, form of the received signal is

$$u(t) = \sigma \exp(j[-\omega_0(t - \frac{2d}{c})]) \sum_{m=0}^{M-1} v_p(t - \frac{2d}{c} - mT_r)$$

where σ is the reflection coefficient. The received signal is mixed (multiplied) with the complex-conjugate of the transmitted signal. If the constant distance on the transmitted signal is properly compensated only one component of the received signal can be considered. Since, the time shift $2d/c$ after range compensation is small the product of the components (1) for the different chirps is a sinusoid at a very high frequency, which can be filtered out. By substituting values for the signal components $v_p(t)$ and omitting the constant phase shift, the received signal can be modeled as:

$$q(m,t) = \sigma \exp(j\omega_0 \frac{2d}{c}) \exp(-j2\pi B f_r (t - mT_r) \frac{2d}{c}). \quad (3)$$

The ground range resolution of the radar is defined as a minimum range separation of the two points that can be distinguished by the system [5], it is equal to $R_g = c/(2B\sin\theta)$. The azimuth (cross range) resolution is given in the literature [5] as $R_c = \pi c/(\omega_0 T_c \omega_R)$, where ω_R is the equivalent target angular velocity.

SAR geometry is shown in the Fig.1. Define a Cartesian coordinate system in which x axis will be along the azimuth direction, y axis is the range direction, and z axis is the direction perpendicular to the xy plane. Imagine an aircraft with the radar, having velocity V, as point A and suppose that its position at the beginning of the observation time is $[x_A, y_{A,} z_A]$. Let B represents the point target inside the beam with the position $[x_B, y_B, z_B]$. The distance from the target to the aircraft is a function of time:

$$d(t) = \sqrt{(x_A + vt - x_B)^2 + (y_A - y_B)^2 + (z_A - z_B)^2} \quad (4)$$

Consider the rotating target case with a constant angular velocity ω_R. Let B represents center of the point target rotation with the position $[x_B, y_B, z_B]$. The target rotates in the xy plane. The target position will changes during the observation time t:

$$x(t) = x_B + A_r \cos(\omega_R t);\ y(t) = y_B + A_r \sin(\omega_R t). \quad (5)$$

The target is on the ground, $z_B = 0$, and the rotation radius is A_r. The distance from the aircraft to the rotating target will be function of time and can be obtain by substituting the constant coordinates x_B and y_B used in (4), in the case of motionless target, with the rotating target coordinates change in time as it is given in (5).

The other case studied in this paper will be the vibrating target, which will be described as the target changing the position z by the rule $z = A_r \sin(\omega_V t)$. A_r is the oscillation amplitude, and ω_V the oscillation angular frequency.

3. FOURIER TRANSFORM IN SAR

The two-dimensional Fourier transform of the received signal is:

$$Q(m', n') = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} q(m,n) \exp(-j[\frac{2\pi}{M} mm' + \frac{2\pi}{N} nn'])$$

where time is discretized $t - mT_r = nT_s$. The coordinate axes should be scaled with the resolution parameters. The periodogram

$$P(m', n') = |Q(m', n')|^2 \qquad (6)$$

represents an SAR image.

In order to analyze the cross range nonstationarities in the Fourier transform, consider a single point target and only the Doppler component part of the received signal, as it is usually done in the literature on ISAR and SAR,

$$e(t) = \sigma \exp(j \frac{2\omega_0}{c} d(t)). \qquad (7)$$

The Fourier transform of $e(t)$ is:

$$E(\omega) = \int_{-\infty}^{\infty} w(t) e(t) \exp(-j\omega t) dt, \qquad (8)$$

where $w(t)$ is a window defined by the considered time interval (CIT).

For the time-varying $d(t)$ a Taylor series expansion of $d(t)$ around $t = 0$ can be written as:

$$d(t) = d_0 + d'(0)t + ... = \sum_{n=0}^{\infty} \frac{1}{n!} d^{(n)}(0) t^n, \qquad (9)$$

where $d^{(n)}(0)$ is the n-th derivative of the distance at $t = 0$.

Fourier transform (FT) of (7) with respect to (9) is of the form [4]:

$$E(\omega) = \int_{-\infty}^{\infty} w(t) \exp(j \frac{2\omega_0}{c} \sum_{n=0}^{\infty} \frac{1}{n!} d^{(n)}(0) t^n - j\omega t) dt.$$

Now, by omitting the constant term d_0, and with $\Delta\omega_d = 2\omega_0 d'(0)/c$ we obtain:

$$E(\omega) = W(\omega - \Delta\omega_d) *_\omega FT \left[\exp(j \frac{2\omega_0}{c} \sum_{n=2}^{\infty} \frac{1}{n!} d^{(n)}(0) t^n) \right],$$

where $*_\omega$ denotes convolution in frequency. Thus, the Fourier transform is located at and around the Doppler shift $\omega = \Delta\omega_d$. It is spreaded by the factor

$$S_{FT}(\omega) = FT\left[\exp(j\frac{2\omega_0}{c}\sum_{n=2}^{\infty}\frac{1}{n!}d^{(n)}(0)t^n)\right]. \qquad (10)$$

This factor depends on the derivatives of the distance, starting from the second order (first order derivative of the Doppler shift), i.e., the spread factor depends on $s_{FT}(t) = \frac{1}{2}d^{''}(0)t^2 + \frac{1}{6}d'''(0)t^3 + \ldots$ It can significantly degrade the periodogram image (6).

