IMPROVING SEPARABILITY OF EEG SIGNALS DURING MOTOR IMAGERY WITH AN EFFICIENT CIRCULAR LAPLACIAN

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ABSTRACT

This paper reports on a new EEG re-referencing scheme, known as the circular Laplacian, for processing multichannel EEG signals. The new reference signals are derived from the average potentials on the circles around the electrodes. The radii of the circles can be adjusted to achieve spatial filtering of EEG at different frequencies. Evaluation with motor imagery recordings suggests that the circular Laplacian leads to a maximum of 5% improvement over the traditional discrete Laplacian in a motor imagery classification task.

1. INTRODUCTION

Imagination of body movements (motor imagery) generates measurable changes in the scalp-recorded electroencephalograph (EEG). Depending on the part of the body imagined moving, these changes concentrate on different regions of the brain. This phenomenon has been extensively employed in brain-computer interfaces (BCI) [1]. These systems train subjects to associate their imaginations with their intentions. EEG generated by the imaginations are then decoded into control signals for driving wheelchairs or computer applications. The independence of BCI systems from muscle control makes it promising for helping those severely paralyzed [2].

BCI systems usually record EEG signals with multiple channels according to the international standard placement. This standard setting approximates the head with a sphere and determines the electrode positions by dividing the great circles and small circles into 5%, 10% or 20% portions [3]. Due to these different portions, the electrodes are not evenly spaced on scalp. Especially when only a subset of the electrodes are used, it is not always possible to find equidistant neighbors surrounding a given electrode.

EEG potentials can be viewed as a blurred version of the cortical activities. This is due to the diffusion of the skull and the skin. To recover the focal activities, EEG potentials

are commonly re-referenced with a discrete Laplacian [4, 5], which approximates the two dimensional Laplacian operator $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Suppose that the EEG potential at an electrode e_0 is $U(e_0)$ (e_0 is also used to denote the position vector). Then its discrete Laplacian LU can be expressed as

$$LU(e_0) = -U(e_0) + \frac{1}{|\mathcal{N}|} \sum_{e \in \mathcal{N}} U(e),$$
 (1)

where \mathcal{N} is the set of neighboring electrodes of e_0 , and $|\mathcal{N}|$ is the number of elements in \mathcal{N} . Ideally, these neighboring electrodes should surround e_0 and be equidistant from e_0 (see right side of Fig 1). However, as mentioned above, such an arrangement is not always possible for the standard electrode placement. Especially for the border electrodes, all of their neighbors will clutter to one side and this results in poor approximation of the Laplacian [6].

To avoid this problem, some methods first interpolate EEG potentials on the scalp, and then derive the analytical Laplacians using the interpolation bases [7, 8, 9]. One of the most commonly used scheme is proposed by Perrin *et al.*[7] They used Legendre polynomials as the bases for interpolation, which leads to a straightforward expression of the Laplacian. Compared to the discrete Laplacian, the neighborhood in these methods effectively shrinks to an infinitesimal one. Such a Laplacian operator behaves like a high-pass filter, preserving only highly localized and sharp changes in the potentials. The neural electrical activities generated by the brain, however, contain various spatial frequencies. Potentially useful information from the middle frequencies may be filtered out by the analytical Laplacian.

The proposed circular Laplacian of an electrode e_0 is defined as

$$LU(e_0) = -U(e_0) + \frac{\oint_{\mathcal{C}} U(e) \mathrm{d}s}{\oint_{\mathcal{C}} \mathrm{d}s},$$
 (2)

where the integration is along a circle C around e_0 . This definition changes the neighbourhood of e_0 to a circle on the sphere (see left side of Fig. 1). The Laplacian is computed

^{*} National ICT Australia is funded by the Australian Government's Backing Australia's Ability initiative, in part through the Australian Research Council.

by combining the potential at e_0 and the average potential along the circle. The radius of the circle will be referred to as the angle ϕ , which is the angle between e_0 and the position vector of a point on the circle. Varying the radius of the circle will allow us to focus on neural information at different spatial frequencies. In the following sections, we will first introduce Perrin's spherical spline interpolation; then we will show how the circular Laplacian can be computed efficiently based on the spherical splines.



Fig. 1. Comparison between the discrete Laplacian and the circular Laplacian. The discrete Laplacian (on the right side) re-references the EEG potential at e_i with electrodes surrounding e_i , while the new circular Laplacian re-references the EEG with values from a circle (on the left side).

