A ROBUST VARIABLE STEP SIZE ALGORITHM FOR LMS ADAPTIVE FILTERS

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ABSTRACT

This work presents a modified version of the variable step size Kwong and Johnston's algorithm (VSS) for LMS adaptive filtering. The new proposal, called Robust Variable Step Size (RVSS), presents less sensitivity to the power of the measurement noise with only a very small increase in the computational complexity. A theoretical analysis demonstrates the main properties of the new algorithm. For white Gaussian input signals the RVSS presents the same performance than the original VSS in a noise free environment. Simulation results are provided, showing the better performance of the new algorithm. The RVSS should find application, for example, in telephony applications when double talking interferences are significant.

1. INTRODUCTION

Adaptive filters have been extensively applied to the telecommunication area, such as in hands-free telephony, hearing aids and audio and videoconference systems [1,2]. The Least Mean Square (LMS) adaptive filter family is attractive for implementation of real-time echo canceling and noise suppression systems due to its low computational complexity and robustness [3,4]. One important member of this family is the NLMS algorithm, which combines the simplicity of the conventional LMS algorithm with the robustness to the input signal power variation [5].

It is well known that the performance of LMS adaptive filters depends directly on the choice of the step size parameter. Larger step sizes lead to faster adaptation (convergence speed) at the expense of a larger misadjustment. Smaller step sizes can provide improved steady-state performance (smaller misadjustment) at the cost of a slower adaptation.

Variable step size strategies are common solutions for obtaining both fast tracking and good steady-state performance. However, in order to remain attractive for real-time applications, these strategies must be implemented with a minimum increase in the computational cost.

Several low cost step size adjustment criteria can be found in the literature. Among them, the most promising ones are based on the instantaneous square error [6,7], on signal changes of successive gradient estimations [8] and on the correlation between input and error signals [9,10]. However, experimental results show that the steady-state performance provided by these techniques can be highly dependent on the measurement noise power level. This high sensitivity can be explained by a residual steady-state term in the step size update equation that is proportional to the noise power. As a result, these algorithms tend to provide poor performance for low signal to noise ratios (SNR). A practical example occurs in network echo cancellation subjected to severe double-talking [2,7]. To overcome this problem, a double-talking detector must be used to stop the adaptation process in low SNR situations.

The variable step size (VSS) algorithm developed by Kwong and Johnston [6] provided an interesting strategy for LMS step size adjustment. Later on, authors of alternative variable step size algorithms have claimed a better performance than VSS [7,11]. More recently, the work in [12] demonstrated that VSS provides the step size sequence that is the closest to the optimum sequence when adequately designed. This result revived the interest in VSS. So far, VSS appears to lead to the best results in terms of convergence speed and misadjustment, even considering its intrinsic large sensitivity to the noise power.

This work proposes a modified version of the VSS algorithm that is less sensitive to the measurement noise, at the price of a small increase in computational cost. The new algorithm is called Robust Variable Step Size (RVSS).

Section 2 presents a brief review of the VSS algorithm. Section 3 introduces the RVSS algorithm and provides an analysis of its mean behavior. Section 4 compares the performances of VSS and RVSS for correlated and white signals. Section 5 presents simulations that corroborate the main theoretical results. Finally, Section 6 presents the main conclusions.

2. THE VSS ALGORITHM

The basic adaptive system block diagram is shown in Fig. 1. Here, n is the discrete time, $x(n)$ is the input signal, zero mean, Gaussian with power r_x. $d(n)$ is the desired signal, $y(n)$ is the output of the adaptive filter, $e(n)$ is the error signal and $z(n)$ is the measurement noise, independent of $x(n)$ and with power r_z. $\mathbf{w}(n) = [\ w_0(n)\ w_1(n)\ \ldots\ w_{N-1}(n)\ ]^T$ is the adaptive weight vector and $\mathbf{w^o} = [\ w^o_0\ w^o_1\ \ldots\ w^o_{N-1}\ ]^T$ is the impulse response of the unknown system. The error signal is given by

$$e(n) = z(n) - \mathbf{v}^T(n)\mathbf{x}(n) \tag{1}$$

where $\mathbf{x}(n) = [\ x(n)\ x(n-1)\ \ldots\ x(n-N+1)\ ]^T$ is the input signal vector and $\mathbf{v}(n)=\mathbf{w}(n)-\mathbf{w^o}$ is the weight error vector ($\mathbf{v}(n) = [\ v_0(n)\ v_1(n)\ \ldots\ v_{N-1}(n)\ ]^T$).

