Spectral Correlation of the embolic blood Doppler signal

Jean-Marc Girault, *Member*, *IEEE*, Mathieu Biard*, Denis Kouamé, *Member*, *IEEE*, Aurore Bleuzen*, and François Tranquart*

Abstract— **In a previous study we have shown that take into account the quasi-cyclostationary properties of the blood Doppler signal is useful to detect embolus (red blood cell aggregates). In this latter approach, we have first considered a simple "off-line" synchronous detector. As results were very interesting, we have thought that the correlation spectrum could be an interesting alternative to the synchronous detection. As the correlation spectrum of *in vivo* signal seems to be too complicated, we propose here to elucidate this apparent complexity by analytically computing the correlation spectrum of Doppler signal model with or without embolus.**

Keywords— **Doppler, ultrasound, emboli detection, spectral correlation, cyclostationarity.**

I. INTRODUCTION

CE rebral vascular accidents, particularly cerebral embolisms, represent more than two thirds of all ischemic strokes. Indeed, several insoluble bodies (fat, red cell aggregation, clots ...) foreign to blood composition, called emboli, can move into intracranial arteries and can even block them. An illustration is given in figure(1). TransCranial Doppler ultrasound (TCD) systems have for several years been the most commonly used techniques in detecting and counting emboli. Detection of micro-emboli [1], [2], [3] (small size emboli) is important for several reasons such as preventing cerebrovascular accidents, finding the cause of embolism and validating the effectiveness of treatment. The underlying phenomenon of the embolism explains that the embolic Doppler signature is an unpredicted high intensity transient signal (HITS) superimposed on the Doppler signal backscattered by the blood. The information on which embolus detection must be based can therefore be the energy. This involves the combined use of an energy estimator and an energy detector. The standard techniques implemented in TCD systems seem to be sufficient to detect most of micro-embolic events. Nevertheless during clinical examinations, it sometimes happens that a medical expert observes micro-embolic signatures not detected by the system. This concern has led our team to analyze the signals in another way. By assuming that the Doppler signal is cyclostationary, we hypothesize that energy is statistically periodic. If we periodically take and compare the values of energy at different time points in the cardiac cycle, we can therefore detect the presence of non-periodic events such as micro-emboli. In a previous study [4] we have shown that we can considerably improve micro-emboli detection by using a synchronous detector.

The authors work at François Rabelais University, LUSSI FRE CNRS, Tours, France. * INSERM U619, Bretonneau Hospital, Tours, France. E-mail: jean-marc.girault@univ-tours.fr.

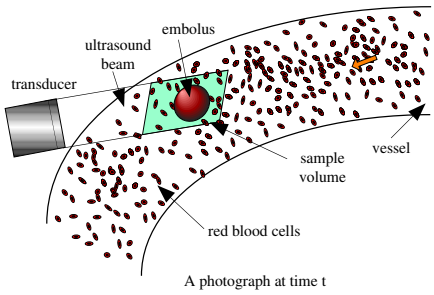

Fig. 1.
Sketch of a solid embolus in the sample volume.

We think that informative parameter extracted from the correlation spectrum can improve the embolus detection compare to the synchronous detector. This technique could be thus an interesting alternative to the synchronous detector. At first sight, the correlation spectrum of *in vivo* blood Doppler signal seems to be too complex to permit any informative parameters identification. In order to reach our goal, we propose an analytical modeling of the blood Doppler signal (with and without embolus). From this analytical model we analytically calculate the corresponding cyclic spectrum. The spectral correlation has proved its interest in another field like in gear faults diagnosis [5] but also in engine vibration analysis [6].

II. ANALYTICAL MODELING OF THE EMBOLIC BLOOD DOPPLER SIGNAL

A. Analytical modeling of the blood Doppler signal

The simplest and most common manner to model the blood Doppler signal is to consider that this latter corresponds to a narrow band filtered stochastic process. For example, in the case of the mean cerebral artery, the central Doppler frequency is around 450 Hz, the minimum and maximum frequencies are close to 250 HZ and 650 Hz, respectively. This non-stationary stochastic process, synchronized with the heart beating is both modulated in amplitude and in frequency. For the sake of simplicity, we consider that the blood Doppler signal is a deterministic signal. This simplification is equivalent to consider this random process in a average point of view.

