ESTIMATION OF FM PARAMETERS USING A TIME-FREQUENCY HOUGH TRANSFORM

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ABSTRACT

An estimator for the phase parameters of mono- and multicomponent FM signals, with both good numerical properties and statistical performance is proposed. The proposed approach is based on the Hough transform of the pseudo Wigner-Ville time-frequency distribution (PWVD). It is shown that the numerical properties of the estimator may be improved by varying the PWVD window length. The effect of the window time extent on the statistical performance of the estimator is delineated. Experimental data is used for validation of statistical properties.

1. INTRODUCTION

The problem is to estimate the phase parameters of mono- or multicomponent FM signals from noisy observations. The multicomponent signal model is $s(t) = \sum_{k=1}^{K} A_k e^{j\varphi(t;\boldsymbol{\theta}_k)}$ where K is the number of components, $\{A_k\}$ are complex-valued arguments and $\{\varphi(t;\boldsymbol{\theta}_k)\}$ are the phase functions parameterised by $\{\boldsymbol{\theta}_k\}$ and containing no constant term with respect to time. The instantaneous frequency (IF) of component k is defined as $\omega(t;\boldsymbol{\theta}_k) = d\varphi(t;\boldsymbol{\theta}_k)/dt$. Given N noisy samples of $s(t)$, the problem is to estimate $\{\boldsymbol{\theta}_k\}$.

In this paper, we consider the time-frequency Hough transform (TFHT) approach proposed in [1]. This method is applicable to general nonlinear FM models and multicomponent signals. However, the implementation can be computationally intensive, since many possible signal trajectories must be evaluated. It was suggested in [2] that this problem may be approached by first decimating and lowpass filtering the TFD, before applying the Hough transform, in order to broaden the peak centered about the true parameter values. While this may reduce the number of trajectories needed to find the global maximum, each trajectory implies a computational cost higher than that of the unfiltered TFD, which is already of order N^2.

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In this work, we propose a numerically efficient and attractive implementation of the TFHT based on the Hough transform of the pseudo Wigner-Ville distribution (PWVD), which is computed by windowing the local auto-correlation function in the lag domain. By virtue of the windowing, this distribution is relatively computationally efficient, even for long data lengths, and is applicable to nonlinear FM signals, specifically to those generated by simple harmonic motions of rotating or vibrating targets. Using a smaller window also serves to broaden the peaks of interest in the Hough transform. This property is exploited to improve numerical efficiency, in the proposed estimation algorithm. We also generalise the statistical analysis of the WHT given in [2] (for linear FM), to include arbitrary FM models with the PWVD.

2. THE PSEUDO WIGNER-HOUGH TRANSFORM

The pseudo Wigner-Hough transform (PWHT) of a signal $s(t)$ is defined as the line integral through the PWVD of $s(t)$, along the IF model; $\omega(t;\boldsymbol{\theta})$. The PWHT is therefore a mapping from the time domain to the parameter domain of $\boldsymbol{\theta}$. In the discrete-time case the PWHT is calculated from N samples $\{s(n)\}_{n=0}^{N}$ of $s(t)$ by

$$P_s(\boldsymbol{\theta}) = \sum_{n=M}^{(N-M-1)} \sum_{l=-M}^{M} s(n+l)s^*(n-l)e^{j2\omega(n;\boldsymbol{\theta})l} \quad (1)$$

where M is a parameter defining the odd PWVD window length; $L = 2M + 1$. We have defined the PWHT as the summation over the $N - L + 1$ points in the center of the PWVD, leaving out the rising and falling edges of the distribution, assuming that $L << N$. The inner sum implies using constant number of data bilinear products, for each lag, l. Defining $R_s(n,l) = s(n+l)s^*(n-l)$ as the local auto-correlation function, we see that the PWHT is a sum over the center rectangle of the support of $R_s(n,l)$, whereas the WHT sums over the full diamond-shaped support. Compared with the WHT, the computational order is reduced by a factor of approximately $2L/N$ for $L << N$.