4. S-METHOD BASED IMPROVEMENTS OF THE RADAR IMAGES

The S-method is defined [6] as:

$$SM(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} E(\omega+\theta)E^*(\omega-\theta)d\theta. \qquad (11)$$

By replacing $E(\omega)$ from (8) into (11) it follows:

$$\begin{aligned} SM(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} w(t_1)w^*(t_2) \\ \times \exp(j\frac{2\omega_0}{c}\sum_{n=0}^{\infty}\frac{1}{n!}d^{(n)}(0)(t_1^n - t_2^n)) \\ \times \exp(-j(\omega+\theta)t_1)\exp(j(\omega-\theta)t_2)dt_1dt_2d\theta. \end{aligned}$$

The part of the integrand depending on the θ is $\exp(-j\theta(t_1 + t_2))$. Integration over θ results in $2\pi\delta(t_1+t_2)$. Integration of a function $g(t_1)g(t_2)\delta(t_1+t_2)$ over t_1 results in the function $g(t)g(-t)$ for $t_1 = -t_2 = t$. Hence, the previous equation can be written [4] as:

$$SM(\omega)=W_e(\omega-\Delta\omega_d)*_\omega FT\left[\exp(j\frac{2\omega_0}{c}(\frac{1}{3!}d^{'''}(0)t^3+ ...))\right]$$

where similar calculations as in the Fourier transform case are done.

The S-method based image is located at the same position as the Fourier transform based image, $\omega = \Delta\omega_d$, but with the spreading term

$$S_{SM}(\omega) = FT\left[\exp(j\frac{2\omega_0}{c}(\frac{1}{3!}d^{'''}(0)t^3 + \ldots))\right].$$

Its exponent starts from the third derivative $d^{'''}(0)$, $s_{SM}(t) = \frac{1}{6}d'''(0)t^3 + \frac{1}{120}d^{(5)}(0)t^5 + \ldots$ In the Fourier transform based image the spreading terms started from the second derivative $d^{''}(0)$ (10). It means that in the S-method the points with linear Doppler changes:

$$\frac{2\omega_0}{c}d'(t) = \Delta\omega_d(t) = \Delta\omega_d + at$$

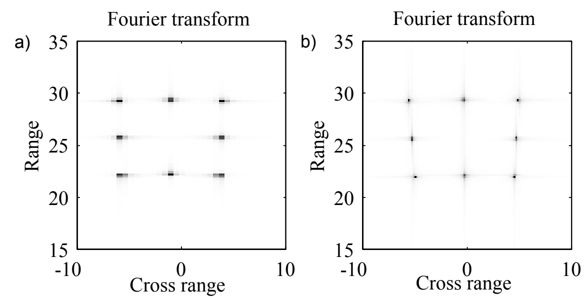

Fig. 2. 2D Fourier transform of radar signal, representing eight motionless point scatterers for aircraft velocity and altitude: a) $V = 200\,\mathrm{m/s}$, $H = 100\,\mathrm{m}$, b) $V = 500\,\mathrm{m/s}$, $H = 20\,\mathrm{m}$.

will be without any spread, since here $s_{SM}(t) = 0$. Note that $\Delta\omega_d$ without argument denotes the constant part of $\Delta\omega_d(t)$, i.e., $\Delta\omega_d = \Delta\omega_d(0)$ while $W_e(\omega)$ is the Fourier transform of the window $w(t/2)w^*(-t/2)$.

Discrete form of the (11) is:

$$\begin{aligned} SM(k) = \sum_{i=-L}^{L} E(k+i)E^*(k-i) = \\ |E(k)|^2 + 2\,\mathrm{real}\left\{\sum_{i=1}^{L} E(k+i)E^*(k-i)\right\}. \qquad (12) \end{aligned}$$

The first term in (12) is the periodogram, while the remaining terms in the sum are used to improve the periodogram concentration in the case of time-varying Doppler shift. The S-method improvement can be achieved starting with the already obtained radar image, with the additional simple matrix calculation according to (12). Thus, it can improve the image concentration in a numerically very simple and efficient way.