2. PERRIN'S SPHERICAL SPLINE

Perrin's method [7] models the head as a unit sphere and employs a subset of the spherical harmonics as the bases for the interpolation. This subset, known as Legendre polynomials, can be expanded as:

$$P_n(x) = \frac{1}{2^n} \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k \binom{n}{k} \binom{2n-2k}{n} x^{n-2k}.$$
 (3)

Using these bases, the potential at a point e on the sphere can be interpolated as:

$$U(e) = c_0 + \sum_{i=1}^{N} c_i g(\cos(e, e_i)),$$
(4)

$$g(\cos(e, e_i)) = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{2n+1}{(n(n+1))^m} P_n(\cos(e, e_i)), \quad (5)$$

where the e_i s are the position of the N recording electrodes, and $\cos(\cdot, \cdot)$ computes the cosine of the angle between two positions on the sphere. The c_0 and c_i are the interpolation coefficients determined by the potentials $U(e_i)$ from the electrodes. Two constraints govern the solution of these coefficients: (i) The interpolated function has to pass the measured potentials at e_i ; (ii) the sum of c_i has to be zero. These constraints can be formulated into a system of linear equations and solved efficiently using singular value decomposition (SVD). The analytical Laplacian can be easily computed from equation (4) and (5), since the Laplacian of a Legendre polynomial is simply a multiple of itself, i.e.

$$\Delta P_n = -n(n+1)P_n. \tag{6}$$

Substituting this into equation (5) results in the analytical Laplacian:

$$LU(e) = \sum_{i=1}^{N} c_i h(\cos(e, e_i)),$$
(7)

$$h(\cos(e, e_i)) = \frac{1}{4\pi} \sum_{n=1}^{\infty} \frac{2n+1}{(n(n+1))^{m-1}} P_n(\cos(e, e_i)).$$
 (8)

Perrin chose m = 4 based on simulations and the sum of the first 20 terms in (8) for the computation (this guarantees an precision of 10^{-6} for $h(\cdot)$). Furthermore, since the Legendre polynomials are only evaluated in the range [-1, 1], they are tabulated for values regularly spaced between -1 and 1. This reduces the computation of this Laplacian scheme.

3. CIRCULAR LAPLACIAN

3.1. Mathematical Derivation

In order to compute the circular Laplacian, the EEG potentials along the paths of the integrations need to be available. We use Perrin's spherical spline to obtain the potentials along these paths. Thus, the integrations in equation (2) can be expressed as:

$$\oint_{\mathcal{C}} U(e) ds = c_0 \oint_{\mathcal{C}} ds + \sum_{i=1}^{N} c_i \oint_{\mathcal{C}} g(\cos(e, e_i)) ds,$$

$$\oint_{\mathcal{C}} ds = 2\pi \sin \phi.$$
(9)

Now, the problem becomes how to integrate $g(cos(e, e_i))$ and thus $P_n(cos(e, e_i))$ efficiently. Suppose that the circle is around a point $e_0 = (x_0, y_0, z_0)$ on the sphere, then points on the circle can be parameterized as (note: the center of the circle, $e_0 \cos \phi$, is inside the sphere):

$$e = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} A & D & G \\ B & E & H \\ C & F & I \end{pmatrix} \begin{pmatrix} \cos t \\ \sin t \\ 1 \end{pmatrix}$$
(10)

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where $t \in [0, 2\pi)$ and F = 0,

$$A = \frac{x_0 z_0}{\sqrt{x_0^2 + y_0^2}} \sin \phi, \qquad B = \frac{y_0 z_0}{\sqrt{x_0^2 + y_0^2}} \sin \phi,$$
$$C = -\sqrt{x_0^2 + y_0^2} \sin \phi, \qquad D = -\frac{y_0}{\sqrt{x_0^2 + y_0^2}} \sin \phi,$$
$$E = \frac{x_0}{\sqrt{x_0^2 + y_0^2}} \sin \phi, \qquad G = x_0 \cos \phi,$$
$$H = y_0 \cos \phi, \qquad I = z_0 \cos \phi.$$

Then $\cos(e, e_i)$, the inner product $e \cdot e_i$ for points on a unit sphere, can be denoted by:

$$\cos(e, e_i) = (x_i, y_i, z_i) \begin{pmatrix} A & D & G \\ B & E & H \\ C & F & I \end{pmatrix} \begin{pmatrix} \cos t \\ \sin t \\ 1 \end{pmatrix}$$
$$= \alpha_i \cos t + \beta_i \sin t + \gamma_i.$$

