SMOOTHNESS CONSTRAINT FOR THE ESTIMATION OF CURRENT DISTRIBUTION FROM EEG/MEG DATA

Wakako Nakamura[1,2]*, Sachiko Koyama*[3]*, Shinya Kuriki*[3] *and Yujiro Inouye*[1,2]

[1]Interdisciplinary faculty of science and engineering, Shimane university,
1060 Nishikawatsu-cho, Matsue 690-8504, Japan
[2] Lab for Advanced Brain Signal Processing, Brain Science Institute, RIKEN,
2-1 Hirosawa, Wako, Saitama 351-0198, Japan
[3]Lab of electromagnetic sensing, Research institute for electronic science, Hokkaido university,
Kita 12 Nishi 6, Kita-ku, Sapporo, 060 0812, Japan

ABSTRACT

Separation of EEG (Electroencephalography) or MEG (Magnetoencephalography) data into activations of small dipoles or current density distribution is an ill-posed problem in which the number of parameters to estimate is larger than the dimension of the data. Several constraints have been proposed and used to avoid this problem, such as minimization of the L1-norm of the current distribution or minimization of Laplacian of the distribution. In this paper, we propose another biologically plausible constraint, sparseness of spatial difference of the current distribution. By numerical experiments, we show that the proposed method estimates current distribution well from both data generated by strongly localized current distributions and data generated by currents broadly distributed.

1. INTRODUCTION

EEG and MEG are fundamental recording methods to study activities of neurons in the brain noninvasively and with fine time resolution. Three-dimensional identification of the area in the brain where an evoked potential is generated is an important problem for EEG/MEG data analysis. Researchers often use estimation of current density distribution in the brain for the purpose.

It is well known that the estimation of current distribution is an ill-posed problem. Usually we assume that there are several thousands or several ten thousands current dipoles in the cortex and estimate their activities. The dimension of data, that is, the number of EEG electrodes or MEG sensors is at most several hundreds. We need an additional assumption on activities of current dipoles in order to obtain them uniquely.

Several assumptions have been proposed for the purpose. Based on the assumption of minimum energy, [1] and [2] proposed the minimum L2-norm criteria. Later, [3] proposed minimization of L1-norm, which leads to a small number of active dipoles. [4] proposed a method called LORETA, which is also widely used for the estimation of current distribution. It assumes that activities of current dipoles are smoothly distributed in space.

The assumption of continuous activity of neurons of LORETA seems biologically plausible. "Continuous" means that neurons located close to each other have similar activations. Besides continuity, LORETA assumes that activation of neurons is as smooth as possible. However, considering studies on neural network models in which neurons are activated synchronously, a different type of continuity seems more plausible. Often in these models using dynamical systems, in most part of the cortex, the activity of a neuron is similar to those of neurons nearby. However, in some part, there is a gap between two neuron groups and the gap is not necessarily small.

In this paper, we propose a new kind of estimation method for current density distribution consistent with this kind of continuity, that is, the distribution changes at only a small number of areas and these changes can be large. We simulate MEG data and analyze them with the proposed method and conventional methods for estimation of current density distribution. We demonstrate that our proposed method can give satisfactory estimation both for a strongly localized distribution and for a broad distribution.

2. ESTIMATION OF CURRENT DENSITY DISTRIBUTION

In this section, we summarize previously proposed estimation methods of current density distribution. First, we describe the data generation model for MEG or EEG data. We define $\boldsymbol{b}$ as recorded MEG or EEG data, $\boldsymbol{x}$ as a current density distribution and L as a lead field. $\boldsymbol{x}$ consists of activities of current dipoles $\boldsymbol{x}_i$ and $\boldsymbol{x}_i$ is represented in three dimensional Cartesian coordinate including the dipole's direction.

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{pmatrix}, \qquad \boldsymbol{x}_i = \begin{pmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \end{pmatrix}$$

Data generation process can be modeled as follows.

$$\boldsymbol{b} = L\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{n}$ is Gaussian noise.

2.1. Minimization of L2-norm of the density

This method seeks the current density that is consistent with the obtained data $\boldsymbol{b}$ and that has the minimum L2-norm. Practically, we allow a certain level of noise on EEG or MEG channels and use the following cost function.

$$\|\boldsymbol{b} - L\boldsymbol{x}\|^2 + \alpha\|\boldsymbol{x}\|^2 \tag{2}$$

The value of $\|\boldsymbol{x}\|$ that minimizes the above cost function is adopted as the estimation of $\boldsymbol{x}$[1, 2].

