Sigma-Delta Converters as a SP Teaching Tool

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ABSTRACT

An undergraduate course in SP doesn't usually teach analog to digital converters. We think that the concepts involved in SD converters are not only appropriate for senior students, but also a way of developing SP skills when it is assisted by a laboratory assignment. For that purpose we will show how this task can be done and we will highlight its advantages according to the results we were able to measure from student's reports.

1. INTRODUCTION

Signal Processing is such a wide subject that it can be taught in many different ways. In order to introduce an innovation, we must point out that essential topics, including signals and systems theory, should always be addressed to students. Laboratory experiments are also a must, but not necessarily in the same course.

Regardless the curricula, SP applications are constantly growing, from telecommunications to computer music, from solid state circuits to DSPs with FPGAs .

How a Signal Processing course is able to adapt to a fast-paced changing environment? In most Universities this is something that cannot be done until a major change in the curricula is performed. In spite of such obstacle, there are some tools that help keeping the dynamics of the system, without following the long process of reforming the whole program.

In that context, course objectives have to be achieved despite the intended changes. Ideas should be introduced in a way that the synergy between theory and practice is not disturbed.

It is the purpose of this paper to show how some topics are suitable to create a motion into modern and global SP-related concepts.

We are SP professors at ITBA (Instituto Tecnologico de Buenos Aires), a private University in Buenos Aires, Argentina. Our course was described in [1], but since then it has been slowly adapting to new challenges.

The mentioned tools we have been developing during the last ten years can be classified in three categories:

- Renewing laboratory experiments
- Introducing subjects that mix SP with other attracting concepts
- Inserting research assignments, that include advance SP topics.

The first category is a regular practice among commendable professors, the second is seldom found in SP curricula and the latter is very unusual.

Our approach involves laboratory assignments where students implement acquisition circuits and program digital filters [2], work with mixed signal systems [3], perform digital music synthesis [4] and study diverse research subjects [5].

A solid background in signal and systems, a strong experience in an analog electronic lab and a true willingness to learn are the instruments we need to reach our goals.

Our students fulfill these requirements. They also follow a five years engineering program that seeks a certain level of expertise in many areas. So the idea of mixing SP topics with other EE subjects fits perfectly well in our course.

In that line of thought, converters have an important role. Many issues that affect Signal Processing are due to conversion constraints. This fact supports the introduction of Analog to Digital Converter (ADC) technologies in the syllabus. We give special emphasis to ΣΔ converters, because the intrinsic theory has the requirements we need.

2. ΣΔ CONVERTER CONCEPTS

ADCs can be classified according to different parameters. One of them is the implementation technology. When the conversion is performed at multiple sampling frequencies and the quantization error has a frequency dependent power spectral density, we are working with ΣΔ converter concepts. [6] [7]

Conversion is achieved after having accomplished the following steps:

- Oversampling the analog signal at a rate much higher than the Nyquist frequency.
- Modulating the sampled signal thru a feedback loop called ΣΔ Modulator, that outputs a low precision (low number of bits) digital signal.
- Digital filtering the low precision signal to remove quantization and noise.
- Downsampling the digital stream to increase the number of bits and return to the Nyquist sampling rate.

Figure 1 shows a block diagram of conversion process.

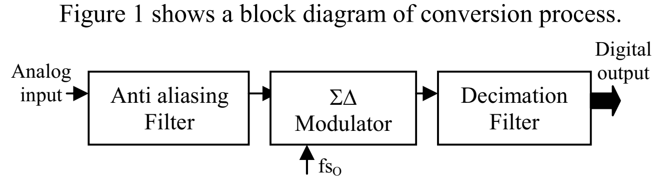

Figure 1: ADC ΣΔ Conversion Steps

Any analog signal must go thru three stages in order to become digital. It is band limited by an analog lowpass filter. Subsequently, it is sampled. Finally it is ready to be converted. SD converters follow the same path, nevertheless introducing notorious originalities.

As we mentioned earlier, sampling is executed at very high rate. If the maximum frequency of the incoming signal is f_B, thus the Nyquist frequency being $f_{sN} = 2\ f_B$, then the oversampling frequency becomes $f_{sO} = L\ f_{sN}$. Typically, L is an integer of values {32, 64, 128 or more}.

High rate sampling brings an advantage with the anti aliasing filter (AAF). Transition band is much wider than usual, as Figure 2 exhibits, allowing a simple first order RC as a common solution.

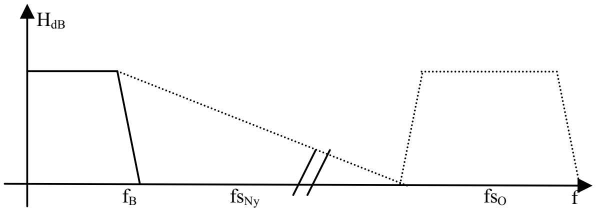

Figure 2: Relaxed AAF constraint

Despite the released analog constraint, there is an incoming problem to deal with. Technology is not yet advanced enough to manage very high sampling rates with a large number of bits. Typical audio applications require 20 bits or more, something impossible to combine with oversampling frequencies 64 times the standard Nyquist rate of 44KHz. Although it is possible to interchange sampling frequency for resolution, see formula (1), calculation of number of bits Δn gained due to increased oversampling rate L in Table 1 shows that it is not enough.

$$\Delta n = 0.5\ \log_2 L \qquad (1)$$

L	*Δn*
16	2
32	2.5
64	3
128	3.5

That is why classical ADC topology is not appropriate. An answer to this matter is the ΣΔ modulator.

A feedback loop like the one displayed in Figure 3 offers a low resolution – high sampling frequency first order conversion system.

