VENTRICULAR AND ATRIAL ACTIVITY ESTIMATION THROUGH SPARSE ECG SIGNAL DECOMPOSITIONS

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ABSTRACT

This paper explores a novel approach for ventricular and atrial activities estimation in electrocardiogram (ECG) signals, based on sparse source separation. Sparse decompositions of ECG over signal-adapted multi-component dictionaries can lead to natural separation of its components. In this work, dictionaries of functions adapted to ventricular and atrial activities are respectively defined. Then, the weighted orthogonal matching pursuit algorithm is used to unmix the two components of ECG signals. Despite the simplicity of the approach, results are very promising, showing the capacity of the algorithm to generate realistic estimations of atrial and ventricular activities.

1. INTRODUCTION

Atrial fibrillation (AF) is the most common type of human arrhythmia and it is responsible for about one third of hospitalizations for arrhythmia problems. AF is more frequent in elderly, as its prevalence doubles with each decade of age, from 0.5% at ages between 50-59 years to almost 9% at ages between 80-89 years. AF is an important clinical entity because of the increased risk of morbidity and mortality. The most frequent consequences are hemodynamic function impairment (loss of atrial synchronized contraction, irregular and inadequately rapid ventricular rate), atriogenic thromboembolic events and tachycardia induced atrial and ventricular cardiomyopathy. AF diagnosis has been assessed for years by visual inspection of the surface electrocardiogram. On the ECG, the AF signals are characterized by continuous, apparently disorganized, fibrillatory waves (F-waves). Due to the much higher amplitude of the electrical ventricular activity (VA) on the surface ECG, isolation of the atrial activity (AA) component in the ECG is crucial for the study of AF.

Some methods used to solve this problem are based on average beat subtraction (ABS). These methods are built on the assumption that the AA is uncoupled with the VA. An average of the ventricular complexes (QRST complexes) is then used to subtract VA [2]. Other approaches are blind source separation methods based on independent component analysis (ICA). They try to find independent components in an instantaneous linear mixture [3]. A major difficulty in ABS approaches is the limitation imposed by the use of a small number of VA average templates for general VA approximation. In present ICA based approaches, a major gap is that only statistical *priors* are considered without taking into account the structural nature of signals. In order to circumvent these problems, a possible direction to explore is the use of sparse source separation approaches based on signal adapted redundant dictionaries.

During the past decade, important advances have been achieved in nonlinear signal approximation methods for sparse decompositions over redundant dictionaries (e.g. [4, 5, 6]). In many applications, these techniques offer better performances than those based on orthonormal transforms or direct time domain processing, thanks to their good capacity for efficient signal modeling. In this paper, we present a novel approach for VA and AA estimation. We explore a source separation method based on sparse decomposition of ECG signals on a redundant multi-component dictionary. Such an approach allows for the consideration of *priors* on the structural nature of the different class of signals we are willing to separate. The multi-component dictionary is composed by functions specially designed to match the main structural characteristics of VA and AA signals. We also present Weighted Orthogonal Matching Pursuit (Weigthed-OMP) [6] as a tool for generating ECG sparse approximations for source separation.

2. UNDERDETERMINED SPARSE SOURCE SEPARATION

Let $f_{mix_j}(t): \; j \in [0, M-1]$ be a set of M signal mixtures generated by the weighted superposition of N source signals $f_i(t): \; i \in [0, N-1]$ such that:

$$f_{mix_j}(t) = \sum_{i=0}^{N-1} a_{j,i} \cdot f_i(t) + n_j(t), \tag{1}$$

where $n_j(t)$ represents some additive noise.

Source separation is a classical problem in many fields like acoustics, radio or medical signal and image processing. Many applications exist where the retrieval of the different additive components, forming a set of complex signals, is required. The use of sparse signal representations in source separation problems was proposed in [7] in order to exploit

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some *prior* knowledge about the structural characteristics of each $f_i\,(t)$. Many signals can be sparsely represented by the superposition of a limited number of atoms from an adapted dictionary of functions ($\mathcal{D} = \{g_l(t) :\ l \in \Omega\}$):

$$f_i\,(t) = \sum_{l \in \Lambda} b_l^i \cdot g_l^i\,(t) + \mathcal{R}_{f_i(t)}^{|\Lambda|}, \qquad (2)$$

where b_l^i are the atom coefficients, $\Lambda \subset \Omega$ and $\mathcal{R}_{f_i(t)}^{\Lambda}$ is an eventual residual that depends on Λ.