Apart from reducing the computational complexity, the PWHT has another important advantage over the WHT. In Figure 1, we plot the PWHT and WHT functions for a linear FM signal observed in additive white Gaussian noise (AWGN) at an SNR of 0dB, with $N = 128$ and $M = 5$. Clearly the peak width in the parameter space is much larger for the PWHT than for the WHT. This implies improved numerical properties with respect to the WHT, when performing optimization. Subsequently, the required accuracy for initializing an efficient gradient based search is reduced. Of course, there is also a disadvantage in that a smaller window length implies reduced statistical accuracy. We therefore propose a grid search of Equation (1) using an initially small value of M, to reduce the number of trajectories needed. We then use this estimate to initialize optimization of Equation (1) for successively larger values of M in order to improve statistical accuracy. The proposed algorithm is summarised in Table 1.

1. Define $M_1 < M_2 < \cdots < M_p$.
2. Perform grid search of $P_x(\boldsymbol{\theta})$, using $M = M_1$, and define $\hat{\boldsymbol{\theta}}_0$ to be the location of the maximum value. Set $i \leftarrow 1$.
3. Obtain $\hat{\boldsymbol{\theta}}_i$, via gradient-based optimization of (1) using $M = M_i$, with initial location $\hat{\boldsymbol{\theta}}_{i-1}$.
4. Set $i \leftarrow i + 1$. While $i \leq p$ repeat from 3.
5. Take the final estimate: $\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}_p$.

Table 1. Estimation algorithm based on the PWHT.

The appropriate choice of $M_1, \ldots, M_p$ will depend on the particular IF model used. Based on the general result given in the following section, it is possible to find the value of M which minimizes the estimator variance. This value can be chosen for $M = M_p$. The initial value of $M = M_1$ should be chosen as low as possible for the given SNR. We have found that the SNR performance threshold for the PWHT is approximately $O(1/L)$, which means that when the SNR is above 0 dB, one may use $M_1 \geq 1$ and expect the global maximum of the PWHT to correspond to the location of the signal peak. For lower SNR, one may have to increase the value of M_1 such that the signal peak is above the noise floor.

In the case of multicomponent signals, there will be a number of peaks within the parameter space of the PWHT, as illustrated in Figure 1. In this case, we propose sequential estimation of each component. One estimates the 'strongest' component from the largest peak of the PWHT, using the method outlined in Table 1. The complex amplitude is then estimated, for example using a simple least-squares approach, and the reconstructed component is then subtracted from the observations. This is repeated until all components have been estimated. This approach does not require the components to have different amplitudes, as necessary in sequential phase based methods [3]. If the number of components is unknown, one may construct a test to determine when the residual term contains no more signal components, though this is not elaborated upon here.

3. STATISTICAL ACCURACY

For the case of additive noise, the observations may be modelled as $x(n) = s(n) + v(n), n = 0, \ldots, N-1$, where $s(n)$ is the signal of interest and $v(n)$ is a complex random process. In the following we perform statistical analysis of the PWHT estimator, based on the assumption that the signal IF is approximately linear within all time intervals of length L, over the entire observation period. We also assume that the noise is a complex white Gaussian process of variance σ_v^2.

The mean and variance of the PWHT estimator were derived via a generalisation of the perturbation approach taken in [2], for analysis of the WHT. Under the additive noise model, we may express the PWHT of the observations as the sum of $P_s(\boldsymbol{\theta})$, being the PWHT of the noise-free signal, and a perturbation $\delta P(\boldsymbol{\theta})$, composed of cross signal-noise and noise only terms. Given the piece-wise linearity assumption, the maximum of $P_s(\boldsymbol{\theta})$ occurs at $\boldsymbol{\theta}_0$, but in the presence of noise the maximum shifts to a location $\boldsymbol{\theta}_0 + \delta\boldsymbol{\theta}$ due to the influence of $\delta P(\boldsymbol{\theta})$. The bias and variance of the estimator are therefore determined by the mean vector and covariance matrix of $\delta\boldsymbol{\theta}$. As a first order approximation, one can show[1] that the bias $\mathsf{E}[\delta\boldsymbol{\theta}] = 0$ and the covariance matrix $\boldsymbol{\Gamma}_{\delta\boldsymbol{\theta}} = \mathsf{E}\left[\delta\boldsymbol{\theta}\delta\boldsymbol{\theta}^T\right] = \boldsymbol{C}^{-1}\boldsymbol{B}\boldsymbol{C}^{-T}$, where

