ANALYSIS OF TIME-FREQUENCY TRANSIENT COMPONENTS USING PHASE CHIRPING OPERATOR

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ABSTRACT

The instantaneous frequency law (IFL) is a very important item when the physical parameters of the corresponding signal have to be evaluated. Radar, sonar, mechanical diagnostic are just three domains where the signal's non-stationarity imposes the IFL estimation.

There are several cases where the IFL is composed by fast variations. Digital phase modulations or signals emitted by electrical switches are typical examples of IFLs having fast transient parts.

To deal with such kind of signals, we propose a new method based on the chirping of the phase transitions. Namely, the Phase Chirping Operator (PCO) transforms a fast IFL variation in a chirp component. This chirp contains all the parameters about the initial variation : time, duration, covered bandwidth, etc. Results for some physical data will highlight the benefits of the PCO compared with wavelet transform and Wigner-Ville distribution.

1. INTRODUCTION

Analysis of the signals characterized by a complex time-frequency behaviour is a challenging topic, due to the richness of the information carried by the IFL. In a large number of applications the analysis of the time-frequency (T-F) content provides an efficient solution to the problems arising in these fields [1]. The signals associated to real applications are generally characterized by many time-frequency structures usually considered as short-time stationary structures. The connections between these stationary parts are often subject of fast transitions whose estimation can be of great value (estimation of digital modulations in a COMmunication INTelligence – COMINT context [2], for example). On the other hand, time-frequency transitory parts are usually "talking" about the events happened during the analysed process (mechanical faults [3]).

The estimation of the transient parts is frequently done with help of either linear (Gabor transform, wavelet-based techniques, etc) or bi-linear time-frequency transforms (Cohen's class methods) [2,3]. One aspects of common problems when dealing with complex signals are related to the resolution trade off and, respectively, to the cross terms. For this reason, the analysis of phase singularities is complex and very often limited to simple configuration.

In this paper we propose a method which is aimed at transforming a phase singularity in a chirp component. In this way, we can easily use the conventional time-frequency representation (TFRs) for displaying or monitoring the phase parameters. This concept can be applied everywhere the phase singularities occur.

This paper is organized as follows. In the section 2 a short mathematical description of signal characterized by phase discontinuities is presented. This establishes the framework for the PCO concept, introduced in section 3. Some results, provided for few physical signals, are depicted in section 4. We will close with "Conclusion".

2. MODELING SIGNAL PHASE SINGULARITIES

Let consider a non-stationary signal expressed as

$$s(t) = A\exp\left[j\phi(t)\right] \tag{1}$$

where A is the amplitude and ϕ is its *instantaneous phase (IP)*. The derivative of the IP relates to the *instantaneous frequency law* (IFL)

$$IFL(t) = \frac{1}{2\pi}\frac{d}{dt}\phi(t) \tag{2}$$

From the signal issued from modulation processes, the IP is expressed as

$$\phi(t) = 2\pi f_0 t + m(t) \tag{3}$$

where f_0 stands for the carrier frequency and $m(t)$ is the modulation function. For simplicity reason we consider $f_0=0$ (which corresponds to the base-band signal).

In the case of smoothed modulations laws (micro-Doppler effect, sinusoidal frequency modulation, etc), the polynomial phase modeling provides a satisfactory model of the instantaneous phase [4]. In this paper, we will focus on the signal whose instantaneous phase is characterized by many abrupt transitions (also called phase singularities).

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This model is appropriate for digital phase modulations (Phase Shift Keying-PSK), for speech attacks [5] or a signal issued from rotating machinery when some faults occur. Generally, for a such signal, the IP can be expressed as

$$\phi(t)=\sum_k a_k h_{D_k}\left(t-\tau_k\right) \qquad (4)$$

where a_k is the amplitude of k^{th} phase transition and h is the rectangular window of length D_k and center τ_k. Figure 1.a. shows an example of a phase function corresponding to a PSK modulation.

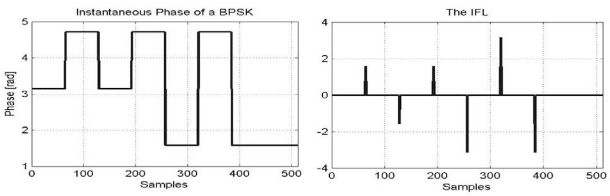


Fig. 1. Phase modulation and its IFL

The derivative of this function provides the IFL which is composed by a sum of weighted Dirac functions (we ignore the 1/2π term) :

$$IFL(t)=\frac{d}{dt}\phi(t)=\sum_k a_k\left[\delta(t-\tau_k)-\delta(t-\tau_k-D_k)\right] \qquad (5)$$

As illustrated in the next figure, the analysis of such IFLs via classical TFRs is not obvious. We considered the representation of the BPSK previously presented by the Wigner-Ville Distribution (WVD) and Wavelet Transform with a complex Morlet analyzing wavelet.

