DOPPLER SIGNAL DETECTION AND PARTICLE TIME OF FLIGHT ESTIMATION USING WAVELET TRANSFORM FOR ACOUSTIC VELOCITY MEASUREMENT

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ABSTRACT

This paper discusses a processing technique for Laser Doppler Velocimetry (LDV) data, enabling to detect and localize in time domain the presence of Doppler bursts in case of acoustic excitation. A joint detection-estimation scheme based on the use of the Wavelet Transform (WT) realized in the time-scale domain is proposed. The performances of the detector are characterized with ROC (Receiver Operating Characteristic) curves. Finally, the estimator performances are studied by means of Monte Carlo trials obtained from synthesized LDV signals.

1. INTRODUCTION

LDV is an optical technique based on interferometry [1]. In the dual beam mode, two laser beams are crossed and focused at a point called probe volume, made of dark and bright fringes. The basic principle of LDV consists in measuring the frequency of the light scattered by tracers introduced in the fluid, the estimation of this frequency leading to the estimation of the tracer velocity. For acoustic excitation, the analyzed signal, called Doppler signal, is frequency modulated and the frequency modulation gives information about the acoustic particle velocity using signal processing methods developped over the last ten years [2]. When many tracers cross the probe volume at different random times, the estimation of the flow and acoustic velocities needs to establish if a particle crosses the probe volume (detection / decision problem) on the one hand and to estimate the central time and the time of flight of each particle on the other hand (estimation problem). The aim of this work is to develop and validate a technique for detecting burst and for estimating the central time and the time of flight of each particle. This article is organized as follows. General LDV principles are presented in section 2 and the application of LDV to acoustics in explained in section 3. The detectionestimation scheme based on WT is presented in Section 4. Finally, in Section 5, numerical examples illustrate the performances of the detector with ROC (Receiver Operating Characteristic) curves. The estimator performances are given with a statistical analysis of the estimations, by means of Monte-Carlo simulations.



Fig. 1. Optical set-up of the LDV system.

2. LDV PRINCIPLES

Principle of LDV is described in figure 1. Two laser beams are crossed and focused to define a probe volume in which an interference pattern, made of bright and dark fringes, appears. The light scattered by a particle crossing the probe volume is modulated in intensity with a frequency

$$F_D = \frac{V}{i} = DV = \frac{2\sin(\theta/2)}{\lambda}V,$$
 (1)

where V is the component of the velocity \vec{V} perpendicular to the fringes, *i* is the space between two adjacent bright fringes, D = 1/i is the sensitivity of the set-up, λ is the laser light wavelength, and θ is the angle between both beams. In order to avoid any ambiguity on velocity sign, the interference fringes are frequency shifted with the help of a Bragg cell at frequency F_B (40 MHz). If only one particle *q* crosses the measuring volume, the electrical signal delivered by the photo-detector, called burst, is

$$s_q(t) = A_q(t) \{ M + \cos[2\pi F_B t + 2\pi D x_q(t) + \phi_0] \},$$
(2)

where $A_q(t) = Ke^{-\{\beta x_q(t)\}^2}$ represents the magnitude of the Doppler signal related to particle q, β being related to the probe geometry, M is due to the positive sign of light intensity, $x_q(t)$ is the position of the particle q in the probe volume frame and ϕ_0 is an initial phase due to optics. Thus the signal is roughly a sine-wave frequency modulation of a carrier frequency F_B with a time varying amplitude. The offset component M is eliminated by an electrical high-pass filter. If many particles cross the probe volume, the signal is the sum of multiple burst with random central times



Fig. 2. Scheme of quadrature demodulation technique.

[1]. Signal processing methods [2] need to define the analytic signal $z_q(t)$ of $s_q(t)$, written as $z_q(t) = A_q(t)e^{j\phi_q(t)}$ where $\phi_q(t) = 2\pi D x_q(t) + \phi_0$ (zero carrier frequency). In practice, $z_q(t)$ is approximated by $z_q(t) = s_{q1}(t) + js_{q2}(t)$ where

$$s_{q1}(t) = A_q(t)\cos\phi_q(t) + b_1(t),$$
 (3)

$$s_{q2}(t) = A_q(t)\sin\phi_q(t) + b_2(t),$$
 (4)

 $b_1(t)$ and $b_2(t)$ being sequences of zero-mean independant Gaussian random variable. $s_{q1}(t)$ and $s_{q2}(t)$ are obtained thanks to a Quadrature Demodulation (QD) technique (figure 2) which shifts down the carrier frequency of the Doppler signal to zero.