Combining Eq. (1) and Eq. (2), one derives that, in order to solve the separation problem, the set of mixing coefficients $a_{j,i}$ and a sparse set of expansion coefficients b_l^i must be recovered while $n_j(t)$ are kept as small as possible. Indeed, accurate sparse models can efficiently capture the structural nature of signals, leading to better source separation results as exposed in [7].

A challenging form of sparse source separation problem is when there are fewer mixtures than sources. An example is the separation of the different components from a single ECG channel trace. This is studied in this work through the use of a novel sparse source separation approach. In order to do this, we adapt the two stage separation process proposed in [7] to the particular case of ECG components estimation: First, we *a priori* design an overcomplete dictionary where sources are assumed to be sparsely representable. Second, the sources are unmixed by exploiting their sparse representability.

3. ECG COMPONENTS SEPARATION

In this section, we formulate the estimation of ECG activities according to the signal models and sparse source separation strategy described in Sec. 2. The ECG signal (f_{ECG}) is modeled as a noisy mixture of the two cardiac activities of interest (f_{AA} and f_{VA}):

$$f_{ECG} = f_{AA} + f_{VA} + n, \qquad (3)$$

where n stands for the noise.

The generation of good sparse models for f_{AA} and f_{VA} requires the use of basis functions fitting their particular structures. As shown in the following, f_{AA} and f_{VA} have quite different characteristics, and this is what enables us to separate them. The approach we propose is based on the decomposition of f_{ECG} on a redundant dictionary ($\mathcal{D}$) composed by the union of two sub-dictionaries: $\mathcal{D}_{VA}$ suited for representing the ventricular activity and $\mathcal{D}_{AA}$, better adapted for representing the atrial activity. In the following D, D_{VA} and D_{AA} stand for the synthesis matrices of $\mathcal{D}, \mathcal{D}_{VA}$ and $\mathcal{D}_{AA}$ respectively, where each column represents an atom of the dictionary. Hence,

$$f_{ECG} \simeq D \cdot \mathbf{b} = D_{AA} \cdot \mathbf{b}_{AA} + D_{VA} \cdot \mathbf{b}_{VA}. \qquad (4)$$

Given the noisy nature of f_{ECG} and the high complexity of each of its components, we exclusively consider sparse approximations in this work. According to Eq. (4), $\mathbf{b}$ is composed of two parts ($\mathbf{b}_{AA}$ and $\mathbf{b}_{VA}$), each one containing the coefficients related to D_{AA} and D_{VA}. The approach we explore is simple: one generates a sparse approximation of f_{ECG} on $\mathcal{D}$ and then, estimates for f_{AA} and f_{VA} are reconstructed by just using the components from the appropriate dictionary:

$$f_{AA} \simeq D_{AA} \cdot \mathbf{b}_{AA} \quad \textit{and} \quad f_{VA} \simeq D_{VA} \cdot \mathbf{b}_{VA}.$$

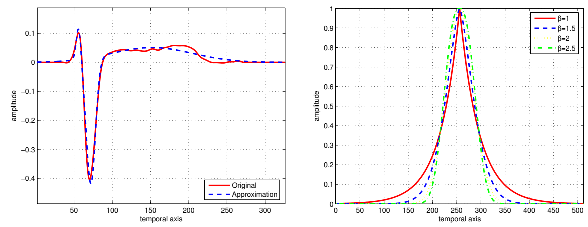

Fig. 1. Left: QRST VA complex and its approximation using just 3 atoms. Right: Effect of β on the GGF (see Eq. (5)).

