INSTANTANEOUS FREQUENCY RATE ESTIMATION BASED ON THE ROBUST CUBIC PHASE FUNCTION

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ABSTRACT

The cubic phase function (CPF) is recently proposed to estimate the instantaneous frequency rate (IFR) for the polynomial phase signals (PPS) in a Gaussian noise environment. However, for an impulse noise environment, the performance of the standard CPF degrades significantly. In addition, the resulting noise in the CPF is a mixture of the Gaussian and impulse noise even for a Gaussian input noise. Hence, a modified robust CPF algorithm based on the α-trimmed form of L-estimation is proposed in this paper. Extension to the robust higher-order phase function (HPF) is also derived. Simulation results demonstrate that the robust CPF outperforms the standard CPF in impulse noise and is also valid to estimate the IFR in Gaussian noise.

1. INTRODUCTION

Numerous signals used in technological applications, such as radar, sonar and communications, can be modeled as polynomial phase signals (PPS) with constant or slowly time-varying amplitude. Many techniques have been proposed for estimating parameters in this kind of signals. These methods can be categorized as nonparametric and parametric estimations. The first type consists of the time-frequency distributions (TFDs) [1,2], which display the signal over a jointly time-frequency plane. Among these TFDs, the Wigner-Ville distribution (WVD) and the polynomial WVD (PWVD) [3] have received much attention due to the excellent concentrations along the instantaneous frequency (IF) of a signal. The parametric approaches may be divided into the maximum likelihood estimation (MLE) [4] and the methods based on rank reduction. The direct implementation of the MLE for high-order PPS requires extensive computation. In contrast, the rank reduction has been associated with the polynomial phase transform (PPT) [5], the product high-order ambiguity function (PHAF) [6], the integrated generalized ambiguity function (IGAF) [7], the method based on the stationary higher-order moments [8] for computational implementation. Recently, a bilinear transform named as cubic phase function (CPF) [9] has been proposed to reveal the instantaneous frequency rate (IFR) of the PPS with order is not exceeding 3. For higher-order PPS, the higher-order phase function (HPF) from CPF is derived in the same way as the PWVD from the WVD.

The above mentioned methods are effective in the Gaussian noise environment. However, the assumption of Gaussian distribution of the input noise is not valid in some areas. For instance, the natural (from atmosphere or underwater phenomena) and the man-made disturbances have impulse characteristics with heavy-tailed distributions. For this kind of noise, the standard transform or time-frequency distributions fail to produce accurate results. Therefore, various robust forms of the unitary transforms and TFDs have been proposed in [10-13]. These robust estimators are based on the M-estimation [10,11] and the L-estimation, respectively [12,13].

In this paper, we develop robust forms of the CPF that are able to produce accurate IFR estimation for additive impulse noise environment. In particular, the α-trimmed form is proposed. The standard and median-based CPF can be treated as special cases of the α-trimmed form. For higher-order phase signal, the robust HPF is also discussed.

The rest of this paper is organized as follows. In Section 2, a brief review of the CPF is provided. The robust forms of CPF/HPF as IFR estimation tools for the impulse noise environment are developed in Section 3. Section 4 presents the simulation results that validate the proposed robust estimator. Some discussions are provided in Section 5. Concluding remarks are given in Section 6.

2. THE CUBIC PHASE FUNCTION

The CPF is defined as a two-dimensional (2-D) bilinear transform efficient for estimating the IFR [9]. Estimation of the IFR can be used as an initial step in estimating other phase parameters. The IFR of a signal $s(n)$ with phase $\phi(n)$ is defined as

$$\mathrm{IFR}(n) = d^2\phi(n)/dn^2 \ . \qquad (1)$$

The discrete CPF for a signal $s(n)$ is given by

$$CPF(n,\Omega) = \int_0^{+\infty} x(n+\tau)x(n-\tau)e^{-\mathrm{j}\Omega\tau^2} d\tau \ , \qquad (2)$$

where Ω represents the IFR. For a quadratic FM signal as, for example,

$$s(n) = Ae^{\mathrm{j}(a_0+a_1n+a_2n^2/2+a_3n^3/6)}, \ n \in \psi \qquad (3)$$

where $\psi = [-(N-1)/2 : (N-1)/2]$, N is odd, the CPF results in

$$CPF(n,\Omega) = A^2\xi(n)\int_0^{+\infty} e^{\mathrm{j}[(a_2+a_3n)-\Omega]\tau^2} d\tau$$
$$= \begin{cases} A^2\xi(n)\sqrt{\dfrac{\pi}{8\left|(a_2+a_3n)-\Omega\right|}}(1+j), & (a_2+a_3n) > \Omega \\ A^2\xi(n)\sqrt{\dfrac{\pi}{8\left|(a_2+a_3n)-\Omega\right|}}(1-j), & (a_2+a_3n) < \Omega \end{cases} \qquad (4)$$