The VSS step size update equation is given by the following recursive equation [6]

$$\beta_{VSS}(n+1) = \alpha_{VSS}\beta_{VSS}(n) + \gamma_{VSS}e^2(n) \tag{2}$$

where α_{VSS} and γ_{VSS} are the control parameters. At each iteration, the result of (2) is bounded by predefined limits $[\beta_{MIN},\beta_{MAX}]$ in order to prevent unstable behavior and to maintain the tracking capability.

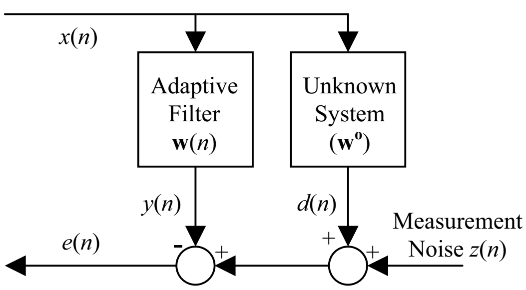

Fig. 1. Adaptive system.

2.1. Mean Behavior of the VSS

The mean behavior of the VSS is given by [6, Eq.(11),(15)]

$$E\{\beta_{VSS}(n+1)\} = \alpha_{VSS}E\{\beta_{VSS}(n)\} + \gamma_{VSS}\boldsymbol{\lambda}^T\tilde{\mathbf{k}}(n) + \gamma_{VSS}r_z \tag{3}$$

where $E\{\cdot\}$ means statistical expectation. $\boldsymbol{\lambda}=diag\{\boldsymbol{\Lambda}\}$, $\tilde{\mathbf{k}}(n)=diag\{\mathbf{Q}^TE\{\mathbf{v}(n)\mathbf{v}^T(n)\}\mathbf{Q}\}$ and $\boldsymbol{\Lambda}$ and $\mathbf{Q}$ are respectively the eigenvalue and eigenvector matrixes of the input signal correlation matrix ($\mathbf{R_{xx}}=E\{\mathbf{x}(n)\mathbf{x}^T(n)\}=\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^T$). $diag\{\mathbf{A}\}$ is a vector containing the main diagonal elements of matrix $\mathbf{A}$. The second order moments of the weight error vector depend on the chosen adaptive algorithm.

Eq. (3) was derived assuming statistical independence between the variable step size and the input signal. This assumption is only valid for slow adaptation. The measurement noise is assumed white.

3. RVSS UPDATE EQUATION AND ANALYSIS

The measurement noise influences the VSS behavior through the two last terms in the r.h.s. of (3). The first term (from left to right) is determined by the misadjustment of the adaptive algorithm. Practical adaptive filters are designed for small steady-state misadjustment. Thus, this term is not the main performance degradation factor. The rightmost term is proportional to the measurement noise power, which is independent of the adaptation process. This term can be reduced if (2) is modified to

$$\beta(n+1) = \alpha\beta(n) + \gamma\left[k\ \mathbf{x}^T(n)\mathbf{x}(n) - 1\right]e^2(n) \tag{4}$$

where k, α and γ are the control parameters. The effect of this modification on the algorithm behavior will become apparent after the analysis in the next subsection.

Comparing the computational complexities of (2) and (4), the latter requires only two extra multiplications and three extra additions per iteration, assuming that $\mathbf{x}^T(n)\mathbf{x}(n)$ is evaluated recursively. For normalized algorithms such as NLMS (which already requires the evaluation of $\mathbf{x}^T(n)\mathbf{x}(n)$), the computational cost increases only by one multiplication and one addition.

