PARALLEL NLMS FILTERS WITH STOCHASTIC ACTIVE TAPS AND STEP-SIZES FOR SPARSE SYSTEM IDENTIFICATION

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ABSTRACT

Within the framework that two filters are working in parallel, Stochastic Taps NLMS (ST-NLMS) effectively chooses only active taps for adaptation, resulting in a good transient behavior when identifying long, sparse, echo path like systems. However, ST-NLMS still suffers from the inherent limitation of LMS. This necessitates a compromise between the opposing fundamental requirements of fast convergence rate and small misadjustment. Following the same block diagram as ST-NLMS, the Stochastic Step-size NLMS (SS-NLMS) scheme is proposed and integrated into the ST-NLMS framework. The combination leads to a novel algorithm called STS-NLMS, which adjusts step-size and active taps simultaneously. Extensive experiments demonstrate that substantial improvements in the speed of convergence are achieved by using the proposed algorithm in stationary environment outperforming both NLMS and ST-NLMS with the same small level of misadjustment. In addition, the proposed algorithm shows superior tracking capability when the system is subjected to an abrupt disturbance. Furthermore, if nonstationary environment is considered, the performance of the proposed algorithm is still satisfying.

1. INTRODUCTION

In various identification applications, the unknown plant is characterized by an impulse response consisting of a region of nonzero response with the remainder insignificant. Examples like "sparse" channels usually include network echo path [1]. Though network echo cancellers now have echo path of 64 ms in length, the active part of the echo path is usually less than 6 ms long. Additional taps of the filter are used to cover the unknown "flat delay" in the long distance network between echo canceller and hybrid/local-loop circuit [2].

Exact localization of nonzero parameters may accelerate the convergence of NLMS, as well as save computations wasted at the redundant taps. One of the earliest algorithms to identify sparse response is the so called adaptive delay filter [3], which iteratively finds each active tap and adapts tap-weight value. There are also different approaches in [4, 5, 6, 2]. A recently proposed active taps localization algorithm is ST-NLMS [7], which adapts two filters with different active taps in parallel. Their estimation errors are compared. The location and tap-weights of auxiliary filter's active taps are used for re-initializing the primary filter, if the latter converge faster than the former. The novelty of the ST-NLMS is to select the new active taps' position for the auxiliary filter in a stochastic manner.

An obvious disadvantage of the NLMS algorithm as well as its variants with active taps adaptation, is that compromise must be made between the transient (convergence and tracking) speed and the steady-state misadjustment. To deal with this problem, many variable step-size LMS algorithms have been developed. In addition, the so-called Complementary Pair LMS (CP-LMS) [8], Parallel Adaptation LMS [9], and Parallel Variable Step-size LMS [10] are proposed within the similar framework of multi-filters working in parallel. However, the above mentioned algorithms are not readily port to ST-NLMS. In this paper, following the same block diagram as ST-NLMS, Stochastic Step-size NLMS (SS-NLMS) scheme is proposed and integrated into the ST-NLMS framework. The combination leads to a novel algorithm called STS-NLMS, which adjusts step-size and active taps simultaneously. Extensive experiments demonstrate that substantial improvements in the speed of convergence are achieved by using the proposed algorithm in stationary and nonstationary environments outperforming both NLMS and ST-NLMS with the same small level of misadjustment.

The rest of this paper is organized as follows. ST-NLMS is briefly reviewed in section 2. SS-NLMS and STS-NLMS are introduced and analyzed in section 3 and section 4, respectively. In section 5, four sets of experiments are performed with the results presented in detail. Conclusions are summarized in section 6.

2. STOCHASTIC TAPS NLMS

Fig.1 shows the block diagram of the ST-NLMS algorithm, where two adaptive filters with different active taps are employed working in parallel.

Let x, d, e, and w_i, $0 \leq i < L$ denote the input data, desired response, estimate error, and tap-weights of the primary filter, respectively, where L is the maximum unknown system taps. For the primary filter, $\mathbf{w}_{\mathrm{A}} = [w_M, w_{M+1}, \cdots, w_N]^{\mathrm{T}}$ and $\mathbf{x}_{\mathrm{A},t} = [x_{t-M}, x_{t-M-1}, \cdots, x_{t-N}]^{\mathrm{T}}$ are defined as the active tap-weights and corresponding input data, respectively, where $[M, N]$ denotes the active taps' location. $\mathbf{w}$ and $\mathbf{x}$ denote the whole tap-weights vector and input vector for concision. For the auxiliary filter, the variables use the same denotations as that of the primary filter, but with a superscript "$\prime$". The recursion of ST-NLMS is displayed in Fig.2, where $\theta \sim \mathcal{U}(a, b)$ denotes that θ is a random number chosen from a uniform distribution on the interval $[a, b]$.