3. LDV APPLIED TO ACOUSTICS

In case of acoustic sinusoïdal excitation at frequency F_{ac} , the velocity of a tracer q can be written

$$v_q(t) = V_{f,q} + V_{ac}\cos(2\pi F_{ac}t + \phi_{ac}),$$
 (5)

where V_{ac} is the amplitude of the acoustic velocity, ϕ_{ac} the acoustic velocity phase and $V_{f,q}$ the flow velocity of particle q, supposed as constant in the probe volume. Typically the flow velocity leads between 50 and 400 mm/s for useful acoustic measurement without forced flow. The position of the particle in the probe volume frame is defined by

$$x_q(t) = V_{f,q}(t - t_{c,q}) + \frac{V_{ac}}{2\pi F_{ac}}\sin(2\pi F_{ac}t + \phi_{ac}), \quad (6)$$

where $t_{c,q}$ is the time at which the particle reaches the center of the probe volume without acoustic excitation, called central time. The aim of signal processing in LDV is to estimate V_{ac} , ϕ_{ac} and $V_{f,q}$ from $s_{q1}(t)$ and $s_{q2}(t)$. For this, the analytic signal $z_q(t)$ corresponding to single burst q is analysed by time frequency transforms for $t \in [t_{c,q} - T_{f,q}/2, t_{c,q} + T_{f,q}/2]$, where $T_{f,q}$ is the time of flight of particle q defined by

$$T_{f,q} = 4\sigma_q,\tag{7}$$

with

$$\sigma_q = \frac{1}{\sqrt{2\beta}V_{f,q}}.$$
(8)

For multiple bursts, the estimation of the three parameters V_{ac} , ϕ_{ac} and $V_{f,q}$ can only be performed if each burst is detected and if the central time $t_{c,q}$ and the time of flight $T_{f,q}$ are estimated.

4. WAVELET DETECTION-ESTIMATION

This section presents the detection of burst in a Doppler signal and the estimation of central time and time of flight using the Wavelet Transform. This approach is based on previous work concerning Doppler signal analysis [3, 4, 5].

4.1. Continuous Wavelet Transform (CWT)

Continuous Wavelet Transform (CWT) is defined by

$$W_{s}(t_{0},a) = \langle s, \psi_{t_{0},a} \rangle = \int_{-\infty}^{+\infty} s(t) \frac{1}{\sqrt{a}} \psi^{*}(\frac{t-t_{0}}{a}) dt,$$
(9)

where the base atom ψ is a zero average function with a finite energy. All the basic vectors can be obtained from this function ψ by scaling and time-shifting [6]:

$$\psi_{t_0,a}(t) = \frac{1}{\sqrt{a}}\psi(\frac{t-t_0}{a}),$$
(10)

with a the scale parameter and t_0 the shifting parameter. The square modulus of the wavelet coefficients $|W_s(t_0, a)|^2$ are used to represent the CWT on a time-scale representation (t_0, a) , called scalogram (figure 3.b). An analogy in time-frequency can be realized with $f \equiv f_0/a$, where f_0 is the central frequency of the Fourier Transform $\hat{\psi}_{t_0,a}(f)$.

According to its time and scale resolution properties, CWT is well-suited to analyse long burst linked to low frequency and short burst linked to high frequency [6].

Morlet wavelet [6], $\psi(t) = (\pi t_0)^{-1/4} e^{\frac{1}{2}(\frac{t}{t_0})^2} e^{j2\pi f_0 t}$, has been chosen as analysing wavelet, according to its similarities with the expression of the burst signal $z_q(t)$ (§2).

4.2. Proposed method

The detection-estimation scheme leads to localize the bursts in the time domain thanks to an estimation of the central time $\hat{t}_{c,q}$ and the time of flight $\hat{T}_{f,q}$ (figure 3.a).

4.2.1. Detection

The burst detection problem is a binary hypothesis problem. It consists in determining if one or multiple bursts are present (hypothesis H_1) or not present (hypothesis H_0) in an signal $s_{T_w}(t)$ observed in a noisy environment during



Fig. 3. (a) : Synthetized Doppler burst, (b) : Scalogram of the Doppler burst ($F_{ac} = 4000 \text{ Hz}$, $\alpha = 0.01$ and $V_{f,q} = 200 \text{ mm/s}$).