3.1. Mean Behavior of the RVSS

The design of the RVSS algorithm (4) requires the determination of the parameter k in order to compensate for the influence of the measurement noise in the mean step size behavior. Taking the expectation of (4) we obtain

$$\begin{aligned} E\{\beta(n+1)\} &= \alpha E\{\beta(n)\} - \gamma E\{e^2(n)\} \\ &\quad + k\gamma E\{\mathbf{x}^T(n)\mathbf{x}(n)e^2(n)\} \end{aligned} \tag{5}$$

where two expected values must be evaluated. The first one can be found in [6, Eq.(15)]:

$$E\{e^2(n)\} = tr\{\mathbf{R_x}\mathbf{K}(n)\} + r_z \tag{6}$$

The second expected value can be evaluated using the Gaussian moment factoring theorem [1], resulting in:

$$\begin{aligned} &E\{e^2(n)\mathbf{x}^T(n)\mathbf{x}(n)\} \\ &= E\{\mathbf{v}^T(n)\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{v}(n)\} \\ &-2E\{\mathbf{v}^T(n)\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{x}(n)\} \\ &+E\{z^2(n)\mathbf{x}^T(n)\mathbf{x}(n)\} \\ &= 2tr\{\mathbf{R_x}\mathbf{R_x}\mathbf{K}(n)\} + r_xN\ tr\{\mathbf{R_x}\mathbf{K}(n)\} + r_xNr_z \end{aligned} \tag{7}$$

Substituting (6) and (7) in (5) leads to

$$\begin{aligned} E\{\beta(n+1)\} &= \alpha E\{\beta(n)\} + 2k\gamma\ tr\{\boldsymbol{\Lambda}\boldsymbol{\Lambda}\tilde{\mathbf{K}}(n)\} \\ &\quad + \gamma(kr_xN-1)tr\{\boldsymbol{\Lambda}\tilde{\mathbf{K}}(n)\} + \gamma(kr_xN-1)r_z \end{aligned} \tag{8}$$

which can be expressed in vector form as

$$\begin{aligned} E\{\beta(n+1)\} &= \alpha E\{\beta(n)\} + 2k\gamma\ \boldsymbol{\lambda}_2^T\tilde{\mathbf{k}}(n) \\ &\quad + \gamma(kr_xN-1)\boldsymbol{\lambda}^T\tilde{\mathbf{k}}(n) + \gamma(kr_xN-1)r_z \end{aligned} \tag{9}$$

where $\boldsymbol{\lambda}_2=diag\{\boldsymbol{\Lambda}\boldsymbol{\Lambda}\}$.

3.2. Compensation of the Noise Influence

Examination of (9) shows that the last term in the r.h.s. is the main responsible for the effect of the noise power on the mean step size behavior. Differently from VSS, this effect can be minimized for RVSS through the appropriate choice of the free control parameter k. Assuming

$$k = 1/\left(r_x N\right) \tag{10}$$

and using (10) in (9) we obtain

$$E\left\{\beta\left(n+1\right)\right\} = \alpha E\left\{\beta\left(n\right)\right\} + \frac{2\gamma}{r_x N}\boldsymbol{\lambda}_2^T \tilde{\mathbf{k}}\left(n\right) \tag{11}$$

In nonstationary applications, the parameter k can be periodically estimated. For white input signals, (11) simplifies to

$$E\left\{\beta\left(n+1\right)\right\} = \alpha E\left\{\beta\left(n\right)\right\} + \frac{2\gamma r_x}{N}\sum_{i=0}^{N-1} k_i\left(n\right) \tag{12}$$

Eqs. (11) and (12) demonstrate that the proposed strategy is able to cancel the direct influence of the measurement noise power on the mean behavior of the RVSS algorithm.

4. COMPARISON BETWEEN VSS AND RVSS

For simplicity of analysis, assuming $\alpha=\alpha_{VSS}$ e $\gamma=N\gamma_{VSS}/2$ in (3) and (11), we obtain

$$\begin{cases} E\left\{\beta\left(n+1\right)\right\} = \alpha_{VSS} E\left\{\beta\left(n\right)\right\} + \gamma_{VSS}\sum_{i=0}^{N-1}\frac{\lambda_i^2}{r_x}\tilde{k}_i\left(n\right) & \text{(13a)} \\ E\left\{\beta_{VSS}\left(n+1\right)\right\} = \alpha_{VSS} E\left\{\beta_{VSS}\left(n\right)\right\} + \gamma_{VSS}\sum_{i=0}^{N-1}\lambda_i\tilde{k}_i\left(n\right) + \gamma_{VSS} r_z & \text{(13b)} \end{cases}$$

Comparing Eqs. (13a) (RVSS) and (13b) (VSS), notice that there is no direct power noise influence in the RVSS update (13a). This property confers to RVSS its extra robustness to the measurement noise power.