For neophyte, notice that the mean frequency of the Doppler signal corresponds to the mean velocity of the blood flaw. The blood velocity is time varying with the cardiac beating. This implies that the blood Doppler signal is a frequency modulation signal. As the mean Doppler

frequency is quasi-cyclic, we can calculate a Fourier series. A simple case, illustrated in the figure (2), is to consider the frequency modulation as a sinus law. The spectral complexity, i.e. the number of harmonics depends of the vascular observed site. For the mean cerebral artery, a reasonable number to well describe the mean Doppler frequency is around 4 harmonics. In a previous study [4], we have shown that the mean cardiac cycle for healthy patients is near of one second with a standard deviation around 20%. Here we assume that mean Doppler frequency is periodic, involving that the Doppler signal is purely cyclostationary.

Here, we propose that the analytical modeling of the Doppler signal $y(t)$ is a combination of three components:

$$y(t) = A(t)B(t)C(t). \quad (1)$$

The first component $A(t)$, named "carrier", is a monofrequency signal:

$$A(t) = e^{j\omega_d t}, \quad (2)$$

where ω_d is the mean Doppler frequency.

The second term $B(t)$ modulates the amplitude the first component. This amplitude modulation term is periodic (cardiac cycle) and can be expressed as a Fourier series:

$$B(t) = \sum_{k=0}^{\infty} a_k e^{jk\omega_c t}, \quad (3)$$

where ω_c is the cyclic fundamental frequency and where a_k are the amplitudes of the different harmonics, $a_0 = 1$. These amplitude coefficients play the role of amplitude modulation indexes.

The third component modulates the frequency of the last two terms. This frequency modulation term is periodic (cardiac cycle) and can be expressed as a Fourier series:

$$C(t) = e^{j\sum_{k=1}^{\infty} a_k sin(k\omega_c t)}, \quad (4)$$

where ω_c is the cyclic fondamental frequency and where a_k are the amplitudes of the different harmonics, $a_0 = 1$. These amplitude coefficients play the role of modulation frequency indexes. Note that the frequency modulation term can also be expressed by a Fourier series:

$$e^{ja_1 sin(\omega_c t)} = \sum_{u=-\infty}^{\infty} J_u(a_1) e^{ju\omega_c t},$$

where $J_u(a_1)$ is the first kind of the Bessel function of order u. We can rewrite the component $C(t)$ as followed:

$$C(t) = \prod_{k=1}^{\infty} \sum_{u=-\infty}^{+\infty} J_{k,u}(a_k) e^{jku\omega_c t}, \quad (5)$$

where $J_{k,u}(a_k)$ is a first kind of Bessel function of order u of the harmonic k.

B. Analytical modeling of the embolic blood Doppler signal

The finite dimension of the insonified sample volume implies that the embolic signature is limited in time. Its duration is inversely proportional to the speed of the blood flow. Furthermore the energy scattered by the embolus is much more higher than the one scattered by red blood cell. This is simply justified by indicating that the embolus size is much higher than red blood cells. A simple way to take into account the presence of an embolus in the insonified area is to superimpose to the blood Doppler signal a higher amplitude of limited duration:

$$z(t) = y(t)D(t), \quad (6)$$

where

$$D(t) = 1 + \gamma E(t), \quad (7)$$

and where $E(t)$ is a function of limited duration as for example a rectangular signal or a Hamming function. γ is a weighted coefficient indicating the over-intensity.

As an illustration, we have reported in the figure (2) a simulated blood embolic Doppler signal. We can see both the amplitude and frequency modulations and the high intensity signature due to the embolus presence.

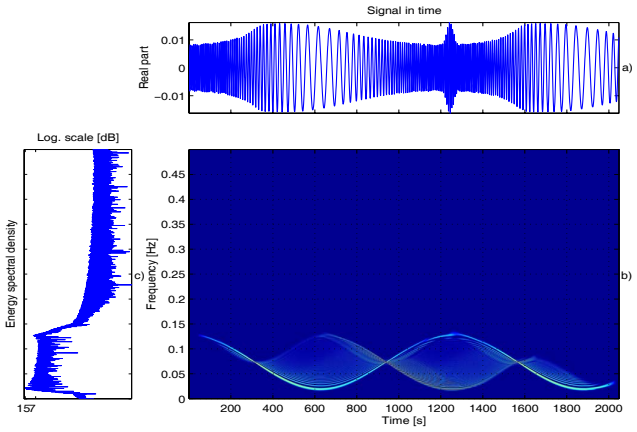

Fig. 2.
Some representations of the simulated embolic Doppler signal.a) Time representation. b) Wigner-Ville representation. c) Fourier spectrum

Having indicated the analytical relationships of the embolic Doppler signal, focus now to its corresponding correlation spectrum.

III. CORRELATION SPECTRUM OF THE EMBOLIC BLOOD DOPPLER SIGNAL

To a better understanding of the cyclic spectrum of the embolic Doppler signal, we propose to calculate the cyclic spectrum in several steps.