In ST-NLMS, only the active taps of these two filters are used for estimation and adaptation. Every K iterations, the primary filter will

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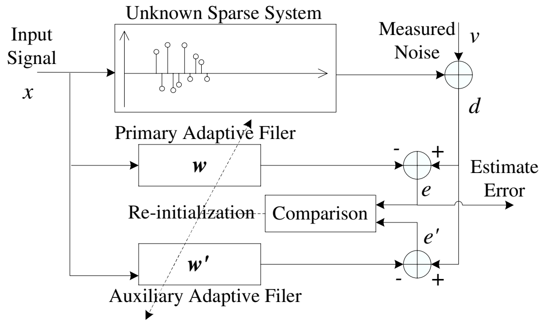

Fig. 1. The ST-NLMS algorithm block diagram.

Given: $L > 0$, $K = L$, $J = K/2$, $0 < \mu_{\min} < \mu_{\max} \leq 1$;
Initial: $\mathbf{w}_0 = \mathbf{w}'_0 = \mathbf{0}$, $M = 0$, $N = L - 1$,
$M' = \lfloor \theta \sim \mathcal{U}(0, L/2 - 1) \rfloor$,
$N' = \lfloor \theta \sim \mathcal{U}(L/2, L - 1) \rfloor$;
for $t = 1, 2, \cdots$
(1) $e_t = d_t - \mathbf{x}_{\mathrm{A},t}^{\mathrm{T}} \mathbf{w}_{\mathrm{A},t}$;
(2) $\mathbf{w}_{\mathrm{A},t+1} = \mathbf{w}_{\mathrm{A},t} + \mu \dfrac{e_t \mathbf{x}_{\mathrm{A},t}}{\mathbf{x}_{\mathrm{A},t}^{\mathrm{T}} \mathbf{x}_{\mathrm{A},t}}$;
(3) $e'_t = d_t - \mathbf{x}'^{\mathrm{T}}_{\mathrm{A},t} \mathbf{w}'_{\mathrm{A},t}$;
(4) $\mathbf{w}'_{\mathrm{A},t+1} = \mathbf{w}'_{\mathrm{A},t} + \mu \dfrac{e'_t \mathbf{x}'_{\mathrm{A},t}}{\mathbf{x}'^{\mathrm{T}}_{\mathrm{A},t} \mathbf{x}'_{\mathrm{A},t}}$;
if $\mathrm{mod}(t, K) = 0$
if $\sum_{j=0}^{J-1} e'^2_{t-j} < \sum_{j=0}^{J-1} e^2_{t-j}$
(5) $\mathbf{w}_{t+1} = \mathbf{w}'_{t+1}$;
(6) $M = M'$, $N = N'$;
end if
(7) $M' = \left\lfloor \theta \sim \mathcal{U}\left(0, \frac{M+N}{2}\right) \right\rfloor$,
$N' = \left\lfloor \theta \sim \mathcal{U}\left(\frac{M+N}{2} + 1, L - 1\right) \right\rfloor$;
(8) $\forall 0 \leq i < L$, $w'_{i,t+1} = \begin{cases} w_{i,t+1} & \text{if } M' \leq i \leq N'; \\ 0 & \text{otherwise.} \end{cases}$
end if
end for

Fig. 2. The pseudo code implementation of ST-NLMS.

become a *clone* of the auxiliary one, if the averaged estimate error of the latter is less than that of the former. Such condition indicates the auxiliary filter uses more adequate active taps and produces more accurate tap-weights.

The auxiliary filter will also be re-initialized to find the better active taps location, based on that of the primary filter. The basic idea of ST-NLMS is to try different taps in a stochastic way, which improves its efficiency especially in the condition of abrupt system change. The tap-weights of the auxiliary filter are also re-initialized according to the tap-weights of the primary filter, i.e., the auxiliary filter's active tap-weights are copied from that of the primary one and its inactive taps are reset to zeros.

3. STOCHASTIC STEP-SIZE NLMS

ST-NLMS effectively chooses only the active taps for adaptation, thus its transient behavior is very close to NLMS with *a priori* information on where the dominant unknown parameters exactly are. However, it still suffers from the inherent limitation which necessitates a compromise between the opposing fundamental requirements of fast convergence rate and small misadjustment. Based on the framework of two filters working in parallel, SS-NLMS algorithm is proposed in this section. Then via combining ST-NLMS and the new proposed variable step-size algorithm, a novel algorithm which adjusts step-size and active taps simultaneously will be presented in the following section.