Assuming white input signals (13) can be expressed in closed form as

$$\begin{cases} E\left\{\beta\left(n\right)\right\} = \alpha_{VSS}^n \beta\left(0\right) + \gamma_{VSS} r_x \alpha_{VSS}^n \sum_{i=0}^{n-1}\sum_{j=0}^{N-1}\frac{k_j\left(i\right)}{\alpha_{VSS}^i} & \text{(14a)} \\ E\left\{\beta_{VSS}\left(n\right)\right\} = \alpha_{VSS}^n \beta_{VSS}\left(0\right) + \gamma_{VSS} r_x \alpha_{VSS}^n \sum_{i=0}^{n-1}\sum_{j=0}^{N-1}\frac{k_j\left(i\right)}{\alpha_{VSS}^i} & \text{(14b)} \\ \qquad +\left(1-\alpha_{VSS}^n\right)\frac{\gamma_{VSS}}{1-\alpha_{VSS}} r_z & \end{cases}$$

From (14a) and (14b), it is clear that RVSS has the same performance as VSS with r_z=0.

5. SIMULATIONS

In order to illustrate the characteristics and properties of the new algorithm three comparative simulations are presented between the VSS and RVSS.

Example 1: NLMS algorithm, white Gaussian input signal and low SNR – Input signal with unitary power r_x=1. White measurement noise with power r_z=0.15. $\mathbf{w^o}$=[2.8 2.8 2.8 2.8]^T; α_{VSS}=0.995; γ_{VSS}=0.01; $\alpha=\alpha_{VSS}$; $\gamma=N\gamma_{VSS}/2$; [β_{MIN},β_{MAX}]=(0,1] (maximum and minimum allowed step sizes). $\mathbf{w}$(0)=[0 0 0 0]^T $\beta_{VSS}(0)=\beta(0)$=0.8; 10000 runs.

Example 2: NLMS algorithm, white Gaussian input signal and measurement noise with abrupt power variation – Input signal with unitary power. White noise with r_z=10^{-8}. $\mathbf{w^o}$=[0.17 0.5 0.7 0.5 0.17]^T; α_{VSS} =0.99; γ_{VSS}=0.01; $\alpha=\alpha_{VSS}$, $\gamma=N\gamma_{VSS}/2$; [β_{MIN},β_{MAX}]=[0,1]. $\mathbf{w}$(0)=[0 0 0 0 0]^T. $\beta_{VSS}(0)=\beta(0)$=0.1; 10000 runs. At sample 3000 the amplitude of the measurement noise is multiplied by 7000.

Example 3: NLMS algorithm, correlated Gaussian input signal and medium SNR – Same conditions of Example 2 but with r_z=10^{-2}. Input signal generated by a second order autoregressive filter ($x(n)=a_1x(n$-1)+$a_2x(n$-2)+$u(n)$) with a_1=-0.3, a_2=0.8 and r_u=0.35 (input power to the model).

Figs. 2 to 4 show the mean square excess error $\varepsilon(n)$ = $E\{[e(n)$-$z(n)]^2\}$ for Examples 1 to 3. Fig. 5 presents the step size evolution for Example 3.

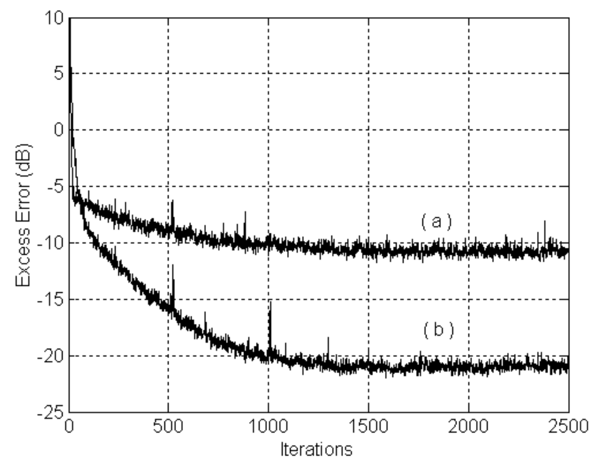

Fig. 2. Mean square excess error for Example 1. Comparisons between (a) VSS and (b) RVSS simulations.