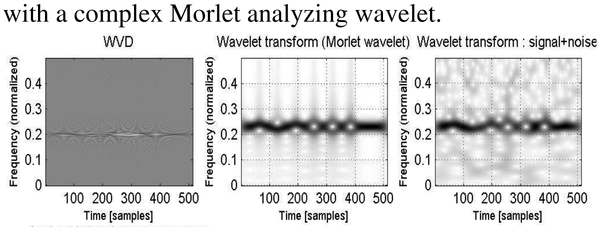


Fig. 2. Analysis of a BPSK modulation by WVD and WT

Clearly, due to the incapacity of the WVD to deal with this type of signals, the TFR provided by the WVD is completely useless when we try to identify the modulation parameters. In the case of the WT, since this technique is appropriate to detect the signal singularities, it is possible to distinguish the time locations of the phase transitions. Nevertheless, an accurate estimation and the retrieval of transition amplitudes require a more complex procedure which is not always easy to design. Moreover, the problem becomes more complicated for the signal corrupted by noise (figure 2, right side).

In the following example we consider the signal recorded from a rotating machinery which changes its angular velocity in three steps (figure 3.a). The wavelet transform of this signal is illustrated in figure 3.b. We can clearly distinguish the three frequency steps. Furthermore, two belt faults during the stationary phases have been introduced (at 1 and 5 seconds). These faults can be modeled as fast phase transitions [3].

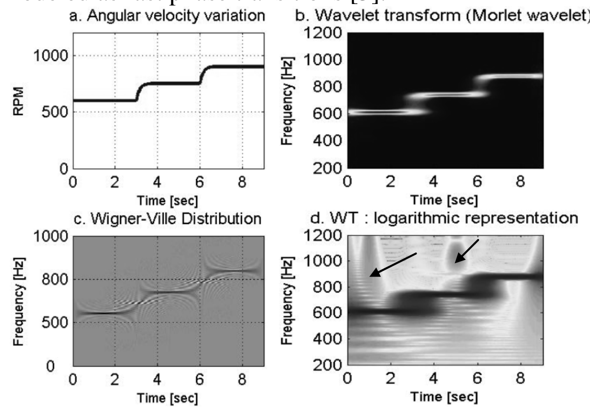


Fig. 3. Phase singularities from a faulty rotating machinery

The figure 3.c. displays the WVD of the signal corresponding to the faulty regime. The inherent cross-terms hide the phase singularities associated to the faults. In the case of the WT, the phase transitions correspond to some wavelet coefficients located in the proximity of the phase transitions (figure 3.d.; the transitions are marked by arrows). Nevertheless, these coefficients are less energetic (visible only in a logarithmic scale representation) than the ones corresponding to the stationary or transitory parts. Moreover, their spreading poses serious problems when we are interested to localize the faults.

These two examples illustrate the limitations of well-known TFRs when the IP is composed by fast transitions.

3. CONCEPT OF PHASE CHIRP OPERATOR

Let us consider a signal whose instantaneous phase is a rectangular window of amplitude A, length D and located at t_0 (figure 4). The phase derivation, which can be done by using the second order instantaneous moment operator [4]

$$\tilde{s}(t)=s(t)\cdot s^*(t-\Delta t) \qquad (6)$$

(* stands for the conjugate operator and Δt is the lag), leads to the following expression of the phase

$$\tilde{\phi}(t)=A\left[\delta(t-t_0)-\delta(t-t_0-D)\right] \qquad (7)$$

Let consider a linear function of length L expressed as

$$g(t)=t;\, t\in\left[-L/2, L/2\right] \qquad (8)$$

An example of such a function is illustrated in figure 4.

The convolution of this function with (7) transforms the two Dirac functions in two linear functions (figure 4) :

$$\varphi(t)=(\tilde{\phi}*g)(t)=\int g(\tau)\tilde{\phi}(t-\tau)d\tau=$$
$$=A(t-t_0)h_L(t-t_0)-A(t-t_0-D)h_L(t-t_0-D) \qquad (9)$$

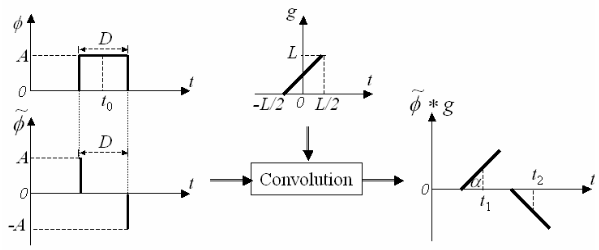


Fig. 4. Time-broadening of Dirac functions

Furthermore, the estimation of this linear functions leads to the parameters of the initial phase :

$$\hat{D} = \hat{t}_2 - \hat{t}_1 ; \hat{t}_0 = \left(\hat{t}_1 + \hat{t}_2\right)/2 \quad \hat{A} = L \tan \hat{\alpha} \tag{10}$$

where ^ stands for estimated, $t_{1,2}$ are the time centers of both functions and α is the function slope.