Substituting it into $P_n(cos(e, e_i))$ and further applying the expansion of the Legendre polynomials from equation (3), we obtain the integration

$$\mathcal{P}_{n} = \oint_{\mathcal{C}} P_{n}(\alpha_{i}\cos t + \beta_{i}\sin t + \gamma_{i})\mathrm{d}s = \frac{1}{2^{n}} \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^{k} \binom{n}{k} \binom{2n-2k}{n} \times \sum_{\substack{l+j+h\\ =n-2k}} \binom{n-2k}{l} \binom{n-2k-l}{j} \alpha_{i}^{l} \beta_{i}^{j} \gamma_{i}^{h} \int_{0}^{2\pi} \cos^{l}t \sin^{j}t \mathrm{d}t,$$
(11)

where

$$\int_0^{2\pi} \cos^l t \sin^j t dt = \begin{cases} \frac{2\Gamma(\frac{1+l}{2})\Gamma(\frac{1+j}{2})}{\Gamma(\frac{2+l+j}{2})}, & \text{for even } l \text{ and } j \\ 0, & \text{otherwise.} \end{cases}$$

3.2. Efficient Implementation

The parameters, α_i , β_i and γ_i , encode the positional information of the center e_0 and the electrodes e_i . Their values remain unchanged irrespective of the measured potentials. Hence they need to be computed only once. From equation (11), it is clear that the integrations \mathcal{P}_n are completely determined by these parameters. Therefore they also need to be computed only once.

Fast implementation of the integrations \mathcal{P}_n can be achieved by exploiting the commonalities between them. For example, the \mathcal{P}_n for even *n* all share 3 terms, e.g.

$$\begin{pmatrix} 2\\2 \end{pmatrix} \begin{pmatrix} 0\\0 \end{pmatrix} \alpha_i^2 \beta_i^0 \gamma_i^0 \frac{2\Gamma(\frac{3}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{4}{2})}, \begin{pmatrix} 2\\0 \end{pmatrix} \begin{pmatrix} 2\\2 \end{pmatrix} \alpha_i^0 \beta_i^2 \gamma_i^0 \frac{2\Gamma(\frac{1}{2})\Gamma(\frac{3}{2})}{\Gamma(\frac{4}{2})} \\ \begin{pmatrix} 2\\0 \end{pmatrix} \begin{pmatrix} 2\\0 \end{pmatrix} \alpha_i^0 \beta_i^0 \gamma_i^2 \frac{2\Gamma(\frac{1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{2}{2})}.$$

which always appear in the same summation. As n goes to larger numbers, the adjacent \mathcal{P}_n will share more terms. The argument applies similarly to odd n. In our implementation, all these shared terms are pre-computed, summed accordingly, and stored in a list. Subsequent computation of \mathcal{P}_n then reduces to simple additions and multiplications.

In summary, the circular Laplacian can be computed using the following equation:

$$LU(e) = -U(e) + c_0 + \frac{1}{8\pi^2 \sin \phi} \sum_{i=1}^N c_i \sum_{n=1}^\infty \frac{2n+1}{(n(n+1))^m} \mathcal{P}_n.$$
(12)

Similar to Perrin's spherical spline, the inner summation of equation (12) need not go to infinity. The first 20 terms are used in our implementation. The overall computation of the circular Laplacian at a point e_0 involves four steps: (i) Initialize \mathcal{P}_1 up to \mathcal{P}_{20} using the coordinates of e_0 and the electrodes e_i (this need to be computed only once); (ii) Compute c_0 and c_i using Perrin's method; (iii) Compute the interpolated potential $U(e_0)$ using Perrin's method; (iv) Compute the circular Laplacian using equation (12). Compared to the analytical Laplacian in (8), the only additional computation is in step (iv). It involves only additions and multiplications of a few real numbers, which can be done in a short time.

4. EVALUATION

4.1. Data Set

We use the EEG data from the Berlin BCI group [10] to compare the performance of the circular Laplacian, the analytical Laplacian and the discrete Laplacian. In this data set, five healthy subjects were asked to carried out two types of motor imageries (eithor hand or foot movement). Each type of imagination lasted 3.5 seconds and was repeated 140 times. EEG signals for these imaginations were recorded with a cap of 118 channels. The task was to classify the single trial EEGs into two types. The performance was measured with the generalization errors of the classification. The lower the error the better the performance of a method.