3.1. Modeling ECG Ventricular Activity

The sub-dictionary $\mathcal{D}_{VA}$ is generated by all possible translations of the Generalized Gaussian Function (GGF):

$$g_{VA}(t) = C_1 \exp\left(-\left(\frac{|t-p|}{\alpha}\right)^{\beta}\right), \qquad (5)$$

where C_1 is a normalizing constant, α determines the scale and β the peakiness. This waveform allows to well approximate the structure of a VA complex using few atoms. Fig. 1 (left) shows a QRST complex and its approximation by using just 3 atoms. With respect to the Gaussian function, fine tuning β gives us higher flexibility to approximate Q, R and S peaks (see Fig. 1 on the right).

The possible values for α and β have been chosen experimentally after an extensive set of tests: $\alpha \in \{3, 4, ..., 7\} \cup \{49, 50, ..., 54\}$, the first set adapted for Q, R and S waves and the second intended for T wave approximation, while $\beta \in \{1.5, 1.6, ..., 2.2\}$. Together with p, this makes $\mathcal{D}_{VA}$ highly coherent, but also very flexible for VA approximation. However, such dictionary is far from being optimal, and several improvements are still possible, mainly concerning the approximation of T waves.

3.2. Modeling ECG Atrial Activity

The sub-dictionary $\mathcal{D}_{AA}$ is generated by all possible translations of a real Gabor function:

$$g_{AA}(t) = C_2 \exp\left(-\left(\frac{t-p}{\alpha\sqrt{2}}\right)^2\right) \cos\left(\frac{2\pi k(t-p)}{N} - \Delta\psi\right),$$

where C_2 is a normalizing constant, N is the signal length, α tunes the scale, k the frequency and $\Delta\psi$ the phase. This waveform is specially adapted for AA approximation. Indeed, as can be observed in Fig. 2 (left), fibrillating AA is of oscillatory nature, which is a perfect fit for the optimal spatio-temporal frequency localization of Gabor functions (see Fig. 2 on the right).

The values of the Gabor function parameters have been determined through an extensive analysis on several ECG signals (see [1]). During the design of $\mathcal{D}_{AA}$, special care in limiting the maximum correlation between $\mathcal{D}_{AA}$ and $\mathcal{D}_{VA}$ atoms was required. Indeed, an excessive correlation between this two sub-dictionaries translates into a complete failure of the algorithm.

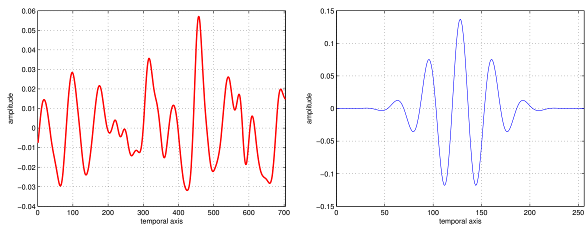

Fig. 2. Left: Example of a simulated AA wave during fibrillation. Right: Gabor atom.

4. ECG SPARSE DECOMPOSITION BY WEIGHTED-OMP

Eq. (4) involves an underdetermined problem that has no unique solution. The search for the sparsest approximation of f_{ECG} requires the exhaustive testing of all coefficient possibilities, i.e. it is a combinatorial problem. Various alternative approaches have been proposed in order to make the retrieval of a solution for $\mathbf{b}$ computationally affordable (see [4] for a review of some of them). These, in general, do not guarantee the recovery of the sparsest solution. Nevertheless, recent results show that under certain conditions on the dictionary and the signal, these sub-optimal methods find the sparsest solution [5, 6]. In our case, Greedy Algorithms appear to be the most appropriate family of decomposition algorithms to handle the very large dictionary we use.

4.1. Weighted Orthogonal Matching Pursuit

Weighted-OMP [6] is the greedy algorithm used to decompose ECG signals. It is a weighted version of Orthogonal Matching Pursuit (OMP), that iteratively builds m-term approximants by selecting at each step the most appropriate term from $\mathcal{D}$ according to a selection rule. Each iteration $k : k \geq 0$ can be seen as a two-step procedure:

1. A selection step where an atom $g_{l_k} \in \mathcal{D}$ is chosen according to:

   $$g_{l_k} = \arg\max_{g_l \in \mathcal{D}} |\langle r_k, g_l \rangle| \cdot w_l,$$

   where $w_l \in [0, 1]$ is a pre-estimated weight that reflects, according to a predefined model, the *a priori* likelihood that atom g_l may be a correct component of the signal.