The block diagram of SS-NLMS is exactly the same as that in Fig.1. Its pseudo code implementation is displayed in Fig.3. Unlike ST-NLMS, all of the tap-weights in SS-NLMS are adapted. Comparing their pseudo codes, one can find the only difference is that step-size, other than active taps location, is the variable to be find in stochastic manner. Simulation results proved that its transient behavior is very close to NLMS with μ_{max}, while keeping the misadjustment as small as NLMS with μ_{min}.

Given: $L > 0$, $K = L$, $J = K/2$, $0 < \mu_{\min} < \mu_{\max} \leq 1$;
Initial: $\mathbf{w}_0 = \mathbf{w}'_0 = \mathbf{0}$, $\mu = \mu_{\max}$, $\mu' = \theta \sim \mathcal{U}(\mu_{\min}, \mu_{\max})$;
for $t = 1, 2, \cdots$
(1) $e_t = d_t - \mathbf{x}_t^{\mathrm{T}} \mathbf{w}_t$;
(2) $\mathbf{w}_{t+1} = \mathbf{w}_t + \mu \dfrac{e_t \mathbf{x}_t}{\mathbf{x}_t^{\mathrm{T}} \mathbf{x}_t}$;
(3) $e'_t = d_t - \mathbf{x}_t^{\mathrm{T}} \mathbf{w}'_t$;
(4) $\mathbf{w}'_{t+1} = \mathbf{w}'_t + \mu' \dfrac{e'_t \mathbf{x}_t}{\mathbf{x}_t^{\mathrm{T}} \mathbf{x}_t}$;
if $\mathrm{mod}(t, K) = 0$
if $\sum_{j=0}^{J-1} e'^2_{t-j} < \sum_{j=0}^{J-1} e^2_{t-j}$
(5) $\mathbf{w}_{t+1} = \mathbf{w}'_{t+1}$;
(6) $\mu = \mu'$;
end if
(7) $\mu' = \theta \sim \mathcal{U}(\mu_{\min}, \mu_{\max})$;
(8) $\mathbf{w}'_{t+1} = \mathbf{w}_{t+1}$;
end if
end for

Fig. 3. The pseudo code implementation of SS-NLMS.

The original intention of SS-NLMS is to some extent similar to CP-LMS [8], both with two adaptive filters of different step-sizes working in parallel. But the stochastic property of SS-NLMS makes it easy to be merged in ST-NLMS, which yields fast convergence performance and small misadjustment to spares system identification.

4. STOCHASTIC TAPS AND STEP-SIZE NLMS

To combine ST-NLMS and SS-NLMS, a hybrid structure is adopted, refer Fig.4. The variables in the periodical re-initialization of the primary filter includes both active taps and step-size, which is an intuitive combination. However, the stochastic generation of new active taps location and new step-size for the auxiliary filter is performed alternatively. The reason for not to run them at the same time is to improve the probability that the auxiliary filter overcomes the primary filter, after the former utilizes the new actives or step-size. In the other words, the alternative implementation can quickly find a *better* step-size or active taps location. In addition, enlarging the constant P in Fig.4 enables successively searching one parameter for more times, which improves the success ratio further. However, a large P may produce unacceptable alteration time and decrease convergence rate. In the experiments, $P = 5$ is found to be a good balance. Though quantitative theoretical analysis on SS-NLMS and STS-NLMS is not included in this paper, extensive experiments are performed to demonstrate their excellent behaviors, both in transient and steady-state, in stationary and nonstationary environments.

