SIMO Blind System Identification and Order Determination

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ABSTRACT

In most of the existing methods for the blind identification of single input multiple output (SIMO) systems, such as the subspace (SS) and Least Squares (LS), the highest order of the unknown channels is assumed to be known or is overestimated. In this paper we propose a new method based on the modified Levinson algorithm which makes no such assumption. This method gives us the exact orders of each of the unknown channels as long as at least one FIR channel is invertible. This algorithm when compared to the existing methods is robust, accurate and computationally simple. This method also exhibits better performance under noisy environments.

1. INTRODUCTION

Blind system identification (BSI) [6] addresses the problem of estimating the channel impulse responses from the output observations only. The unknown system could be multiple input multiple output (MIMO), or single input multiple output (SIMO) [7]. These problems have been studied using second (SOS) or higher-order statistics (HOS). In HOS-based methods large data samples are collected for reliable estimation. This was made simpler using SOS techniques, but at the expense of *priori* channel order information. Working with SOS based methods is extremely robust under time varying environments when compared to HOS. The SOS approach was proposed by Tong *et al.* Many versions of the SOS methods have also been developed. Among such methods are the subspace (SS) [4,5], Least squares (LS) and Linear prediction (LP) [7] methods and their variations. The SS methods are the most robust, but need exact or good estimate of the channel order. For LS and LP methods, the order could be overestimated but are very sensitive to observation noise. In all of these methods only the highest order of the channel is known, or estimated, at the expense of inferior performance. This also leads to the assumption that all the channel impulse responses are of equal length, which means overestimation of the order for some. The BSI (Blind System Identification) for a SIMO system could be arrived at by oversampling the observations at a sensor by a factor *P,* or by having *P* sensors maintaining the same sampling rate. Then the system becomes a SIMO with one source and *P* outputs. In this paper we present a novel method towards solving the SIMO BSI problem. Our method, based on the modified Levinson algorithm, does not require any prior order information of any channel as long as at least one channel modeled as an FIR filter is invertible. This method gives us the exact order for each of the FIR channels from the observations of the modified prediction error, and is robust and less sensitive to noise when compared to the existing SS based methods. It is also computationally inexpensive and extremely simple to implement, and does not require the EVD of any matrix. We start with the simple case when *P*=2, and then extend it for any *P*. We first start with the modified Levinson algorithm (MLA) described in [1], which characterizes an unknown system by a stable ARMA (*n, m*) using both first order and second order information. The method first described in [2] is applicable only when the input is a white noise sequence. In our method we go beyond such assumption and solve the problem for colored input processes. This is achieved by first whitening the input process and then applying the method in [2]. This problem is directly related to the SIMO for *P*=2. Likewise all the channels are assumed finite order FIR filters. This paper is organized as follows. In Section 2 we describe the problem of BSI for SIMO. In Section 3 is discussed in detail the MLA for the colored input process. In Section 4 is discussed the use of the MLA method to solve the BSI for SIMO system. Section 5 discusses the simulation results and shows the performance comparison of our method with the conventional SS [4] and Power of R (POR) [5] methods for noisy observations.

2. PROBLEM STATEMENT

In this section we explain the basic steps for the SIMO based BSI and the approach for a few existing methods. As mentioned earlier a SIMO model can be generated by oversampling or by using multiple sensors. All methods are applicable for both types of data observation. The setting for a single input/ multiple output is shown in Fig. 1.

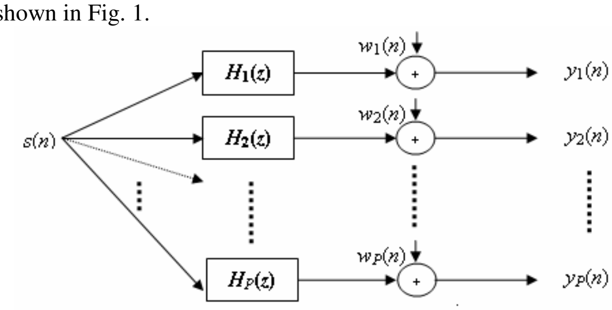

Fig. 1 SIMO model for BSI

Consider the mathematical model for any arbitrary source

$$y_i(n) = \sum_{j=0}^{M_i} h_i(j)\, s(n-j) + w_i(n)\,,\;\; 1 \le i \le P,\, 0 \le n \le N \qquad (1)$$

where, $w_i(n)$ is zero-mean, unit-variance, Additive white Gaussian noise at the i^{th} channel. M_i is order of the i^{th} channel impulse response and P is number of sensors, number of channels, or oversampling factor. The blind identification is in estimating the unknown vector **h** with dimension $P\,(M^i+1)\times 1$ which is the vector of all channel coefficients from the observations vector **y** only, and where $\mathbf{h}=[\mathbf{h}_1^T,\mathbf{h}_2^T,\ldots\ldots\mathbf{h}_P^T]^T$ with vector $\mathbf{h}_i$ representing coefficients of the i^{th} channel. This problem is solved under a few basic assumptions:

1. All of the channels are FIR, at least one of them being invertible.
2. All the channel impulse responses are mutually co-prime, i.e. they have no common zeros.
3. The number of observations *N* is greater than the maximum order of these channel impulse responses.
4. All the channels are of equal order and are known, say *M*.

For our MLA based method, assumption 4 is not required. For the existing SS and POR methods, except the invertible condition all the other assumptions are needed.

3. ARMA MODELLING OF A LTI SYSTEM

Consider a stable, linear time-invariant system $H(z)$ of order (*N*, *M*), characterized by a zero-mean stationary process $u(k)$ to get the output $y(k)$.

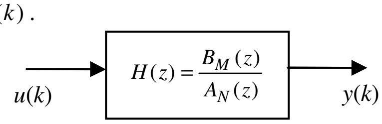

Fig. 2 ARMA model for an LTI system

Where, $A_N(z)$ is minimum phase. Based on the available information about the system $H(z)$, the input and output processes $u(k)$ and $y(k)$, we would like to approximate $H(z)$ by the model $\hat{H}(z)=\frac{Q_{M'}(z)}{P_{N'}(z)}$. If we denote $\hat{y}(k)$, an estimate of $y(k)$ based on the N' past values of $y(k)$ and the $M'+1$ present and past values of $u(k)$ we can write

$$\hat{y}(k)=-\sum_{n=1}^{N'}p(n)\;y(k-n)+\sum_{m=0}^{M'}q(m)\;u(k-m) \qquad (2)$$

The coefficients of the filters $Q_{M'}(z)$ and $P_{N'}(z)$ can be obtained by the minimization of the mean-squared error with respect to the filter coefficients. For the case when $u(k)$ is white noise, then the Levinson algorithm produces stable AR models, while the Roberts and Mullis algorithm [2] produces stable ARMA models for the system $H(z)$. As far as the AR modeling is considered, the Semi-whitening filter (SWF) [1,3] can be considered as an alternative to the conventional whitening filter. It is the special properties of the SWF that allows us to generalize the Robert and Mullis method for the case when $u(k)$ is a correlated process. In our method we first derive the SWF $F_L(z)=\sum_{l=0}^{L}f(l)\,z^{-l}$ for the input process $u(k)$ [1,3]. Pre-filtering of the correlated signals $u(k)$ and $y(k)$ by the invertible filter $F_L(z)$ will produce processes $s(k)$ and $v(k)$. Finally, minimization of the Mean-Square error $\hat{e}(k)$ using the MLA will result in the stable $\hat{H}(z)$ as the estimate of $H(z)$. The approach is depicted in Fig. 3. If $s(k)$ is zero-mean white process then the cross-correlation of $s(k)$ and $v(k)$ represent

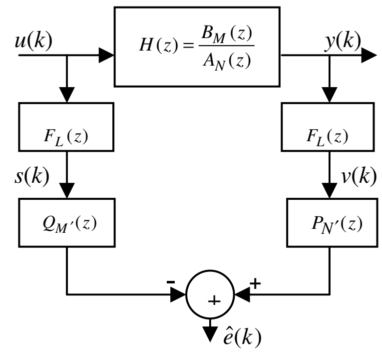

Fig. 3 Modified Robert and Mullis method

samples of the impulse response of $H(z)$ and the algorithm given in [2] can be used directly. In this case the $(n+1)\times(n+1)$ covariance matrix