$$\begin{aligned}\boldsymbol{C} &= |A|^2 \sum_{n=M}^{N-M-1} \sum_{l=-M}^{M} \left[-j2l \left.\frac{\partial^2 \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}\partial\boldsymbol{\theta}^T}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \right.\\ &\qquad \left. +4l^2 \left[\frac{\partial \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} \frac{\partial \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}^T}\right]\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \Bigg], \qquad (2)\end{aligned}$$

$$\begin{aligned}\boldsymbol{B} &= 8|A|^2\sigma_v^2 \sum_{l=-M}^{M} \sum_{k=-M}^{M} l\, k \sum_{n=M}^{N-M-1} \sum_{m=M}^{N-M-1} \\ &\qquad \left.\left[\frac{\partial \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} \frac{\partial \omega(m;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}^T}\right]\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \delta(n-m+l-k) \\ &\quad +\frac{4}{3}\sigma_v^4 M(M+1)(2M+1) \\ &\quad \times \sum_{n=M}^{N-M-1} \left.\left[\frac{\partial \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}} \frac{\partial \omega(n;\boldsymbol{\theta})}{\partial\boldsymbol{\theta}^T}\right]\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}. \qquad (3)\end{aligned}$$

[1] The full derivation of (2) and (3) is omitted here due to space limitations.

Defining the input SNR as $SNR_{in} = |A|^2/\sigma_v^2$, one can easily show from (2) and (3) that $\mathbf{\Gamma}_{\delta\boldsymbol{\theta}} = \frac{1}{SNR_{in}}\boldsymbol{D} + \frac{1}{SNR_{in}^2}\boldsymbol{E}$, where the matrices $\boldsymbol{D}$ and $\boldsymbol{E}$ depend on M, N and the first and second order derivatives of the IF model at $\boldsymbol{\theta}_0$. The expression for $\mathbf{\Gamma}_{\delta\boldsymbol{\theta}}$ therefore has the same form as the variance of the WHT estimator given in [2], for linear FM signals. Further, for particular IF models such as PPS, one may find from (2) and (3) that the variance is independent of the true signal parameter value $\boldsymbol{\theta}_0$.

4. RESULTS

4.1. Simulation

In the first example, the linear FM signal has a mean frequency $a_0 = 0.12/T_s$ and chirp rate $b_0 = 0.23/(NT_s)$, where T_s denotes the sample period. The root mean square error (RMSE) of the estimators for a and b is simulated and compared with the theoretical variance expressions and the CRB, as shown in Figure 2, with $N = 128$. The algorithm of Table 1 is applied, with the window parameter M varied from 3 to 13 in steps of 2. The initial estimate is computed by a grid search of the PWHT with $M = 3$. We precalculate a total of 50 trajectories within the non-aliased parameter range. In optimization of the PWHT, we have used an efficient gradient based technique proposed by Fletcher and Powell (FP) [4] for an SNR of 0 dB and above. It was found that for an SNR below 0 dB, the FP algorithm did not always converge. In these cases, we have used a more robust, albeit more complex, algorithm proposed by Nelder and Mead (NM) [5], which was found to produce good results down to about -5 dB SNR. The simulation results have been obtained by averaging 500 Monte Carlo runs.