Given: $L > 0$, $K = L$, $J = K/2$, $P = 5$, $0 < \mu_{\min} < \mu_{\max} \leq 1$;
Initial: $\mathbf{w}_0 = \mathbf{w}'_0 = \mathbf{0}$, $M = 0$, $N = L - 1$,
$M' = \lfloor \theta \sim \mathcal{U}(0, L/2 - 1) \rfloor$,
$N' = \lfloor \theta \sim \mathcal{U}(L/2, L - 1) \rfloor$,
$\mu = \mu_{\max}$, $\mu' = \mu_{\max}$;
for $t = 1, 2, \cdots$
(1) $e_t = d_t - \mathbf{x}_{\mathrm{A},t}^{\mathrm{T}} \mathbf{w}_{\mathrm{A},t}$;
(2) $\mathbf{w}_{\mathrm{A},t+1} = \mathbf{w}_{\mathrm{A},t} + \mu \dfrac{e_t \mathbf{x}_{\mathrm{A},t}}{\mathbf{x}_{\mathrm{A},t}^{\mathrm{T}} \mathbf{x}_{\mathrm{A},t}}$;
(3) $e'_t = d_t - \mathbf{x}'^{\mathrm{T}}_{\mathrm{A},t} \mathbf{w}'_{\mathrm{A},t}$;
(4) $\mathbf{w}'_{\mathrm{A},t+1} = \mathbf{w}'_{\mathrm{A},t} + \mu \dfrac{e'_t \mathbf{x}'_{\mathrm{A},t}}{\mathbf{x}'^{\mathrm{T}}_{\mathrm{A},t} \mathbf{x}'_{\mathrm{A},t}}$;
if $\mathrm{mod}(t, K) = 0$
if $\sum_{j=0}^{J-1} e'^2_{t-j} < \sum_{j=0}^{J-1} e^2_{t-j}$
(5) $\mathbf{w}_{t+1} = \mathbf{w}'_{t+1}$;
(6) $M = M'$, $N = N'$;
(7) $\mu = \mu'$;
end if
if $\mathrm{mod}\left(\left\lfloor \dfrac{t}{PK} \right\rfloor, 2 \right) = 0$
(8) $M' = \left\lfloor \theta \sim \mathcal{U}\left(0, \frac{M+N}{2}\right) \right\rfloor$,
$N' = \left\lfloor \theta \sim \mathcal{U}\left(\frac{M+N}{2} + 1, L - 1\right) \right\rfloor$;
else
(9) $\mu' = \theta \sim \mathcal{U}(\mu_{\min}, \mu_{\max})$;
end if
(10) $\forall 0 \leq i < L$, $w'_{i,t+1} = \begin{cases} w_{i,t+1} & \text{if } M' \leq i \leq N'; \\ 0 & \text{otherwise.} \end{cases}$
end if
end for

Fig. 4. The pseudo code implementation of STS-NLMS.

5. SIMULATIONS RESULTS

A comparison of the STS-NLMS, SS-NLMS, ST-NLMS with NLMS and Exact Taps NLMS (ET-NLMS) algorithm is demonstrated here for several cases using correlated stationary and non-stationary input data in both stationary and non-stationary environment. ET-NLMS, whose active taps are manually set according to that of the unknown system, can considered as the "optimum" solution for reference. Both NLMS and ET-NLMS run with $\mu_{\max} = 1$ and $\mu_{\min} = 0.1$, respectively. In all simulations of system identification we assume that $L = 256$ and the additive noise at observation is white Gaussian with zero mean. All of the mentioned adaptive algorithms are performed independently for 100 times, where the squared residuals are averaged and then processed by a low-pass Butterworth filter to provide a smooth expression.

Case 1: Stationary correlated input. In this case, the unknown impulse response is the echo path model 7 of ITU-T recommendation G.168 [1], delayed by 80 taps and tailed by 80 zeros, see Fig.5(a). The input data excites adaptive filter and unknown system is generated and followed by normalization, $x_{t+1} = \rho x_t + s_t$, where $\rho = 0.9$ and s_t is white Gaussian noise with zero mean. The variance of measured noise is 1E-3. The leaning curves of these algorithms are shown in Fig.6. While retaining the same level of misadjustment as NLMS ($\mu_{\min}$) and ET-NLMS ($\mu_{\min}$), SS-NLMS and STS-NLMS, clearly, provides the close speed of convergence as NLMS ($\mu_{\max}$) and ET-NLMS ($\mu_{\max}$), respectively.

Case 2: Abrupt change of unknown system. There is an abrupt change in the unknown impulse response, see Fig.5(b) and (c). Initially, the unknown channel is the echo path model 7, delayed by 40 taps and tailed by 120 zeros. At iteration number 5E4, it is replaced by the echo path model 8, delayed by 128 taps and tailed by 30 zeros. The high correlated input data is generated using the same method in the first case with $\rho = 0.98$. The variance of measured noise is 1E-2. The other settings are the same with the first case. Fig.7 shows the tracking performance of SS-NLMS is exact the same with its convergence behavior. As to STS-NLMS, there is, but only a little, loss in tracking speed. The active taps position of ST-NLMS and STS-NLMS are shown in Fig.8. Both these two algorithms can fleetly and exactly locate and run after the active taps. Fig.8 also shows how the step-sizes of SS-NLMS and STS-NLMS initially decrease and then immediately increase to the large value as a response to the abrupt change of unknown impulse response to provide a fast speed of tracking, as well as small misadjustment.

Case 3: Non-stationary input. The non-stationary input data in this case is generated similarly to the previous cases, but ρ is a time varying random number uniformly distributed between -0.5 and 0.5. All of the other parameters are as in the first case. The similar performance with stationary input is attained by ST-NLMS, SS-NLMS, and STS-NLMS as shown in Fig.9.