where $\xi(n) = e^{\mathrm{j}2(a_0+a_1n+a_2n^2/2+a_3n^3/6)}$. Obviously, the CPF achieves maxima along the IFR $\Omega = a_2 + a_3 n$. By exploiting the dependence of IFR on time, an algorithm for parameter estimation for PPS signals with order is 3 is proposed in [9]. This algorithm is able to estimate other parameters using two slices of CPF. The selection of time positions is also discussed: $n = 0$ is used to reduce the variance of the third- and second-order phase coefficients and $n \approx 0.11N$ is used to lower the mean-square error (MSE) of the third-order phase coefficient.

3. THE ROBUST CUBIC PHASE FUNCTION

In this section, the robust CPF is proposed in order to perform the IFR estimation for impulse noise environments. The *M*- and *L*-estimation based forms of the robust CPF are developed.

3.1. *M*-estimation of CPF

Katkovnik first introduced the *M*-periodogram [10] based on the optimization of the loss function $F(e) = |e|$. Since this type of function does not produce a closed-form solution, the iterative procedure approach is used. In [11], the median filter approach was proposed without iterative procedures. In particular, the vector median and marginal median filters are implemented. Both approaches have similar performance. However, since the marginal median filter requires less calculation it is preferred in most practical applications.

Direct application of marginal median filter to the CPF yields:

$$\begin{aligned} &CPF_M(n,\Omega) = \\ &\mathrm{median}\{\mathrm{Re}\{x(n+m)x(n-m)\exp(-j2\pi\Omega m^2)\} : m \in \psi\} \qquad (5) \\ &+ j\mathrm{median}\{\mathrm{Im}\{x(n+m)x(n-m)\exp(-j2\pi\Omega m^2)\} : m \in \psi\}. \end{aligned}$$

It is worthy noting that the median-based CPF can introduce the spectral distortion effect, since only two modulated samples are used. Moreover, for a Gaussian input noise, resulting noise in CPF is inherently a mixture of Gaussian and impulse noise, due to bilinear nature [13]. In this case, the *L*-estimation based CPF can produce better results with respect to the *M*-estimation based CPF. Hence, the *L*-estimation based CPF is proposed in the next subsection, to estimate the IFR for a signal embedded in an impulse or a mixture of Gaussian and impulse noise.

3.2. *L*-estimation of CPF

By analogy with the *L*-estimation based TFDs [13] of complex-valued signal, the *L*-estimation based CPF can be defined as

$$\begin{aligned} CPF_L(n,\Omega) &= \mathrm{Re}\{CPF_L(n,\Omega)\} + j\,\mathrm{Im}\{CPF_L(n,\Omega)\} \\ &= \textstyle\sum_{i=0}^{N} a_i R_i(n,\Omega) + j\sum_{i=0}^{N} a_i I_i(n,\Omega) \end{aligned} \qquad (6)$$

where $\sum_{i=0}^{N} a_i = 1$, $R_i(n,\Omega)$ and $I_i(n,\Omega)$ are the values from the sets: $R_i = \{\mathrm{Re}\{x(n+m)x(n-m)\exp(-j2\pi\Omega m^2)\} : m \in \psi\}$ and $I_i = \{\mathrm{Im}\{x(n+m)x(n-m)\exp(-j2\pi\Omega m^2)\} : m \in \psi\}$, respectively, sorted into the non-increasing sequences. The coefficients a_i , for odd N , are given by

$$a_i = \begin{cases} \dfrac{1}{[\alpha(2-2N)+N]}, & i \in [\alpha N, N(1-\alpha)] \\ 0 & , \text{ elsewhere} \end{cases} \qquad (7)$$

where $0 \le \alpha \le 0.5$. Note that the standard form and median-based form of CPF can be obtained as special cases of (7) with α=0 and α=0.5, respectively.

1) The standard CPF is derived from (7) with $a_i = 1/N$, $i \in [0, N]$.