2. An orthogonal projection step where an approximant $f_{k+1} \in span(g_{i_p} : p \in \{0, ..., k\})$, and a residual $r_{k+1} = f - f_{k+1}$ are generated (notice that $r_0 = f$, and $r_{k+1} \perp f_{k+1}\ \forall k$). This stage updates, at every step, the set of scalar expansion coefficients.

The signal representation generated by Weighted-OMP is, thus, of the form of Eq. (2).

Weighted-OMP was recently proved in [6] to outperform OMP when using coherent dictionaries and reliable *prior* information. Weighted-OMP can consider, in the decomposition algorithm, *a priori* models about the behavior of the dictionary in use with the class of signals to decompose. The use of good enough *a priori* models can reduce the instability of Weighted-OMP with respect to OMP when trying to recover sparse approximations/representations.

4.2. Weight Generation: Relation Between ECG *A Priori* Knowledge and the Dictionary

Thanks to the structure of VA, f_{ECG} can be divided in VA periods. In addition, each VA period can be divided in a set of intervals corresponding to the different VA waves (Q, R, S and T) and an interval without ventricular activity. VA intervals can be estimated and identified in practice through the use of QRST point estimators (e.g. see [8], used in this work). This *prior* information can thus be used to generate $w_l\ \forall l$. The *a priori* knowledge obtained from [8] needs to be related with $\mathcal{D}$ in the following way.

$\mathcal{D}$ is divided in $\mathcal{D}_{AA}$ and $\mathcal{D}_{VA}$. Due to dynamics of AF, AA can be found through all the VA period. Hence, $\mathcal{D}_{AA}$ atoms cannot be penalized. This is the reason why in this study we force: $w_l = 1\ \ \forall l :\ g_l \in \mathcal{D}_{AA}$. To the contrary, the selection of $g_l \in \mathcal{D}_{VA}$ can be successfully influenced by the use of the available *a priori* information. $\mathcal{D}_{VA}$, as seen in Sec. 3, is composed of a block optimized for QRS waves (ventricular depolarization) and a block designed for T waves (ventricular repolarization). Depending on the VA interval, w_l can be set to 1 for every $g_l \in \mathcal{D}_{VA}$ belonging to the appropriate kind for that interval. In case a g_l is unsuitable for a given interval, w_l can be set to a penalizing factor $0 \leq \tau < 1$. Thanks to the reliability of the estimators used in this work, it turned out that the best value for τ in our experiments is 0.

5. EXPERIMENTAL RESULTS

5.1. Validation

A biophysical computer model of the atria was used to obtain a realistic atrial electrical activity on the torso [9]. The AF signals that were generated in the 12-lead ECG were added to a clinical 4-second standard 12-lead ECG of an AF paroxysmal patient (78 years old) in sinus rhythm in which the P waves were removed. The clinical ECG was selected to represent the VA in AF as closely as possible. The ratio between the power of the original signal (simulated AA) and the estimation error (estimated AA - simulated AA) was used to evaluate the performance of our method.

5.2. Results

First of all, we want to underline that we validated our choice of Weighted-OMP instead of OMP with these simulated measured 4-second ECG signals. By using Weighted-OMP, we increased the SNR in the recovery of VA (respectively, AA) by 0.81 dB (respectively, 0.65 dB). All the following results were obtained by approximating ECG signals with 50 atoms. Fig. 3 shows the resulting separation of VA and AA for the simulated measured 4-second ECG signal on lead V1. One can see how our method succeeds in approximating each one of the VA episodes and in separating, at the same time, the AA with surprising accuracy.