 $T_W = 10\hat{\sigma}_q$ (eq. 8). This can be written as

$$H_0: s_{T_W}(t) = b(t), H_1: s_{T_W}(t) = b(t) + \sum_q s_q(t).$$
(11)

The detector indicates the presence of signal on the window T_W when the following test is achieved:

$$\left[|W_{s}(t_{0},a)|^{2}\right]_{T_{W},max} > \gamma. \left[|W_{b}|^{2}\right]_{max}, \qquad (12)$$

where $[|W_s(t_0, a)|^2]_{T_W,max}$ is the maximum of scalogram on T_W , γ is a threshold defined by the user and $[|W_b|^2]_{max}$ is the maximum of the scalogram of a noisy portion of the tested signal (reference value).

4.2.2. Estimation

The estimation procedure of the central time $\hat{t}_{c,q}$ and the time of flight $\hat{T}_{f,q}$ is performed in two steps. The first step consists in getting a more accurate localisation of the detected information in time-frequency domain (figure 3.b). The second step leads to an estimation of the time parameters ($\hat{t}_{c,q}, \hat{T}_{f,q}$).

To realize the pre-estimation, two standard deviations $\hat{\Delta}_{t_0}$ and scale $\hat{\Delta}_a$ are calculated in time and scale. The time width \hat{T} and the scale width \hat{A} are determined by searching α in order to obtain:

$$|W_s(\hat{T}, \hat{A})|^2 > \gamma \left[|W_b|^2\right]_{max},\tag{13}$$

with $\hat{T} = \alpha \hat{\Delta}_{t_0}$, $\hat{A} = \alpha \hat{\Delta}_a$ and $|W_s(\hat{T}, \hat{A})|^2$ the scalogram coefficients localized in the time-scale domain (\hat{T}, \hat{A}) . The time standard deviation $\hat{\Delta}_{t_0}$ is calculated as follows:

$$\hat{\Delta}_{t_0}^2 = \frac{\int_{T_W} (t_0 - \hat{t}_0)^2 |\int_a \mathbf{W}_s da|^2 dt_0}{\int_{T_W} |\int_a \mathbf{W}_s da|^2 dt_0},$$
(14)

where

$$\hat{t}_{0} = \frac{\int_{T_{W}} t_{0} |\int_{a} \mathbf{W}_{s} da|^{2} dt_{0}}{\int_{T_{W}} |\int_{a} \mathbf{W}_{s} da|^{2} dt_{0}},$$
(15)

where $\mathbf{W}_s = |W_s(t_0, a)|^2$ and T_W is the scalogram window. The same calculations are carried out for $\hat{\Delta}_a$.

The second step leads to the central time $\hat{t}_{c,q}$ and the time of flight $\hat{T}_{f,q}$ estimations. $\hat{t}_{c,q}$ is calculated from width \hat{T} by derivating the marginal of the scalogram along the time axis

$$\xi_{deriv,t_0} = \frac{d}{dt_0} (\int_a |W_s(t_0, a)da|^2).$$
(16)

Scalogram maximum are localized thanks to zero crossing of the derivative ξ_{deriv,t_0} , and scalogram maximum leads to bursts envelop maximum. Then, zero crossings give an estimation of burst central times $\hat{t}_{c,q}$. The time width $\hat{T}_{f,q}$ is estimated using the derivative of the marginal along the frequency axis $\zeta_{deriv,f}$ given by

$$\zeta_{deriv,f} = \frac{d}{df} \left(\int_{t_0} |W_s(t_0, \frac{f_0}{f}) dt_0|^2 \right), \tag{17}$$

thanks to the analogy $a \equiv f_0/f$ (§4.1). All zero crossings of $\zeta_{deriv,f}$ give an estimation of the mean frequency of the burst $D\hat{V}_{f,q}$. The flow velocity $\hat{V}_{f,q}$ is linked to the standard deviation of the burst envelop $\hat{\sigma}_q$ (eq. 8). So, the frequency axes leads to a time of flight estimation $\hat{T}_{f,q}$ (eq. 7) for each burst.