Case 4: Non-stationary input and environment. The adaptive filter in this case is used to model a time-varying system whose impulse response is generated by a random walk process, $\mathbf{h}_{t+1} = \mathbf{h}_t + \mathbf{g}_t$, where the unknown response $\mathbf{h}$ is initialized the same as that in the first case and $\mathbf{g}$ is white Gaussian vector with zero mean and variance 1E-3. All of the other parameters were as in the third case. Fig.10 shows the learning curves in this case. The steady state misadjustments of all algorithms, especially ET-NLMS, are damaged by nonstationary characteristic, which makes little difference in using large step-size and small step-size. However, STS-NLMS still retained the smallest misadjustment while keeping nearly the fastest transient convergence behavior.

6. CONCLUSION

In this paper, a novel NLMS-based adaptive algorithm with simultaneously varied active taps and step-size is proposed to better balance the trade-off between misadjustment and speed of convergence when identifying long sparse channel. Both of the active taps' location and step-size are stochastically generated according to the estimation errors of the two filters. Compared with the traditional NLMS, especially where the active taps are manually set, the transient and steady-state estimation performance of the proposed method are improved significantly in stationary cases, as well as non-stationary environments. A possible limitation to utilize the proposed algorithm is that dominant sparse taps are required to be concentrated on some parts of the long channel. However, in the applications such as network echo canceller, this constraint is generally satisfied.

7. REFERENCES

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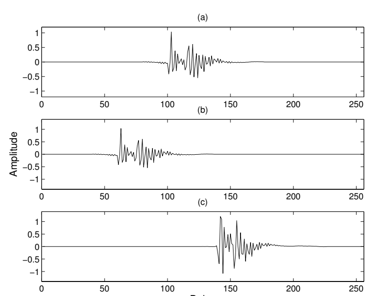

Fig. 5. The simulated unknown impulse responses for identification.

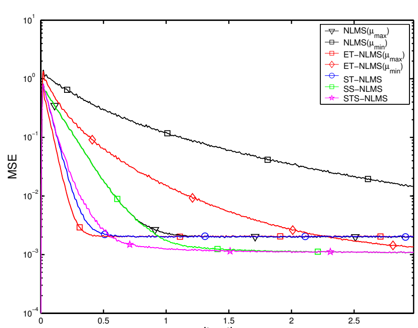

Fig. 6. Learning curves with correlated input.

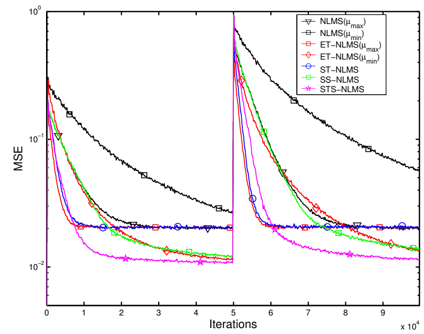

Fig. 7. Convergence and tracking performance for an abrupt disturbance in unknown system.

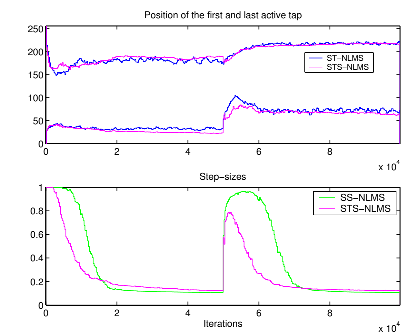

Fig. 8. Variable active taps position and variable step-sizes in convergence and tracking for an abrupt disturbance in unknown system.

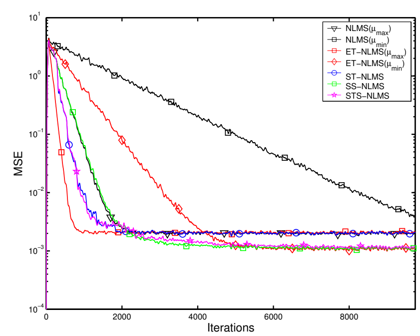

Fig. 9. Learning curves with nonstationary input.

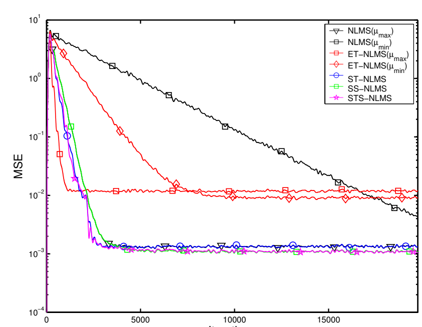

Fig. 10. Learning curves with nonstationary input in nonstationary environment.