In order to study the influence of the AA amplitude on the method, three different simulated AA signals were created; 50%, 100% and 150% of the original simulated AA amplitude. The ratio between the power of the original activity (VA or AA) and the error on the estimated one was evaluated on leads VR, V1 and V4 (see Table 1). We can observe that the quality of AA estimation depends on the lead and its original amplitude. The AA SNRs are much higher with the 150%

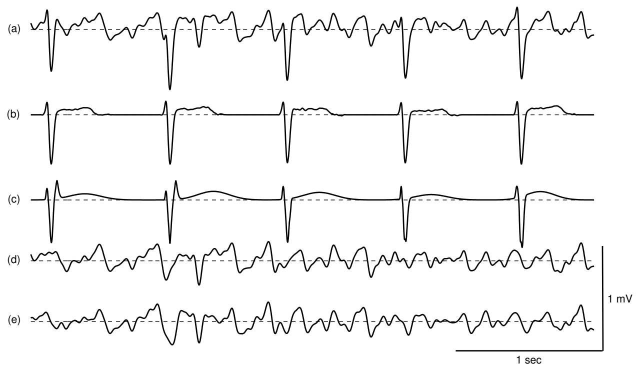

Fig. 3. (a) Simulated measured 4-second ECG signal on V1. (b) Original VA on V1. (c) Estimated VA on V1 (SNR : 8.69 dB). (d) Simulated AA on V1. (e) Estimated AA on V1 (SNR : 6.81 dB).

original amplitude and the overall performance on lead V1 is better than those on the other two leads. Of course, the SNR values are directly related to the signal amplitude. In V1, the AA amplitude is higher compared to other leads. However, we observe a decrease of VA estimation performance in lead V1.

	0.5·AA+VA	1·AA+VA	1.5·AA+VA
		lead VR	
VA SNR(dB)	11.06	10.88	11.08
AA SNR(dB)	-6.94	-1.05	2.61
		lead V1	
VA SNR(dB)	11.13	8.69	2.41
AA SNR(dB)	3.61	6.81	4.28
		lead V4	
VA SNR(dB)	12.33	11.94	11.66
AA SNR(dB)	-6.53	-0.8	2.4

Table 1. Signal-to-noise ratio (dB) on lead VR, V1 and V4. Our method performance is tested on 3 different AA amplitudes (50, 100 and 150 % of the original simulated signal).

Fig. 4 shows the resulting separation of the VA and AA of the clinical 4-second ECG signal on lead V2. Apart from the visually satisfying component separation, the resulting signals were validated using estimated power spectral densities (PSD). The dominant frequency of VA (respectively, AA) is between 1 and 2.5 Hz (respectively, between 3 and 10 Hz). The fact that there is no presence of VA dominant frequencies in the AA estimated PSD demonstrates the quality of our clinical results. Further results can be found in [1].

6. CONCLUSIONS

In this paper, we present a new framework based on sparse source separation that can be used for QRST cancellation. Results appear to be very promising. Additional works are planned for the improvement of the dictionary design, specially concerning T wave modeling. Moreover, more efficient use of *a priori* knowledge can be studied and the number of atoms used for decomposing the signals can be optimized.

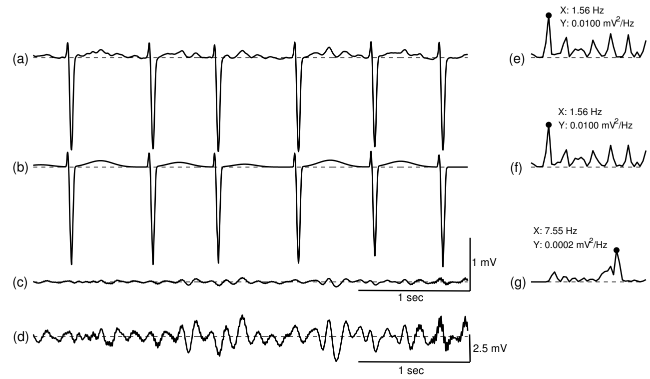

Fig. 4. (a) Clininal 4-second ECG signal on V2 with a dominant frequency of 1.56 Hz (see its PSD (e)). (b) Estimated VA on V2 with a dominant frequency of 1.56 Hz (see its PSD (f)). (c) Estimated AA on V2 with a dominant frequency of 7.55 Hz (see its PSD (g)). (d) Estimated AA on V2 magnified 5 times.

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