We note that the simulated accuracy shown in Figure 2 is consistent with the theoretical analysis, down to about -5 dB SNR. The discrepancy at extremely low SNR is expected as assumptions inherent in the perturbation analysis are no longer valid. While the estimation is clearly not efficient, the performance is still very close to the CRB and the computation time has been greatly reduced when compared to the WHT based estimator. In this case, the need to use the WHT becomes questionable, since the large increase in computational burden provides only a minor improvement in estimation accuracy. However, statistical efficiency is easily achieved, if desired, by optimizing the WHT using the PWHT estimate for initializing the search. The overall approach is still far more computationally efficient than trying to directly optimize the WHT function.

4.2. Experimental

We have also applied the PWHT to experimental data, which has been collected from a 24 GHz radar system, observing a rotating fan. The rotational movement of the scatterer in this experiment results in a sinusoidal Doppler shift with respect to time, termed a micro-Doppler signature. To illustrate the estimation of multi-component signatures, we apply the PWHT estimator to the data collected only from the in-phase baseband channel of the radar system. This effectively produces two "signatures" each π radians out of phase with the other. The baseband signal was sampled at 1000 Hz and we have used an observation interval of 402 samples (~0.4 seconds) to estimate the micro-Doppler signatures. The initial grid search is performed for $B \in [0, 250]$ Hz, $\phi \in [0, 2\pi)$ rad and $\omega_0 \in [1, 10]$ Hz, with 12, 10 and 6 samples along each parameter range respectively (720 total trajectories). In the initial search, $M = 15$ was used to calculate the PWHT, and in the final optimization step, $M = 35$.

In Figure 3, we show the PWHT of the experimental data for $B = 16$ Hz. The figure shows both cases of $M = 15$ and $M = 35$, which clearly illustrates the advantage of widening the main peak, achieved with the smaller window length. In Figure 4 we show the estimated micro-Doppler signatures overlaid on the PWVD of the data (for $M = 35$). In this figure we see both the initial grid search estimates and the final estimated signatures. It is observed that both the final estimated signatures overlap the TF signatures as expected, although the initial grid search yielded somewhat inaccurate results.

5. SUMMARY

We have proposed a numerically efficient approach for estimation of the multicomponent nonlinear FM signals, based on the Hough transform of the PWVD. The approach was greatly reduced the number of FM trajectories to be computed, while still achieving good statistical performance. In the case of linear FM signals, performance close to optimal was demonstrated. An expression for the estimator variance valid for general FM models was also given. The proposed approach was verified for nonlinear FM signals in the application to experimental radar data, where multicomponent micro-Doppler signatures, modelled as sinusoidal FM, were successfully estimated.

6. REFERENCES

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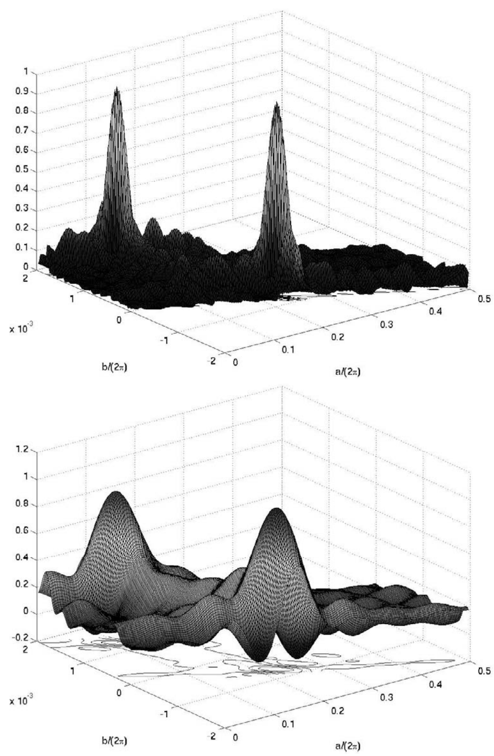

Fig. 1. (top) WHT and (bottom) PWHT of the sum of two chirp signals at 0 dB SNR.