5. EXAMPLES

Example 1 : The target consisted of the eight point scatterers is considered in this example. The coordinate system origin is in the center of the target. Positions, in meters, are given by the vectors $\mathbf{X} = [-5, -5, 5, 5, 0, 0, -5, 5]$, $\mathbf{Y} = [-5, 5, -5, 5, -5, 5, 0, 0]$ and $\mathbf{Z} = [0, 0, 0, 0, 0, 0, 0, 0]$. The high resolution radar operating at the frequency $f_0 = 25\,\mathrm{GHz}$, bandwidth of the linear FM chirps $B = 750\,\mathrm{MHz}$, the pulse repetition time $T_r = 30\,\mu\mathrm{s}$, with $M = 256$ pulses in one revisit and the number of coherently integrated pulses in the synthetic aperture $N = 256$ is used. The platform velocity is $200\,\mathrm{m/s}$. Radar returns from all 8 point scatterers can be obtained by using the superposition principle, as the sum of the individual returns, where individual returns can be obtained by using (3). The 2D Fourier transform of the radar returns is given in the Fig.2 a), where the aircraft altitude is $H = 100\,\mathrm{m}$.

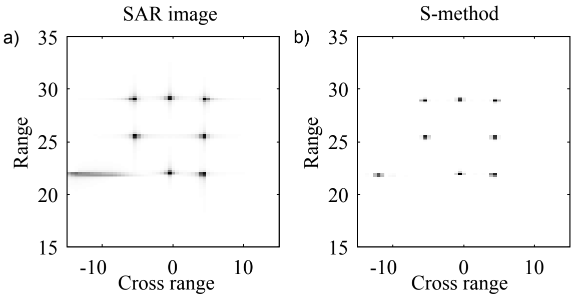

Fig. 3. Radar image in the range, cross range domain, for the case of one rotating and seven stationary target points, obtained by using: a) 2D Fourier transform, b) S-method

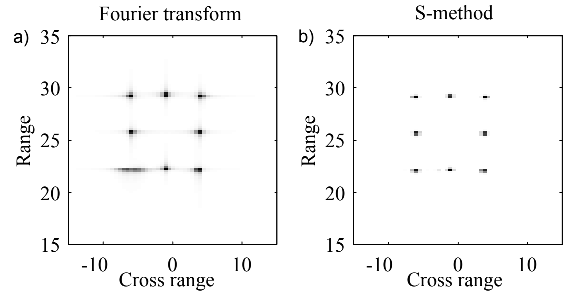

Fig. 4. Radar image in the range, cross range domain, for the case of one vibrating and seven stationary point targets, obtained by using: a) 2D Fourier transform, b) S-method

From the Fig.2 it can be seen that for the motionless targets case the 2D Fourier transform will detect all point scatterers and the positions of the maxima in 2D Fourier transform will correspond to the point scatterers positions represented in the range, cross range domain. The 2D Fourier transform of the same target for the platform velocity $500\,\mathrm{m/s}$, pulse repetition time $40\,\mu\mathrm{s}$, and the platform altitude $H = 20\,\mathrm{m}$ is shown in the Fig.2 b). For the high platform velocity the radar image is deformed.

Example 2 : The same parameters as in the Example 1 are used, except that the first target point is considered to rotate in the xy plane with the constant angular velocity $\omega_R = 2\pi f_R$, $f_R = 30\,\mathrm{Hz}$, and the radius $A_r = 0.1\,\mathrm{m}$. The 2D Fourier transform of the received radar signal is shown in the Fig.3 a), it shows that the 7 motionless target points are well concentrated in the 2D plane, while the position of the moving target point is blurred in the cross range. This is the reason for applying the S-method along this direction. The resulting radar image is shown in the Fig.3 b). Better concentration along the axis defined as cross-range is obvious. The S-method is calculated by using square root of the Hanning window and $L = 3$.

Example 3 : The same parameters as in the Example 2 are used, except that the first target point is considered to oscillate along the z axis. The oscillation amplitude is $A_r = 0.01\,\mathrm{m}$, the angular oscillation frequency $\omega_V = 2\pi f_V$, $f_V = 50\,\mathrm{Hz}$. The 2D Fourier transform will be blurred in the cross range direction (Fig.4 a)), caused by the first target point vibration. After applying the S-method along the cross range direction, Fig.4 b) will be obtained. The vibrating target is better concentrate, as it was expected. In this example $L = 2$ is used.

6. CONCLUSION

It was shown that the 2D Fourier transform is not a good tool for obtaining SAR images in the case when the fast moving or the vibrating target points exist. Namely, it will produce blurred or smeared image. A numerically simple S-method based approach is presented. Since it is obtained by making the slight correction on the results already obtained by the Fourier based algorithm, it is computationally efficient comparing to the time-frequency based and motion compensation methods available in the literature. Analytically and experimentally (by using simulated SAR setup) is proved that this approach can improve readability of the SAR images.

7. REFERENCES

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