2.2. Minimization of L1-norm of the density

This method supposes that the current density which minimizes L1-norm $|\boldsymbol{x}|$ allowing a certain level of noise is an estimation of the true density[3]. We can use the cost function similar to eq. (2). Here, instead, we use the following formulation to make the problem a second-order cone programming problem. Under the following constraint with an appropriate δ,

$$\|\boldsymbol{b} - L\boldsymbol{x}\| < \delta \tag{3}$$

we seek a current density which minimizes $|\boldsymbol{x}|$. The criterion is derived from the assumption that an area with large neural activity is narrow and localized.

2.3. Minimization of the Laplacian(LORETA)

The method minimizes the discrete Laplacian of the current density under the constraint $\boldsymbol{b} = L\boldsymbol{x}$ [4]. For the analysis, we used a modified method called sLORETA(standardized LORETA) [5] considering effects of noise. The cost function to minimize is as follows.

$$\|\boldsymbol{b} - L\boldsymbol{x}\|^2 + \alpha \sum_i \left\| \boldsymbol{x}_i - \frac{1}{6} \sum_{p \in \Phi_i} \boldsymbol{x}_p \right\|^2 \tag{4}$$

The second term is the discrete Laplacian. Here, Φ_i is a set of indices of current dipoles located near the i-th current dipole. In LORETA and sLORETA, smooth spatial change of the current distribution is assumed.

3. INTRODUCING NEW MEASURE OF CONTINUITY OF CURRENT DISTRIBUTIONS

If a neural activity causes electromagnetic field measurable from outside of the skull, then the activity is likely to be a synchronous activity of many neurons. If these active neurons are localized in a very narrow area, the minimization of L1-norm is an appropriate method to estimate the current distribution. In this case, the assumption of a few active points is consistent with the data. However, if these neurons are distributed in an area which is as wide as a few cm^2, the method may not work well. On the other hand, LORETA is the method assuming continuity of the activity in space and it is appropriate when neurons in a relatively wide area are firing. Then, is the other assumption of LORETA, that is, the smoothness of a distribution plausible? Considering properties of neural activities, we expect that there can be a sudden change of the activity at the border of a group of neurons firing synchronously and a group of neurons not joining the activity. It is not necessarily a smooth change as assumed in LORETA.

Therefore, in this paper, we introduce a different measure of the continuity of the current density. Practically, we assume two assumptions. (1) In most points in the brain, the value of the current density is very close to the value around the point (2) Changes of the value of the density occur at a small number of areas and these can be large changes.

We can characterize this kind of continuity by L1-norm of spatial difference. As minimization of usual L1-norm leads to a sparse distribution[6], minimization of L1-norm of spatial difference leads to sparse spatial changes of activity. The actual measure of the continuity is as follows.

$$\sum_{(i,j)\in\Omega} \sum_{k=1,2,3} |x_{ik} - x_{jk}|, \quad \Omega : \begin{array}{l}\text{a set of pairs of dipoles}\\ \text{next to each other}\end{array} \tag{5}$$

The estimation method that we propose is the minimization of the above cost function under the constraint of eq. (3). Since we use L1 norm, the minimization problem can be written in a form of a second-order cone programming problem.

4. EVALUATION OF THE PROPOSED METHOD

We applied various methods to several artificially generated data. Methods that we evaluated are the proposed method using minimization of L1-norm of difference, one using minimization of L2-norm, one using minimization of L1-norm and sLORETA. To solve second-order cone programming problems for the minimization of L1-norm or L1-norm of difference, we used next two programs that are available through World Wide Web. One is SeDuMi[7] and we used it for solving the second-order cone programming actually. The other is YALMIP[8] and we used it mainly as an interface for using SeDuMi.

We defined a set of points on which we placed current dipoles for analysis so that these points covered the cortex. We placed 1657 points on the gray matter only by using MR images of a subject. We simulated three kinds of MEG data.

(A) We chose only one or two points from the set of points and assumed that only one or two current dipoles on these points were active.

(B) We chose one or two groups of several successive points from the set of points and assumed that these points were active.

(C) We defined one or two sets of points which were not related to points previously defined for analysis. In these new sets, points were placed in a more dense way. We assumed that there were current dipoles on these densely placed points and they were active.

We prepared 4 sets of simulated MEG data with each (A)-(C) type of current density. Among these 4 sets, 3 sets have a single activated area at different positions in the brain. The other set have double activated areas. We added Gaussian noise on every channels of every data set. The standard deviation of the noise was set to 1% and 5% of that of the signal. We also generated a data set that consists of noise only and used it to determine the value δ in eq. (3) or regularization coefficients α for the minimization of L2-norm or sLORETA.