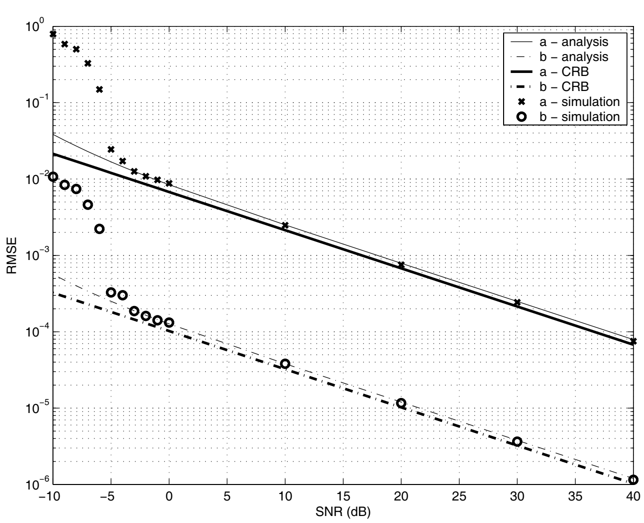

Fig. 2. Single-component linear FM estimation variance, simulated, theoretical and the CRB.

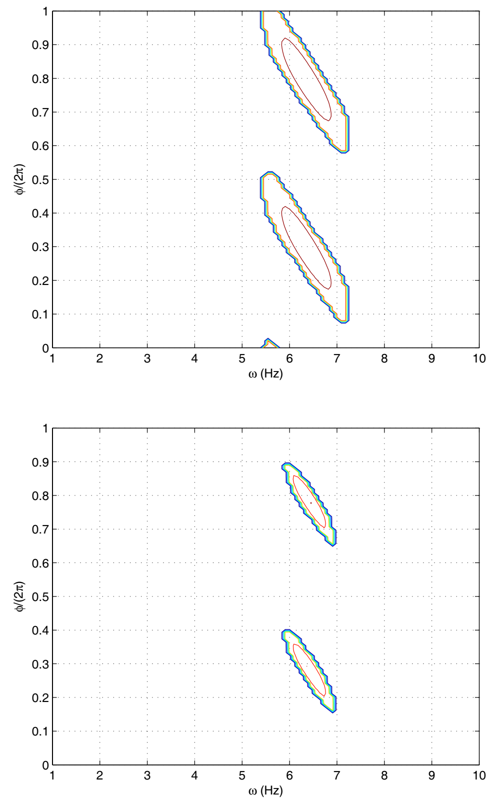

Fig. 3. PWHT for $M = 15$ (top) and $M = 35$ (bottom) of the experimental micro-Doppler data, evaluated for $B = 16$ Hz.

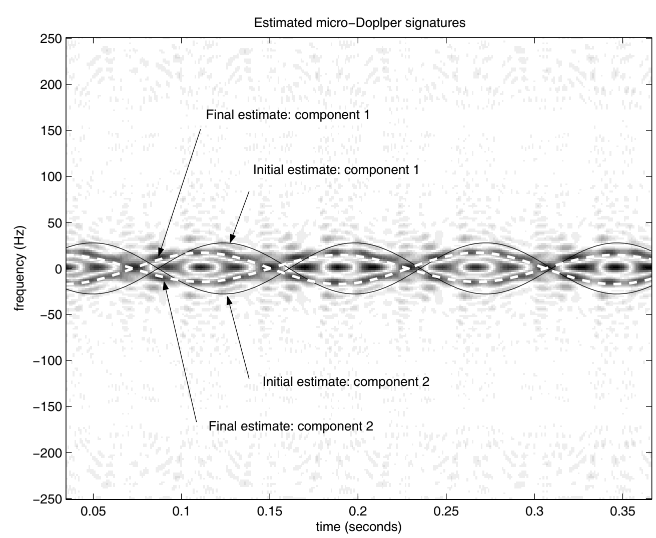

Fig. 4. Estimated micro-Doppler signatures from the initial grid search with $M = 15$ (solid black) and optimization of the PWHT function with $M = 35$ (dashed white), overlaid on the PWVD of the data computed with $M = 35$.