$$\mathbf{K}(m,n)=\mathbf{R}_v-\mathbf{R}_{vs}\mathbf{R}_{vs}^T \qquad (3)$$

is used in implementing the MLA. $\mathbf{R}_v$ is the $(n+1)\times(n+1)$ autocorrelation matrix of $v(k)$, and $\mathbf{R}_{vs}$ is the $(n+1)\times(m+1)$ cross-correlation matrix of $s(k)$ and $v(k)$. Matrix $\mathbf{K}(m,n)$ is positive definite for $n<N$ and $m<M$, and semi-definite for $n\geq N$ and $m\geq M$. The semi-definiteness of $\mathbf{K}(m,n)$ is checked to determine the correct orders *N* and *M* for the estimation as *n* and *m* are incremented from the initial values of 1. In our method, $s(k)$ does not need to be white. It is obtained via the SWF $F_L(z)$ ensuring only first $\hat{M}\leq L$ autocorrelation lags of $s(k)$ to be zero beyond the $r_s(0)$. This property of $s(k)$ has been shown in [1] to be sufficient for applying MLA to obtain a stable $\hat{H}(z)$ and to determine orders *N* and *M* correctly. The proof is given in [1] and involves the following steps. Matrix $\mathbf{R}_{vs}$ is decomposed as $\mathbf{R}_{vs}=\mathbf{U}+\mathbf{C}$ where the first row of the (*n*+1) by (*m*+1) upper triangular Toeplitz matrix **U** is given by $[r_{vs}(0)\;\; r_{vs}(-1)\ldots\ldots r_{vs}(-m)]$ and **C** is a (*n*+1) by (*m*+1) lower triangular Toeplitz matrix with zeros on its main diagonal, and for *m*=*n*, its last row is defined by $[c\,(n)\;\; c\,(n-1)\ldots\ldots c(1)\;\; 0]$, where $c(j)=r_{vs}(j),\; 1\leq j\leq n$. We then derive the matrices $\tilde{\mathbf{W}}(m,n)=\tilde{\mathbf{R}}_n-\mathbf{U}\mathbf{U}^T$, where $\tilde{\mathbf{R}}_n=\mathbf{R}_v-\{\mathbf{C}\mathbf{U}^T+\mathbf{U}\mathbf{C}^T\}$, and

$$\mathbf{K}(m,n)=\tilde{\mathbf{W}}(m,n)-\mathbf{C}\mathbf{C}^T \qquad (4)$$

Matrix $\tilde{\mathbf{R}}_n$, is symmetric, Toeplitz and positive definite. The elements of the singular matrix $\mathbf{C}\mathbf{C}^T$ are in terms of the second powers of $r_{vs}(k)$ for $1\leq k\leq n$. Thus depending on both the process $u(k)$ and the how large the choice of $\hat{M}$ is, the approximation of $\mathbf{C}\mathbf{C}^T$ by a zero matrix becomes more appropriate. If $u(k)$ is an ARMA process, then $\mathbf{C}\mathbf{C}^T$ will quadratically approach zero making $r_{vs}(-k)$, $k \geq 0$, a consistent estimate of the impulse response of $H(z)$. Hence an approximation of (4) by matrix $\tilde{\mathbf{W}}(m,n)$ can be done by throwing away the $\mathbf{CC}^T$ matrix. It is shown in [1] that then the properties of the prediction error $\alpha(m,n)$, associated with the matrix $\mathbf{K}(m,n)$, are preserved for the prediction error $\tilde{\alpha}(m,n)$ which is defined for matrix $\tilde{\mathbf{W}}(m,n)$. Thus the algorithm discussed in [2] can be used to obtain a stable estimate $\hat{H}(z)$.

4. MLA ALGORITHM FOR SIMO BSI

In the previous section was characterization of an arbitrary ARMA model for any colored input process. In this section we shall show how this method can be adapted for solving the BSI problem for SIMO systems. We also show how this method gives us the exact order of all of the channels from the observations of the prediction error only - the $\tilde{\alpha}(m,n)$. For a single-input two-output case in Fig. 1, signals $y_1(n)$ and $y_2(n)$ are received/measured through the channels $H_1(z)$ and $H_2(z)$ of some orders M and N respectively. From Fig. 1, $Y_1(z) = H_1(z)S(z)$ and $Y_2(z) = H_2(z)S(z)$ and

$$\frac{Y_1(z)}{Y_2(z)} = \frac{H_1(z)}{H_2(z)} \tag{5}$$

The equivalence of (5) to Fig. 3 can be done by replacing $B_M(z)$ and $A_N(z)$ by $H_1(z)$ and $H_2(z)$ respectively. Now the BSI problem becomes identical to the ARMA modeling described in Section 3. Thus, we have to come up with the best ARMA estimate for (5) defined by $\hat{H}(z) = \frac{Q_{M'}(z)}{P_{N'}(z)}$. It was shown in [1] that the prediction error $\tilde{\alpha}(M',N') \to 0$ for $M' = M$ and $N' = N$. This property helps us in finding the exact orders of $H_1(z)$ and $H_2(z)$. The algorithm for finding the exact solutions is described below. In this method we identify two channels at a time. For the SIMO case, this can be done by taking two FIR channels at each instant and applying the above method to find which channel is invertible, transfer functions, and correct orders of each channel.