We show the outline of the results of estimation. For data type (A) and 5% noise, where only one current dipole is activated, an example of the estimation is shown in Fig 1. For the data set, the assumption of the minimization of L1-norm suits very well. Therefore, with the method, we could obtain the true current density almost perfectly. The result by the proposed method was the second best. Although small activations were estimated around the true activated point, the result estimated by the proposed method describes the characteristics of the true density well. Other two methods estimated broader activation area than the true answer and they could not detect that the true activation area is strongly localized.

For data type (C) and 5% noise, where broad and continuous activity of small current dipoles are assumed, an example of the estimation is shown in Fig 2. All methods estimated the places of activations accurately. However, the estimated area of activation varied. By the minimization of L1-norm, strongly localized activation was estimated though for this data, the true activated area was wider. The activations estimated by the minimization of L2-norm and sLORETA seem good though they are a little broader than the true activated area. The proposed method estimated narrower area than these two methods and the estimated area is closer to the true activated area.

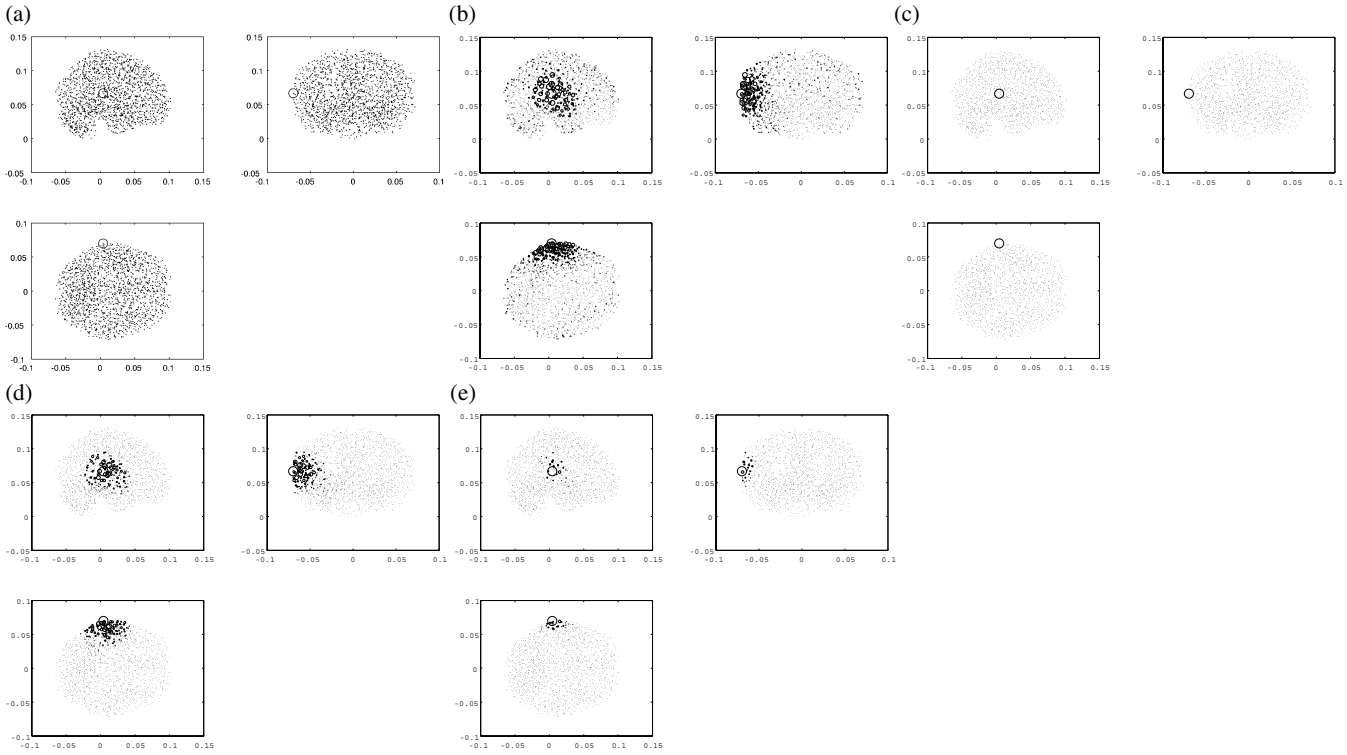

Fig. 1. An example of the estimation of one point activation with type (A) data with 5 % noise. Only magnitudes of current dipoles are shown and they are represented by sizes of circles. (a) The true current density that we used to simulate MEG data. Other graphs are results of estimation by (b) minimization of L2-norm. (c) minimization of L1-norm. (d) sLORETA. (e) proposed method that minimizes L1-norm of spatial difference. upper left:sagittal. upper right: coronal. lower left: transverse.