2) The median-based CPF follows from (7) with

$$a_i = \begin{cases} 1, \ i = (N-1)/2 \\ 0, i \ne (N-1)/2 \end{cases}, \text{ for odd } N . \qquad (8)$$

3.3 Extension to robust higher-order phase function

In [9], the CPF is extended to estimate the IFR of the PPS with order is exceeding 3. It can be defined in the discrete form as

$$HPF_{hL}(n,\Omega) = \textstyle\sum_m K^P(n,m)\exp(-j\Omega m^2) \qquad (9)$$

where P is the order of the signal phase, $K^P(n,m) = \prod_{i=1}^{N}[x(n+c_i m)^{k_i} x(n-c_i m)^{k_i}]^{r_i *}$, and the operator $[\cdot]^{r_i *}$ indicates conjugation of $[\cdot]$ iff $r_i = 1$. The parameters c_i , k_i , r_i and N are selected to yield unbiased

IFR estimates for a phase polynomial of order P in the same way that similar parameters were chosen to give unbiased estimates of the IF in [3].

Therefore, the robust L-estimation based form of HPF can be defined similarly as the robust CPF:

$$\begin{aligned} HPF_{hL}(n,\Omega) &= \mathrm{Re}\{HPF_{hL}(n,\Omega)\} + j\,\mathrm{Im}\{HPF_{hL}(n,\Omega)\} \\ &= \sum_{i=0}^{N} a_i R_{hi}(n,\Omega) + j\sum_{i=0}^{N} a_i I_{hi}(n,\Omega) \end{aligned}, \quad (10)$$

where $R_{hi}(n,\Omega)$ and $I_{hi}(n,\Omega)$ are the values from the sets: $R_{hi} = \{\mathrm{Re}\{K^P(n,m)\exp(-j\Omega m^2)\} : m \in \psi\}$ and $I_{hi} = \{\mathrm{Im}\{K^P(n,m)\exp(-j\Omega m^2)\} : m \in \psi\}$, respectively, sorted into the non-increasing sequences. The coefficient a_i, for odd N, are given by (7).

3.4 IFR estimation

For a noise-free PPS observation, the energy of the CPF/HPF concentrates along the IFR of the signal. Also, the peaks of the CPF/HPF yield unbiased estimate of the IFR. For a noisy PPS, the peaks of the standard CPF/HPF are sensitive to the impulse noise, whereas the robust estimators given by (6) and (10) have better performance in the impulse noise environment. Hence, it is possible to estimate the IFR from the peaks of the robust CPF/HPF. According to (1), the IFR is the second derivative of the phase. The IFR estimation is given as

$$\Omega(n) = \begin{cases} \arg\max\limits_{\Omega}(|CPF_L(n,\Omega)|), & p \le 3 \\ \arg\max\limits_{\Omega}(|HPF_{hL}(n,\Omega)|), & p > 3 \end{cases}. \quad (11)$$

Thus, the IFR at any time point can be obtained by using the argument that maximizes the magnitude of the robust CPF/HPF.

4. SIMULATIONS

Performances of the robust CPF both in the Gaussian and impulse noise are considered in this section. The noiseless signal is generated by (3) and the parameters are chosen to be $A=1$, $a_0=1$, $a_1=\pi/5$, $a_2=\pi/5N$, $a_3=-\pi/8N^2$, and $N=515$. Sampling rate is 1. A mixture of Gaussian and impulse noise corrupts the noiseless signal:

$$x(n) = s(n) + v(n) = s(n) + \beta v_1 + \lambda v_2^{\,3} \quad (12)$$

where v_i, $i=1,2$, are mutually independent complex white Gaussian noises with zero mean and unit variance, while β and λ are the parameters to control the noise distribution.

Example 1 compares the performances of the standard and robust CPF under different noise distributions. Three kinds of distributions are considered, while β is fixed to 0.4. The first one is the white Gaussian input noise with $\lambda=0$, the second is a mixture of Gaussian and impulse noise with $\lambda=0.4$, and the last is the mixed noise with $\lambda=0.8$. The results are shown in Fig. 1. It can be concluded that the standard CPF is valid for estimation only in small Gaussian input noise whereas the robust form is able to present the peaks in all three considered noise environments.