4.1. Algorithm to solve the SIMO BSI

Step 1: Collect N observations of $y_1(n)$ and $y_2(n)$. Generate the autocorrelation matrix $\mathbf{R}_{\mathbf{y}_2}$. Using this, solve the QFIR problem [1,3] choosing a sufficiently large $\hat{M}$ to obtain the SWF $F_L(z)$.

Step 2: Obtain the signals $s(k)$ and $v(k)$, and their autocorrelation and cross-correlation samples by passing $y_2(n)$ and $y_1(n)$ through $F_L(z)$. Set m=1, and define upper bounds $\tilde{M}$ and $\tilde{N}$.

Step 3: If $m \leq \tilde{M}$, set n=1 and go to *Step 4*. Else go to *Step 6*.

Step 4: Derive the symmetric, Toeplitz, positive definite matrix $\tilde{\mathbf{R}}_n$, and construct $\tilde{\mathbf{W}}(m,n)$. Solve the matrix equation $\tilde{\mathbf{W}}(m,n) \cdot \mathbf{p} = \tilde{\alpha}(m,n)\,\boldsymbol{v}$, where $\mathbf{p}^T = [1 \quad p(1) ....p(n)]$ and $\boldsymbol{v}^T = [1 \quad 0.........0]$. Record values of the $\tilde{\alpha}(m,n)$. If these values are not positive and monotonically decreasing, it means that the transfer function $H_2(z)$ is not minimum phase. Interchange the positions of $H_1(z)$ with $H_2(z)$ and $y_1(n)$ with $y_2(n)$ and go to *Step 1*. Else continue to *Step 5*.

Step 5: Set n=n+1, If $n \leq \tilde{N}$ go to *Step* 4. Else set m=m+1 and go to *Step 3*.

Step 6: Looking at the values of all $\tilde{\alpha}(m,n)$ obtained from *Step 4*, choose the minimum pair of (m, n) for which $\tilde{\alpha}(m,n) \to 0$. This gives the estimate of the true order $M' = M$ and $N' = N$.

Step 7: Knowing the exact orders M' and N' derive the symmetric, Toeplitz, positive definite matrix $\tilde{\mathbf{R}}_{N'}$ and $\tilde{\mathbf{W}}(M',N')$. Solve the matrix equation $\tilde{\mathbf{W}}(M',N')\mathbf{p} = \tilde{\alpha}(M',N')\,\boldsymbol{v}$ to obtain the $(N'+1)$ dimension vector $\mathbf{p}$.

Step 8: Determine the $(M'+1)$ dimension vector $\mathbf{q}$ from

$$\begin{cases} q(0) = r_{vs}(0) \\ q(k) = r_{vs}(-k) + \sum_{j=1}^{N'} r_{vs}(-k+j)\; p(j) \end{cases} \quad 1 \leq k \leq M',\; r_{vs}(k) = 0 \tag{8}$$

We can then directly equate $H_1(z) = Q_{M'}(z)$ and $H_2(z) = P_{N'}(z)$. The source $s(n)$ is easily obtained by using the invertible channel or one of the various recovery methods.

5. SIMULATION RESULTS

In this section we show the simulations and results for our method in comparison to SS and POR methods. The SS and POR methods have been developed using the same assumptions used in [4] and [5], i.e. the highest order of the channel transfer functions is assumed to be known. We first show the behavior of the prediction error for the MLA method which determines the exact order. If M is the number of zeros and N the number of poles, then three cases, M= N, M> N and M <N is chosen to show the behavior of the prediction error. It is assumed in all the cases that the transfer function chosen as the denominator is the minimum phase channel. Table 1 shows the result for the true order detection for the case M < N. A huge drop is clearly seen in the value of the prediction error for the right combination of m and n. Hence this method gives us the correct pair (m=3, n=5). Now that we have shown that this method gives us the correct order, we then proceed in solving the BSI for a SIMO. The performance of our method is compared with the conventional SS, and POR method for SNR level of 40dB and 20dB. For the high SNR case all methods perform equally well. For 20dB SNR it is observed that the MLA method performs the best, and all the methods perform badly for SNRs lower than 20dB. The source was chosen to be derived from an AR model with the transfer function:

$$D(z) = \frac{0.0147016845}{(1-0.9z^{-1})(1-0.9e^{j\pi/8}z^{-1})(1-0.9e^{-j\pi/8}z^{-1})}.$$

The simulations have been carried out for the two channel case, with one being a minimum phase system and the other being a mixed phase system. The values of

$H_1(z) = [1 + 0.5\,z^{-1} + 2.2025\,z^{-2} + 1.10125\,z^{-3} + z^{-4} + 0.5\,z^{-5}]$,

$H_2(z) = [1 + 0.3\, z^{-1} - 0.15\, z^{-2} + 0.2\, z^{-3}]$.