We did not show the result of type (B) data due to page limitation. The trend was similar to the type (C) data. The trend of results of estimation for data with 1% noise is also similar to that for data with 5% noise for all data types (A)-(C).

For further evaluation of these results, we calculated a simple estimation error defined by the following equation.

$$\frac{\|\boldsymbol{x}_{\mathrm{true}} - \boldsymbol{x}_{\mathrm{estimated}}\|}{\|\boldsymbol{x}_{\mathrm{true}}\|} \tag{6}$$

For a set of data type (C), this measure cannot be directly defined because places of dipoles are different for $\boldsymbol{x}_{\mathrm{true}}$ and $\boldsymbol{x}_{\mathrm{estimated}}$. For these data sets, we transformed the true activation of dipoles into a corresponding activation of a set of dipoles used for the analysis. Each dipole used to generate data is represented by the nearest dipole in the dipole set for analysis. Activations of dipoles in the set for analysis were defined by sums of activations of dipoles assigned to them to represent.

Values of the measure are summarized in Table 1. For type (A) data, values of the minimization of L1-norm is by far the best. The proposed method is the second best. Though the value of the measure for results by the proposed method is not very small, as we showed in Fig 1, actually, the proposed method often recovered the characteristics of the original distribution well. Among 8 data sets generated by only one or two active current dipoles, with 6 data sets, the method estimated this characteristics of strongly localized activation clearly. For data set (B) and (C), the proposed method clearly outperformed other three conventional methods. These trends are common with two noise levels.

Table 1. Values of estimation error calculated by eq. (6). Each value is a mean for 4 data sets. (A)-(C) corresponds to the type of true current density. 5% and 1% mean the level of noise. The used estimation methods are minimization of L2-norm (L2-norm), minimization of L1-norm (L1-norm), sLORETA and minimization of L1-norm of spatial difference (L1-norm of diff).

	L2-norm	L1-norm	sLORETA	L1-norm of diff
(A) 5%	1.030	0.05731	1.045	0.7292
(B) 5%	1.021	1.621	4.028	0.6281
(C) 5%	1.013	1.058	1.027	0.7962
(A) 1%	1.019	0.009870	1.107	0.3981
(B) 1%	0.9918	1.345	4.369	0.6148
(C) 1%	0.9914	1.003	1.018	0.7835

5. DISCUSSION

We proposed a new criterion for the estimation of the current density from EEG/MEG data, the minimization of the L1-norm of the spatial difference. The criterion is biologically plausible. and it can be formulated as a problem of second-order cone programming. By the analysis of simulated MEG data, we showed that the method gave