The time-broadening procedure illustrated in the figure 4 performs only if we have control of the phase law samples which compose the function (7). In signal processing applications, this situation is hypothetical since we have access **only** to the signal samples. Fortunately, it is possible to adapt this time-broadening procedure to deal with the signal samples. For this purpose, we propose the Phase Chirping Operator (PCO), defined as :

$$\mathbf{PCO}_x[n] = \prod_k \{x[n-k]\}^{g[k]} \tag{11}$$

where L is the length of g. For a signal $x(t) = \exp(j\phi(t))$ the effect of this operator is

$$\mathbf{PCO}_x[n] = \exp\left\{ j \sum_k g[k] \phi[n-k] \right\} \tag{12}$$

which is equivalent to the phase convolution with the function g. Considering a phase function issued by the derivation of (4) (using operator 6), expressed as

$$\tilde{\phi}[n] = \sum_k a_k \{\delta[n-\tau_k] - \delta[n-\tau_k-D_k]\} \tag{13}$$

the effect of the PCO can be analytically described as :

$$\mathbf{PCO}_x[n] = \exp j \left\{ \sum_i g[i] \sum_k a_k \delta[n-\tau_k-i] - g[i] \sum_k a_k \delta(n-\tau_k-D_k-i) \right\} = \exp j \left\{ \sum_k a_k g[n-\tau_k] - \sum_k a_k g[n-\tau_k-D_k] \right\} \tag{14}$$

The equation (14) shows that using the PCO we can transform the sum of Dirac in a sum of functions g which conserves the amplitude and the time parameters of the original phase law. Choosing a quadratic form for the function g, i.e. $g[n] = n^2 ; n \in [-L/2, L/2]$, the PCO provides the chirping of the phase transitions :

$$\mathbf{PCO}_x[n] = \exp j \left\{ \sum_k a_k (n-\tau_k)^2 h_L[n-\tau_k] \right\} \cdot \exp\left\{ -\sum_k a_k (n-\tau_k-D_k)^2 h_L[n-\tau_k-D_k] \right\} \tag{15}$$

where h is the rectangular function defined as

$$h_L[n-n_0] = \begin{cases} 1, & n \in [n_0 - L/2, n_0 + L/2] \\ 0, & otherwise \end{cases} \tag{16}$$

The IFL of the signal issued by the PCO-based transforming is composed by a sum of delayed chirps whose parameters lead to the original phase. We can also notice that the chirps have an alternating sign rate. Actually, estimating the chirp parameters :

n_{ck}^{+} – the time center of the positive rate chirp of index k

n_{ck}^{-} – the time center of the negative rate chirp of index k

α_k – chirp rate of index k

allows for evaluating the phase law parameters according to :

$$\hat{D}_k = n_{ck}^{-} - n_{ck}^{+} ; \hat{\tau}_k = \left(n_{ck}^{+} + n_{ck}^{-}\right)/2 \quad \hat{a}_k = L \tan \hat{\alpha}_k \tag{17}$$

Therefore, by using the PCO concept, it is possible to estimate the parameters of a phase law of type (4) (or alternatively, an IFL given by 5) by estimating the chirps associated to each phase transition. For the purpose of chirp estimation, we have used the method proposed in [6] which adaptively estimates the best chirplets.

Noise robustness. The PCO definition (11) indicates that, in the presence of the additive noise, the corrupting samples will be taken into account during the multiplications implied by the operator. To reduce the noise effect, we apply a sliding window whose parameters are adaptively modified according to *the robust representation concept* [7]. More precisely, the window is modified in order to minimize the error function e which expresses the similarity between the signal and a set of harmonics whose frequencies are close to the signal's IFL.

Using this concept, the definition of the robust PCO is

$$\mathbf{PCO}_x[n] = \prod_k \{\gamma[n-k] x[n-k]\}^{g[k]} \quad \gamma[n] = \frac{w_h[n]}{e[n]} \cdot \left(\sum_{n=-P/2}^{P/2} \frac{w_h[n]}{e[n]} \right)^{-1} \tag{18}$$

where w_h is a Hamming window of length P.