4.2. Feature Extraction and Classification

The feature extraction and classification process is depicted in Fig. 2 (refer to [11] for further details). The only difference for the three schemes is the use of different Laplacian filters. Spatially filtered EEG signals are then passed to the same back-end feature extractor and classifier.

As described in [11], the features are the correlations of the phases between multichannel EEGs, and the classifier is a two-level linear Support Vector Machine(SVM). The phase information can be computed using either wavelet analysis or Hilbert transformation (the latter is used in this study). The correlation of the phases between two channels in a time window τ can be quantified using the phase locking value (PLV), i.e.

$$\operatorname{PLV}_{ij} = \frac{1}{\tau} \left| \sum_{T-\tau}^{T} \frac{\varphi_i[n]}{\varphi_j[n]} \right| \in [0, 1],$$
(13)

where $\varphi_i[n]$ and $\varphi_j[n]$ (both complex valued with unit modulus) are the phases at the sampled time points for channel *i* and *j* respectively (*n* is the discrete time index here). PLV measures the stability of the phase difference between two channels. If the phase of the two channels exhibit correlated behavior, the PLV will be high. PLV encodes information on the interaction between different brain regions, and can be exploited to separate different motor imageries [11].



Fig. 2. Block diagram for the classification of single trial motor imagery. The classifications using the three Laplacian schemes differ only in the first block of the diagram.

4.3. Result

The generalization errors were estimated using a 50×2 -fold cross validation, where the order of the trials is randomized 50 times. In each randomization, the trials are split into two equal halves, each serving as training data once, and the reminder serving as test data. The generalization error is then the average of the prediction errors from each fold. In Table 1, the generalization errors (with standard deviations) for the five subjects are presented. It can be seen that the circular Laplacian scheme always achieves smaller errors than the other two. Most notably, for subject 4, it decreases the error by as much as 5%. Although the performance of the circular Laplacian and the discrete Laplacian is very close for some subjects (e.g. subject 1 and 2), our statistical tests (Student's t-tests with a 0.05 significance level) suggest that the circular Laplacian is always significantly better for all five subjects (the *p*-values are listed in the column of Table 1).

4.4. Discussion

The circular Laplacian is an approximation of the Laplacian of a Gaussian (LoG) operator, i.e. $\Delta(\frac{1}{\sigma\sqrt{2\pi}}\exp(-\frac{x^2+y^2}{2\sigma^2}))$. The circle is equivalent to the ring of positive peaks of LoG, while its center corresponds to the dominant negative peak of LoG. As the σ in LoG varies, different spatial filtering effects can be achieved. The circular Laplacian approximates this by varying the radius of the circle. Since direct convolution of LoG with U(e) is hard to derive, we employ the circular Laplacian as an efficient substitute.

The results for the circular Laplacian (in Table 1) are obtained with subject-specific radii. The best radii (measured as the angle ϕ) are 26, 23, 27, 22 and 26 degrees for the five sub-

Table 1. Generalization errors (%) of the three Laplacianschemes for five subjects (Sb).

Sb	Circular	Analytical	Discrete	р
1	$21.74 {\pm} 3.23$	$31.26{\pm}3.90$	$22.78{\pm}3.60$	3.21e-2
2	$3.51{\pm}1.30$	$5.79{\pm}1.86$	$4.29{\pm}1.29$	3.16e-5
3	$29.40{\pm}3.22$	$34.62{\pm}3.36$	$32.09{\pm}3.85$	2.39e-7
4	$17.78 {\pm} 3.46$	$26.58{\pm}4.25$	$22.86{\pm}4.03$	4.74e-18
5	$5.51{\pm}1.54$	$7.66{\pm}1.88$	$6.79{\pm}2.15$	2.67e-6

jects respectively (determined by the cross validation). This suggests that the most discriminative spatial frequencies are different across subjects. Varying the radius of the circle picks up these optimal signals.

5. CONCLUSION

In this study, we derive an efficient circular Laplacian scheme to re-reference EEG potentials. Varying the radius of the reference circles allows one to select the best filter from a set of filters. The chosen filter picks up subject specific neural signals, which improves the separability of EEG signals during motor imageries in a BCI setting.

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