In practice, the derivated functions ξ_{deriv,t_0} and $\zeta_{deriv,f}$ are calculated with a finite difference method (Euler's method).

4.2.3. Cramer Rao Bounds

Cramer-Rao Bound (CRB) gives the lower bound on the variance of any unbiased estimator. CRB of the central time $t_{c,q}$ and of the time of flight $T_{f,q}$ have been calculated according to Le Duff works [7]:

$$CRB(t_{c,q}) = \frac{1}{4\pi^2} \sqrt{\frac{2}{\pi}} \frac{\beta T_f}{D^2} \frac{1}{V_{f,q}} \frac{1}{SNR},$$
 (18)

$$CRB(T_{ft,q}) = \frac{1}{\pi^2} \sqrt{\frac{1}{\pi}} \frac{\beta^2 T_f}{D^2} \sigma_q \frac{1}{SNR}, \quad (19)$$

with SNR the Signal to Noise Ratio.

5. RESULTS AND CONCLUSIONS

Detector validation and estimations are realized on simulated data for 1000 Monte Carlo simulations (one event for one simulation) at various SNR (Signal to Noise Ratio). Each event contains at least one burst (H_1) or no burst (H_0) (eq. 11).

Different parameters of the LDV system have been tested as shown in table 1.

Signal parameters are $\beta = 4.6407.10^4 m^{-1}$, D = 1.3126. $10^3 m^{-1}$, K = 1, $\phi_0 = \pi$ and $T_e = 10^{-6} s$.

Results presented in this paper are obtained with a set



False Detection Rate

Fig. 4. ROC curves for wavelet detector ($F_{ac} = 4000 \text{ Hz}$, $V_{f,q}=200 \text{ mm/s}$ and $\alpha = 0.01$)

$\alpha = DV_{ac}/f_{ac}$	10^{-2}	1	10
$f_{ac}\left(Hz\right)$	100, 1000, 4000		
$V_{c,q} \ (mm/s)$	20, 200		

Table 1. Numerical values used for the simulations

of parameters corresponding to low acoustic displacement amplitude and high convection flow which corresponds to difficult conditions for the instantaneous frequency estimator [2]. These conditions are defined by $f_{ac} = 4000 Hz$, $\alpha = 0.01$ (acoustic level of 58 dBSPL in free field) and $V_{f,q} = 200 \ mm/s$. ROC curves (figure 4) show the detector behaviour for different SNR (0 dB, 3 dB, 5 dB, 10 dB). Figure 5 presents the estimator bias and variance on flight time $T_{f,q}$ for two values of the threshold γ . As that was expected results $(b_{\hat{T}_{f,q}} \text{ and } \sigma^2_{\hat{T}_{f,q}})$ are better with an optimal threshold γ , determined thanks to ROC curves (figure 4). The error on the standard deviation $\sigma_{\hat{T}_{f,\sigma}}$ for the best estimations, is at worse of $10 \ \mu s$ (SNR of $0 \ dB$) and at best of $3 \ \mu s$ (SNR of 15 dB), with $T_{f,q} = 176 \ \mu s$ for $V_{f,q} = 200$ mm/s. The bias $b_{\hat{T}_{f,q}}$ is at worse of 0.5 μs (SNR 0 dB) and at best of $0.1 \ \mu s$ (SNR 15 dB). The results about the central time estimation \hat{t}_{cq} lead, for the same set of parameters as before, to a standard deviation $\sigma_{\hat{t}_{c,q}}$ which is at worse of 0.7 μs (SNR 0 dB) and at best of 0.1 μs (SNR 15 dB). The bias $b_{\hat{t}_{c,q}}$ is at worse of $0.2 \ \mu s$ (SNR $0 \ \mathrm{dB}$) and at best of $0.05 \ \mu s$ (SNR 15 dB). Results concerning the detection-estimation step in LDV set-up allows to optimize the estimators linked to the caracterization of bursts (V_{ac} , ϕ_{ac} and $V_{f,q}$). According to the experimental condition in which acoustics measures are achieved, results obtained by the detectionestimation procedure are satisfactory and will allow to improve investigation tools dedicated to acoustics.

6. REFERENCES

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Fig. 5. (a) Time of flight bias . (b) CRB and time of flight variance, for $\alpha = 0.01$ and two values of γ ($F_{ac} = 4000 Hz$ and $V_{f,q}=200 mm/s$).

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