First of all, we recall the definition of the correlation spectrum of a signal $y(t)$ [7]:

$$S_{yf}^{\alpha} = \int\int y(t+\tau/2)y^*(t-\tau/2)e^{-j2\pi(f\tau+\alpha t)}d\tau dt. \quad (8)$$

This 2D representation corresponding to this correlation spectrum shows how is distributed the energy for different value of frequency and cyclic frequency.

A. Correlation spectrum of $A(t)$

The correlation spectrum of the "carrier" term $A(t)$, by using (2) and (8) becomes:

$$S_{A}{}_{f}^{\alpha} = \delta_{f-f_d}^{\alpha}. \tag{9}$$

This correlation spectrum is a simple Dirac function centered at $\alpha = 0$ and $f = f_d$. This cyclic spectrum is depicted in the figure (3a). Note that this signal is not cyclostationary because its cyclic spectrum does not have cyclic components for $\alpha \neq 0$. The cyclic spectrum of $A(t)$ is strictly equals to its Fourier spectrum: $S_{A}{}_{f}^{\alpha} = S_A(f) = \delta(f - f_d)$.

B. Correlation spectrum of $B(t)$

The cyclic spectrum of the amplitude modulation term $B(t)$ (see equation (3)) can be expressed by:

$$S_{B}{}_{f}^{\alpha} = \sum_{m=0}^{+\infty}\sum_{n=0}^{+\infty} a_m a_n \delta_{f-(n+m)f_c/2}^{\alpha-(m-n)f_c}. \tag{10}$$

As this correlation spectrum has got non zero components for $\alpha \neq 0$, $B(t)$ is a cyclostationary signal. The cyclic spectrum points out links between each harmonics components. As an example, assume that the Fourier series is composed of only one term ($N = 1$, $m = 0, 1$ and $n = 0, 1$ in (10)), the corresponding cyclic spectrum is:

$$S_{B}{}_{f}^{\alpha} = \delta_{f}^{\alpha} + a_1\delta_{f-f_c/2}^{\alpha+f_c} + a_1\delta_{f-f_c/2}^{\alpha-f_c} + a_1^2\delta_{f-f_c}^{\alpha}.$$

$B(t)$ is a modulated signal centered around the carrier frequency f_c. For $\alpha = 0$, we recognize the classical spectrum of an amplitude modulation signal. This cyclic spectrum is represented in the figure (3b). We can verify that each cyclic component (Dirac function) are spaced by f_c while each frequency component are spaced by $f_c/2$. Note also that the four components are weighted differently.

C. Correlation spectrum of $C(t)$

The cyclic spectrum of the frequency modulation term $C(t)$ (see equation (5)) can be expressed by:

$$S_{C}{}_{f}^{\alpha} = \prod_{k=1}^{\infty}\sum_{u=-\infty}^{+\infty}\sum_{v=-\infty}^{+\infty} J_{k,u}(a_k)J_{k,v}(a_k)\delta_{f-k(u+v)f_c/2}^{\alpha+k(u-v)f_c}. \tag{11}$$

As an example, assume that the Fourier series is composed of only one term ($N = 1$, $k = 1$ in (11)), the corresponding cyclic spectrum is:

$$S_{C}{}_{f}^{\alpha} = \sum_{u=-\infty}^{+\infty}\sum_{v=-\infty}^{+\infty} J_{1,u}(a_1)J_{1,v}(a_1)\delta_{f-(u+v)f_c/2}^{\alpha+(u-v)f_c}.$$

As for a classical Fourier spectrum, the number of components is directly related to the frequency modulation index a_1. Each spectral lines, spaced by f_c in the cyclic axis and $f_c/2$ in the frequency axis, are weighted by the Bessel function. An illustration is given in figure (3c).

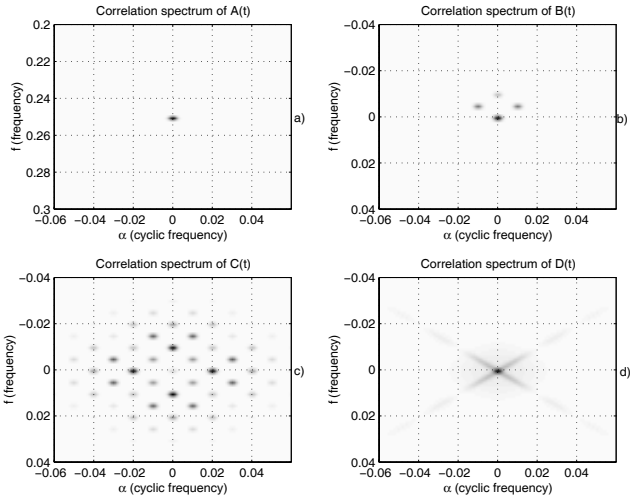

Fig. 3.
Correlation spectrum for the four elementary signals. $Fe = 1$, $f_d = 0.25$, $f_c = 0.01$, $a_1 = 2$.