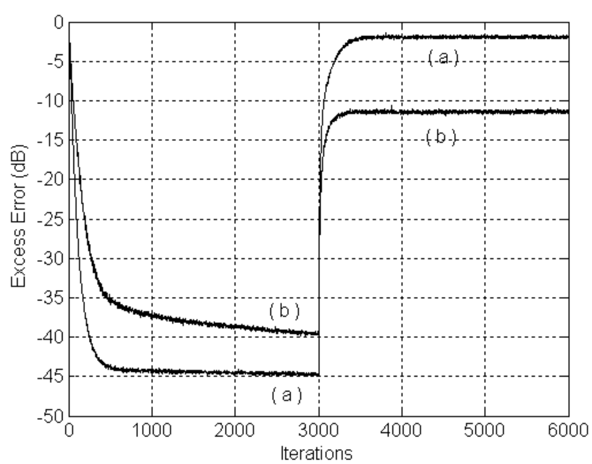

Fig. 3. Mean square excess error for Example 2. Comparisons between (a) VSS and (b) RVSS simulations.

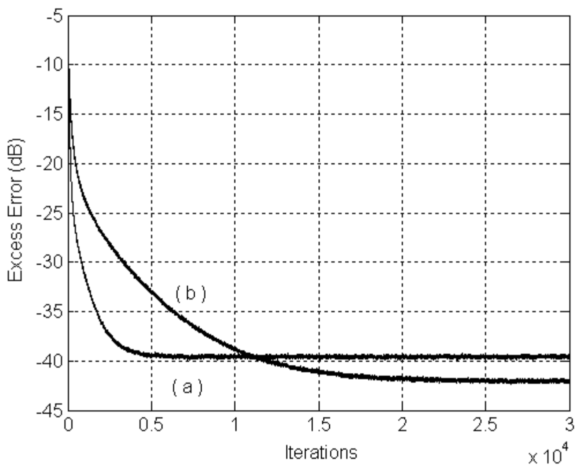

Fig. 4. Mean square excess error for Example 3. Comparisons between (a) VSS and (b) RVSS simulations.

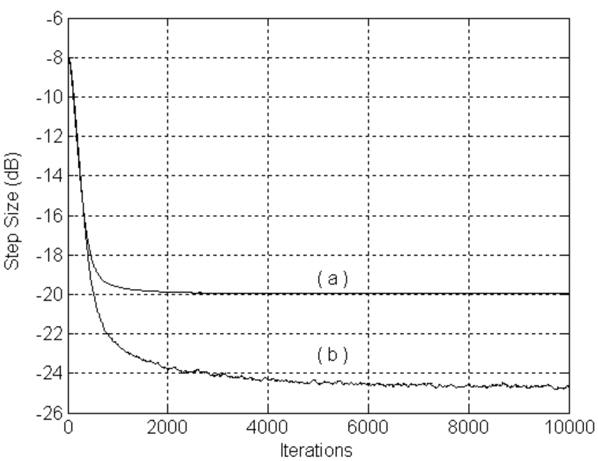

Fig. 5. Step-size evolution for Example 3. Comparisons between (a) VSS and (b) RVSS simulations.

These examples illustrate the ability of the RVSS algorithm to achieve high cancellation levels due to its low sensitivity to the measurement noise power. The improvement in the excess error is about 10 dB for Example 1 and 2.5 dB for Example 3. This difference can be explained by the difference in the SNR ratios (8.2 dB and 20 dB for Examples 1 and 3 respectively). The steady-state performance improvement of RVSS over VSS increases as the SNR gets smaller.

Fig. 3 demonstrates that the VSS original recovery ability for abrupt changes is retained by the RVSS. Note the RVSS had not achieved steady-state conditions before the nonstationary event.

Figs. 4 and 5 show that the main conclusions reached for white input signals are valid also for correlated inputs. The simulation results are in agreement with the theory, which can be used to explain the good properties of the new algorithm.

6. CONCLUSIONS

This work presented a new variable step size algorithm based on the original contribution of Kwong and Johnston's algorithm (VSS). Analysis has demonstrated that the new algorithm is less sensitive to the power of the measurement noise when compared to VSS, at the price of a very small increase in the computational cost. For white Gaussian input signals the RVSS algorithm presents the same performance than the original VSS algorithm in a noise free environment. Monte Carlo simulations illustrated the validity of the theoretical results. The RVSS algorithm is especially attractive for applications with low SNR.

7. ACKNOWLEDGMENTS

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