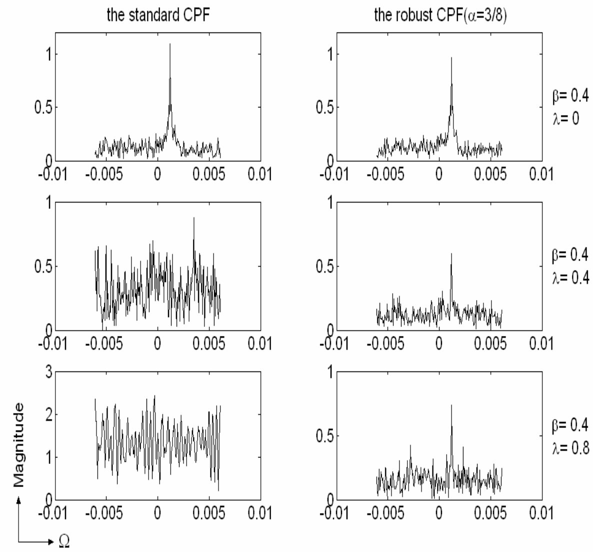

Fig.1. The standard and robust CPF (at the middle of time) of the signal in a mixture of the Gaussian and impulse noise. Top row — $\beta=0.4, \lambda=0$; Middle row — $\beta=0.4, \lambda=0.4$; Bottom row — $\beta=0.4, \lambda=0.8$.

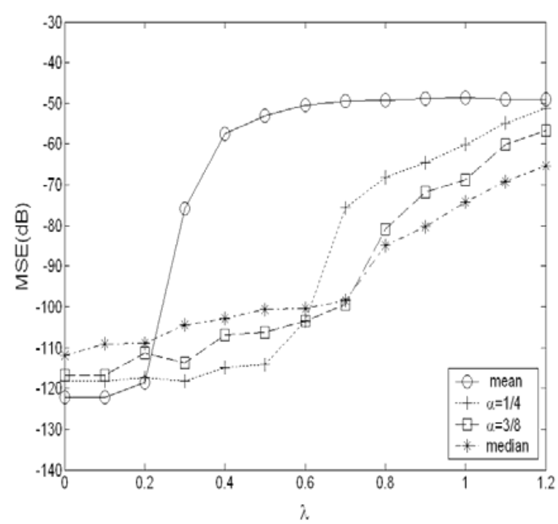

Fig.2. MSEs of IFR estimates (at the middle of time) based on the standard CPF and robust CPF with different α ($\beta=0.2$).

Example 2 evaluates the performance of the standard form and robust forms of CPF. The IFR at $n=0$ (corresponding to the IFR: 0.00122 Hz/s) is estimated according to (10) with 100 Monte-Carlo runs for each value of γ in the cases of $\beta=0.2$. The estimated MSEs are plotted in Fig.2. From this figure, it can be concluded that, the standard CPF is slightly better than the robust CPF for small impulse noise, whereas with an increase of γ, the median and two α-trimmed forms of the CPF are more insensitive to the impulse noise than the standard CPF. It is also shown that, for $0.2 < \gamma < 0.6$, the robust CPF with small α has the best performance among three robust estimators, while the median-based estimator has better performance than the other two α-trimmed mean estimators for $\gamma > 0.6$ in Fig.2.

5. DISCUSSION

From *Example 2*, the α-trimmed mean-based robust CPF (including the standard and median-based CPF) has different performance in different noise distributions. Hence, it is better to adaptively choose the appropriate coefficient α for minimizing the MSEs. An approach for automatic selection of the coefficient α can be found in [13].

In [14], the ML estimator in compound-Gaussian clutter is proposed for chirp signal. This technique can be extended for high-order PPS. However, it requires multidimensional grid search, leading to extensive computation and nonlinear programming. As to the bilinear transform exploiting the second-order cyclostationary, it can be efficiently implemented, but its application has not been developed for higher-order PPS, yet. Moreover, the robust CPF/HPF uses the estimated IFR to estimate the phase parameters simultaneously, that remarkably reduces the error propagation.

6. CONCLUSION

In this paper, the limitation of the standard CPF for impulse noise is pointed out. We considered the α-trimmed mean form of the robust CPF, which can produce better results than the *M*-estimation form since resulting noise in bilinear CPF is mixture of Gaussian and impulse noise when the input noise is Gaussian. Also, the extension to the robust HPF is discussed. Finally, the performances of the robust estimators have been evaluated using Monte-Carlo experiments. The results show that the *L*-estimation of CPF is more robust to the impulse noise influence than the standard CPF whereas the standard CPF is slightly better than the robust CPF for Gaussian noise or small impulse noise environment.

7. ACKNOWLEDGMENT

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