D. Correlation spectrum of $D(t)$

The cyclic spectrum of the embolic signature (see (7)) can be expressed by :

$$S_{D}{}_{f}^{\alpha} = \delta_{f}^{\alpha} + \gamma^2 S_{E}{}_{f}^{\alpha} + \gamma E_{f+\alpha/2}^{\alpha} + \gamma E_{f-\alpha/2}^{*\alpha}. \tag{12}$$

The cyclic spectrum is composed of four terms. The first is equivalent to a "continuous" component. The second term $S_{E}{}_{f}^{\alpha}$ is the correlation spectrum of $E(t)$. In the last two terms, the frequency components depend linearly on the cyclic frequency. Consequently, the last two terms represented by a croze correspond to two straight lines passing by zero and having a slope $\pm\alpha/2$. These lines are modulated in amplitude by the Fourier spectrum $E(f)$. For example, if $E(t) = Rect_T(t)$, the corresponding spectral correlation and the Fourier spectrum can be written respectively by:

$$S_{E}{}_{f}^{\alpha} = \frac{e^{-2\pi j\alpha T/2}}{-2\pi j\alpha}\left(\delta(f-\alpha)e^{-j\pi\alpha} + \delta(f+\alpha)e^{j\pi\alpha}\right),$$

$$E(f) = Tsinc(\pi fT).$$

An illustration is given in figure (3d). Finally, the presence of an embolus appears in the correlation spectrum by a croze centered over each Dirac.

E. Correlation spectrum of the blood Doppler signal

The spectral correlation of the blood Doppler signal becomes:

$$S_{y}{}_{f}^{\alpha} = S_{A}{}_{f}^{\alpha} *_{f,\alpha} S_{B}{}_{f}^{\alpha} *_{f,\alpha} S_{C}{}_{f}^{\alpha}, \tag{13}$$

where $*_{f,\alpha}$ denotes the double convolution over f and α. From the relationships (9), (10), (11), (12), the cyclic spectrum becomes:

$$S_{y}{}_{f}^{\alpha} = \prod_{k=1}^{\infty} \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} J_{k,u}(a_k) J_{k,v}(a_k) \qquad (14)$$
$$\left(\sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} a_m a_n \delta^{\alpha+(k(u-v)+(m-n))f_c}_{f-(f_d+(k(u+v)+(m+n))f_c/2)} \right).$$

For example, by limiting the harmonics number to $N = 1$ ($k = 1$ in (14)), the corresponding cyclic spectrum is:

$$\begin{aligned} Sy_f^{\alpha} = & \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} J_{1,u}(a_1) J_{1,v}(a_1) \delta^{\alpha-(u-v)f_c}_{f-(f_d+(u+v)\frac{f_c}{2})} \qquad (15) \\ & + a_1^2 J_{1,u}(a_1) J_{1,v}(a_1) \delta^{\alpha-(u-v)f_c}_{f-(f_d+(2+u+v)\frac{f_c}{2})} \\ & + a_1 J_{1,u}(a_1) J_{1,v}(a_1) \delta^{\alpha-((1+u-v)f_c)}_{f-(f_d+(1+u+v)\frac{f_c}{2})} \\ & + a_1 J_{1,u}(a_1) J_{1,v}(a_1) \delta^{\alpha+((1+v-u)f_c)}_{f-(f_d+(1+u+v)\frac{f_c}{2})}. \end{aligned}$$

To illustrate, we have reported the correlation spectrum of a simulated blood Doppler signal in the figure (4a).

F. Correlation spectrum of the blood embolic Doppler signal

The correlation spectrum of the embolic blood Doppler signal is finally:

$$S_{z}{}_{f}^{\alpha} = S_{y}{}_{f}^{\alpha} *_{f,\alpha} S_{D}{}_{f}^{\alpha}. \qquad (16)$$

Each cyclic spectral components of the blood Doppler signal are "corrupted" by straight lines coming from the embolic signature.