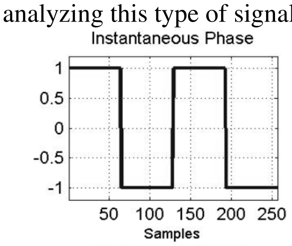
Illustrative results are presented in the next section for synthetic and physical data.

4. RESULTS

First, let us consider a synthetic signal whose analytical form is

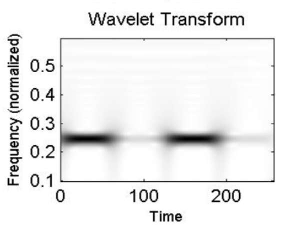
$$s[n] = \exp\left\{ j2\pi \cdot 0.25n + \sum_{i=1}^{4} (-1)^{i+1} h_{64}[n - (64i - 32)] \right\}; n = 0,..,255 \quad (18)$$

Appling the PCO on the signal derived according to(6) we obtain a complete characterization of the phase transitions as indicated in figure 5. In this figure we remark that the signal issued from PCO application contains four chirps associated to the four phase transitions. The Pseudo WVD displays clearly these chirps. At the same time, the wavelet transform shows its limitations in accurately analyzing this type of signal.



Parameters estimation

	i=1	i=2	i=3	i=4	
A_k	+1	-1	+1	-1	Theoretical values
τ_k	32	96	160	224	
D_k	64	64	64	64	
$\hat{A}_k$	.98	-1.01	.99	-1.02	Estimated values
$\hat{\tau}_k$	31	96	159	225	
$\hat{D}_k$	65	64	65	65	



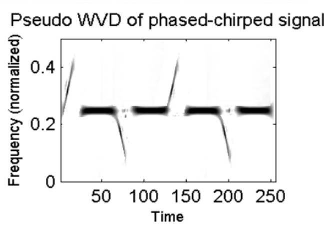
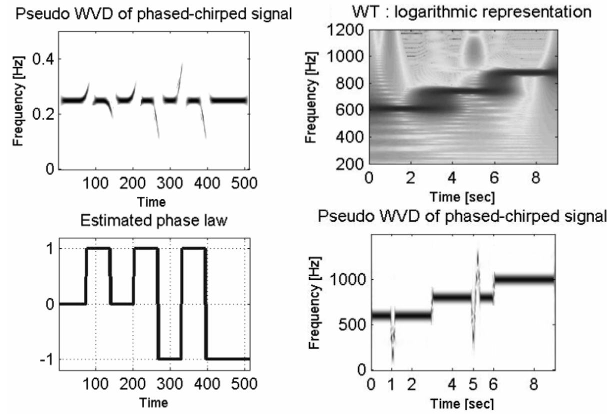


Fig. 5. PCO-based instantaneous phase estimation

By applying the "Chirping Hunting" method [6] we obtain an evaluation of chirp parameters. Furthermore, exploiting these parameters via (16) we obtain the parameters of the phase transitions, depicted in the figure 5. We remark that the estimated values are very closed to the theoretical ones, the small differences being due to the limited numerical precision.

In the next examples we analyze, using PCO, the signals considered in the section 2 : the PSK and the recorded signal from a faulty machinery. These signals were corrupted by a additive noise and the SNR was about 12 dB. The PWVDs of the phase chirped signals are illustrated in the figure 6. In the case of the PSK we display the estimated phase law (figure 6.a). Comparing with the figure 1, we remark that the parameters of the PSK are well estimated. We obtain the same symbol duration and the same differences between phase levels. This is justified in figure 6.a. : the chirp rates corresponding to the transitions 1 to –1 are twice than the chirp rates associated to the transitions 0 to 1.

For the signal issued from a faulty rotating machinery we remark that the phase transitions caused by the faults are well detected in time at 1 and 5 seconds, respectively. Moreover, since the second fault has been provoked by a longer event than the first one, the result shows that the PCO-based analysis is able to inform about the time duration of the phenomenon which generated the phase transitions.



a. PSK modulation b. Signal from a rotating machinery

Fig. 6. PCO-based IP estimation for two physical signals

5. CONCLUSION

While the already existing TFRs are aimed to estimate the IFLs, the analysis of the phase singularities is generally not efficiently done. In this paper we proposed a method for the analysis of the phase singularities. This method is based on the chirping of the phase singularities which enables their estimation using the existing methods for chirp analysis. The results proved the efficiency of the method to provide a complete analysis of the phase singularities in a realistic context (presence of noise and signal issued form real applications).

In the future, our work will be focused on extending this method to the multi-component signals. On the other hand, the noise robustness of the PCO will be also subject of further works.

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