Figures 4, 5 show the performance for an SNR of 20dB. It is seen that all the methods perform very well and the source recovery is close to perfect. For both the channel 1 and channel 2 estimates, the SS and POR method sit on top of each other.

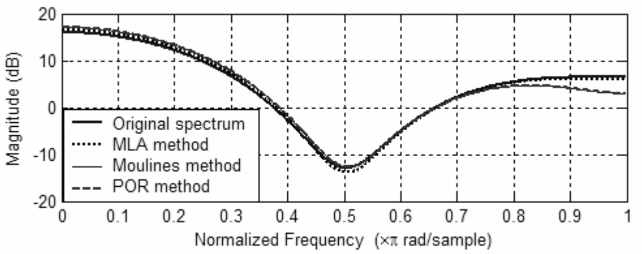

Fig. 4 Estimate of $H_1(z)$ for SNR 20dB

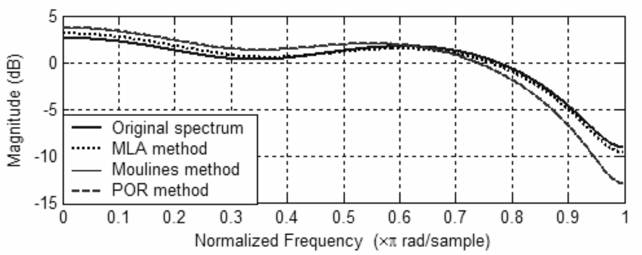

Fig. 5 Estimate of $H_2(z)$ for SNR 20dB

This is because the value to which the inverse of the autocorrelation matrix is raised is chosen to be 4. In [5] it was claimed that for powers $m \geq 3$, the performance of POR is same as SS. It is seen that our method out performs the SS and POR method. It was observed that for low SNRs the channel estimates obtained for MLA method are much closer to the original spectrum, and no ripples are seen in the case of phase recovery of the source, which is not the case for the SS and POR method.

We conclude that the MLA method performs better in the presence of low SNRs. It does not require the order information *apriori* which is the most important assumption for the other two methods to work. The SS needs two sets of EVD computation, which becomes computationally expensive when *N*, the number of observation samples, is large. Although the first EVD is not required in the POR, it is replaced by the inverse computation and matrix exponents. This is also computationally expensive when *N* is large. The MLA method on the contrary, is computationally inexpensive, as it does not depend on the number of observations *N*. All computations for the MLA use Levinson type of recursions, making it extremely robust. The only requirement for the MLA is that one of the channels must be minimum phase.

6. CONCLUSIONS

In this paper we devised a new method to solve the SIMO BSI problem. We derived the stable ARMA modeling for colored input processes and showed its equivalence to solving the SIMO BSI problem. We called this approach the MLA method. We showed how this method gives us the exact order for each of the channels which had not been addressed before in any accessible literature. The simulation results showed the MLA to have better performance as compared to the SS and the POR method for noiseless and noisy observations. We also discussed the complexity of our method to the SS and POR and concluded that our method is certainly less complex. We used the SWF to whiten the data. We have also observed that one could use a conventional whitening filter of large order in place of the SWF.

7. REFERENCES

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Table 1. Prediction errors for a maximum phase system for different (*m, n*)

Order *n*	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Order *m*							
1	0.098721	0.042212	0.018933	0.01798	0.003085	0.003071	0.000375
2	0.062496	0.026191	0.018519	0.017977	0.003077	0.003066	0.000375
3	0.029031	0.014293	0.010916	0.004054	1.25E-10	1.25E-10	1.25E-10
4	0.007643	0.002896	0.001828	0.001106	1.24E-10	7.05E-11	7.05E-11
5	0.0075	0.002737	0.000246	0.000163	1.24E-10	7.05E-11	6.73E-11
6	0.003364	0.002737	0.000142	0.000139	5.72E-12	7.05E-11	6.73E-11
7	0.002943	0.00042	5.27E-05	1.88E-05	4.16E-12	2.76E-12	6.73E-11