$$\begin{aligned} S_{z}{}_{f}^{\alpha} = \prod_{k=1}^{\infty} \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} J_{k,u}(a_k) J_{k,v}(a_k) \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} a_m a_n \qquad (17) \\ (\delta^{\alpha+(k(u-v)+(m-n))f_c}_{f-(f_d+k(u+v)+(m+n))f_c/2} \\ + \gamma^2 a_m a_n S_{E}{}^{\alpha+(k(u-v)+(m-n))f_c}_{f-(f_d+k(u+v)+(m+n))f_c/2} \\ + \gamma a_m a_n E^{\alpha+(k(u-v)+(m-n))f_c}_{f-(f_d+\alpha/2+k(u+v)+(m+n))f_c/2} \\ + a_m a_n E^{*\alpha+(k(u-v)+(m-n))f_c}_{f-(f_d+\alpha/2+k(u+v)+(m+n))f_c/2}). \end{aligned}$$

To illustrate, we have reported the correlation spectrum of a simulated embolic blood Doppler signal in the figure (4b). As forecasted, the correlation spectrum is a spectrum line centered around $\alpha = 0, f = f_d$ as for the example depicted in figure (3b). Note the presence of crozes convolves at each Dirac functions.

It seems that a good way to be followed, to detect the presence of embolic signature is to identify straight lines in the cyclic spectrum. An interesting alternative would be to detect the presence of linear frequency modulation in the dual space of the cyclic spectrum, as for evaluating a signal from the spectrogram. In this case, its seems that the fractional Fourier transform could be a good candidate tool to reach our goal.

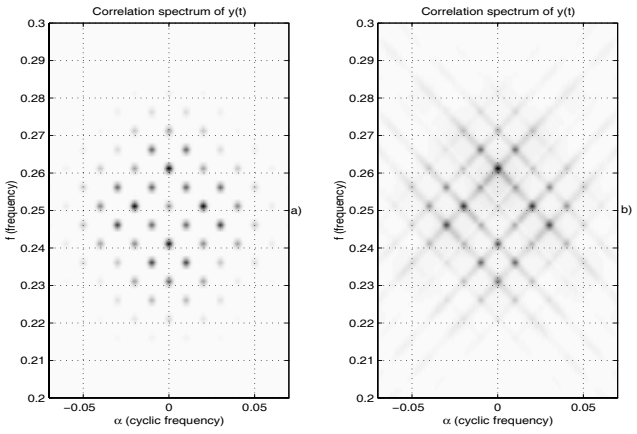

Fig. 4.
Correlation spectrum of the simulated blood Doppler signal without embolus (a) and with embolus (b). $Fe = 1$, $f_c = 0.01$, $f_d = 0.25$, $a_1 = 2$.

IV. CONCLUSION

In this study we have proposed an analytical deterministic model of the blood embolic Doppler signal. We have evaluated the correlation spectrum of the analytical model of the blood Doppler signal without or with an embolus. This study permits to better understand the different spectral components of the Doppler correlation spectrum. It seems that an attractive way to detect the presence of an embolus is to detect the cyclo-frequential straight lines or the "linear frequency modulation" in the dual representation.

V. Acknowlegdement

This study has been supported by the European community (UMEDS Project) and by the French government (CASC Project).

References

[1] M.P. Spencer, G.I. Thomas, S.C. Nicholls and L.R. Sauvage *Detection of middle cerebral artery emboli during carotid endarterectomy using transcranial Doppler ultrasonography.* Stroke, 21, 415–423, 1990.

[2] J. Smith, D. Evans, and R. Naylor, *Analysis of the frequency modulation present in doppler ultrasound signal may allow differentiation between partiulate and gaseous cerebral emboli,* Ultrasound in medecine and biology, 23, pp. 727–734, 1997.

[3] J.M. Girault, D. Kouamé, A. Ouahabi and F.Patat. *Microembolic detection: an ultrasound Doppler signal processing view point.* Ieee Trans. Biomed. Eng., 47, 1431–1439, 2000.

[4] J.M. Girault, M. Biard, D. Kouamé, G. Souchon, A. Bleuzen and F. Tranquart. *Etude de la cyclostationnarité du signal Doppler sanguin pour la détection de micro-emboles : étude préliminaire.* Gretsi, Louvain, 2005.

[5] C. Capdessus, M. Sidahmed et J.L. Lacoume. *Cyclostationary processes: application in gear faults early diagnosis.* Mechanical systems and signal processing, 371–385, 2000.

[6] J. Antoni, J. Daniere et F. Guillet. *Effective vibration analysis of IC engines using cyclostationarity. Part 1 - A methodology for condition monitoring*, Journal of sound and vibration, 815–837, 2002.

[7] W.A. Gardner. *Statistical Spectral Analysis: A non probabilistic theory.* Englewood Cliffs NJ - Prentice-Hall, 1987.