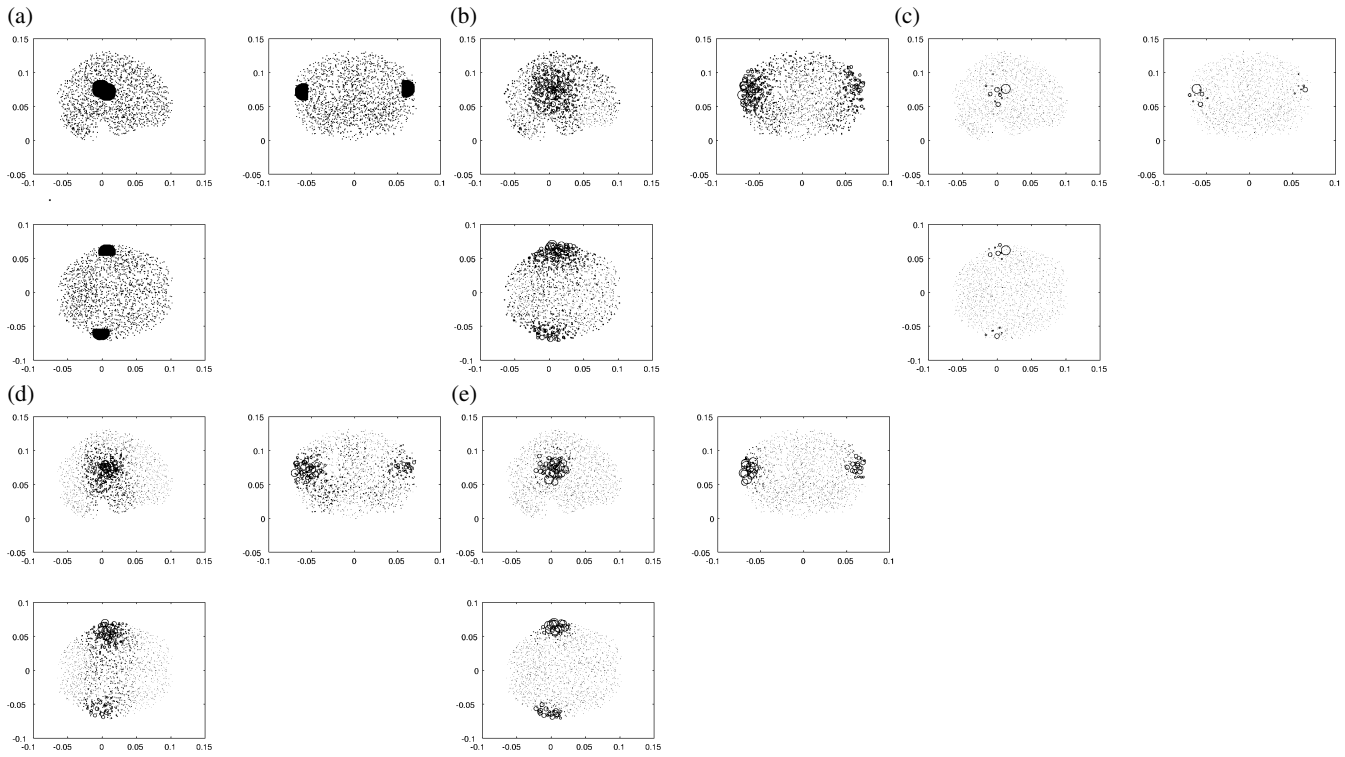

Fig. 2. An example of the estimation of a broad activation. Distributions were estimated from type (C) data made by two actiaved areas and 5 % noise. Only magnitudes of current dipoles are shown and they are represented by sizes of circles. (a) The true current density that we used to simulate MEG data. Dipoles are distributed densely. Other graphs are results of estimation by (b) minimization of L2-norm. (c) minimization of L1-norm. (d) sLORETA. (e) proposed method that minimizes L1-norm of spatial difference. upper left:sagittal. upper right: coronal. lower left: transverse.

satisfactory results both for data with strongly localized current distribution and data with more broadly distributed activation. Three conventional methods that we tested for comparison could estimate the distribution well for only one of two kinds of data.

The method of using the sparseness of difference is a variation of methods introduced in [9]. In [9], difference meant temporal difference and they discussed not only the sparseness but entropy of various kinds of distributions of source signals. The success of our method may be explained by the same entropy minimization framework. Also, the method applied in image processing in [10] is relevant to our proposed method.

6. REFERENCES

[1] J. Wang, S. J. Williamson, L. Kaufman, "Magnetic source images determined by a lead-field analysis: The unique minimum-norm least-squares estimation", IEEE Trans. Biomedical Engineering, vol. 39, pp. 665-675, 1992.

[2] M. S. Hämäläinen, R. J. Illmoniemi, "Interpreting magnetic fields of the brain: minimum norm estimates", Med. & Biol. Eng. & Comput., vol. 32, pp. 35-42, 1994.

[3] K. Matuura, Y. Okabe, "Selective minimum-norm solution of the biomagnetic inverse problem", IEEE Trans. Biomedical Engineering, vol. 42, pp. 608-615, 1995.

[4] R. D. Pascual-Marqui, C. M. Michel, D. Lehmann, "Low resolution electromagnetic tomography: A new method for localizing electrical activity in the brain," International Journal of Psychophysiology, vol. 18, pp. 49-65, 1994.

[5] R. D. Pascual-Marqui, "Standardized low resolution brain electromagnetic tomography(sLORETA): technical details", Methods & Findings in Experimental & Clinical Pharmacology, vol. 24d, pp. 5-12, 2002.

[6] B. A. Olshausen, D. J. Field, "Emergence of simple-cell receptive field properties by learning a sparse code for natural images", Nature, vol. 381, pp. 607–609, 1996.

[7] SeDuMi is developed in Advanced Optimization Laboratory, McMaster University. Available from http://sedumi.mcmaster.ca/

[8] Löfberg, J. YALMIP : A Toolbox for Modeling and Optimization in MATLAB. *Proceedings of the CACSD Conference*, 2004,Taipei, Taiwan. Available from http://control.ee.ethz.ch/˜joloef/yalmip.php

[9] A. Hyvärinen, "Complexity pursuit: Separating interesting components from time series". Neural Computation, vol. 13, pp. 883-898, 2001.

[10] C. Bouman, K. Sauer, "A generalized Gaussian image model for edge-preserving MAP estimation", IEEE Trans. on Image Processing, vol. 2, pp. 296-310, 1993.