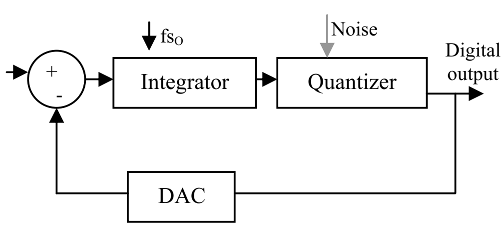

Figure 3: First order ΣΔ Modulator

A sampled signal goes through the system while the quantization noise is highpass filtered. This configuration allows a very low resolution, because noise power spectral density is not longer flat. In this case, the number of bits gained depends not only on L but also on the filter order. Loop architecture also influences filter's selectivity, affecting Δn calculation. Expression (2) represents bits gained (Δn) when the ΣΔ modulator of order p is considered, using the classical topology of Figure 3.

$$\Delta n = (p + 0.5) \cdot \log_2 L - 0.5 \cdot \log_2 \left(\frac{\pi^{2p}}{2p + 1} \right) \quad (2)$$

Then, the white noise spectral density σ_e^2 / f_{sO} is modulated by the hipass filter, obtaining an output $\frac{\sigma_e^2}{f_{sO}} \left| H_N(s) \right|^2$. In Figure 4, it can be seen that the noise remaining for frequencies below $f_B/2$ implies high resolution.

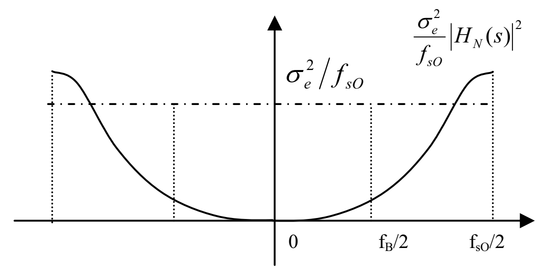

Figure 4: Noise Power Spectral Density

ΣΔ feedback loop can be modeled by both analog and digital signals. Figure 5 represent a digital block diagram of a second order SD modulator [8] implemented in Simulink [9], whose noise transfer function $H_N(z)$ is detailed in (3).

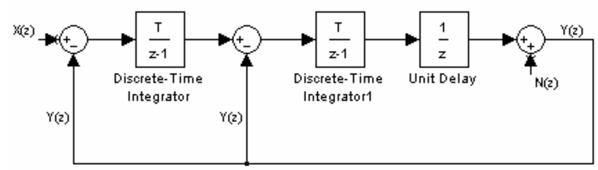

Figure 5: ΣΔ Digital Model in Simulink

$$\frac{Y(z)}{N(z)} = H_N(z) = \frac{z}{z + H(z) + zH(z) * H(z)} \quad (3)$$

$$H_N(z) = \frac{z(z-1)^2}{z^3 - 2z^2 + (1 + T + T^2)z + -T}$$

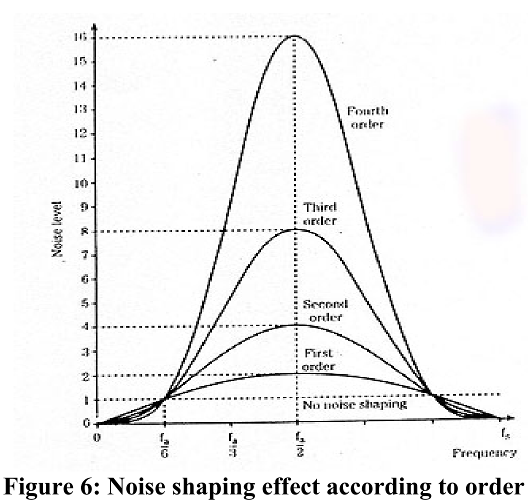

Figure 6: Noise shaping effect according to order p

At this point it is interesting to consider how power spectral noise density is modified by modulator order p, as displayed in Figure 6 (see [10])

Combining modulator order, circuit topology and quantizer resolution, modern ΣΔ ADCs can reach to 24 bits with a sampling frequency of 196 KHz.

After leaving the modulator loop, the digital signal must get rid of the additional quantization noise, and return to a high resolution Nyquist rate condition. These tasks are performed by the decimation filter, a digital FIR filter which is integrated in the same IC.

From the user's point of view, a SD converter is very similar to any other converter, except for the high clock requirement and the surprisingly simple anti aliasing filter.

As SD concepts helps understand mixed signal processing, we implemented a differential second order modulator.

3. The Hardware

We design a PCB with a discrete circuit that implements the block diagram shown in Figure 5 [11] [12]. Each digital integrator is a switched capacitor integrator, as displayed in Figure 7. We used standard operational amplifiers, like the TL081 and CD4066 analog switches.

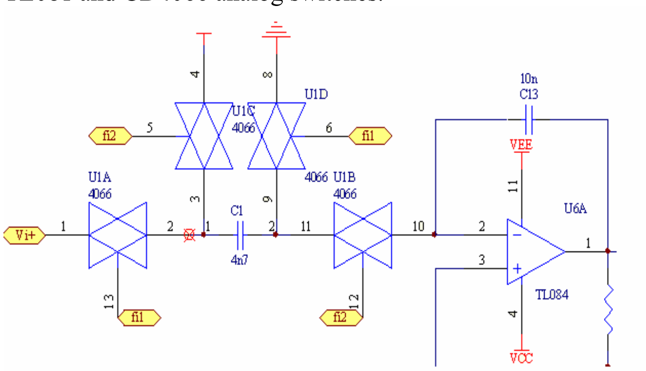

Figure 7: Switched Capacitor Integrator

Oversampling clock frequencies are around 100 KHz, which implies an input signal bandwidth limit of 1 KHz. As the board is used for learning purposes only, this is not a significant constraint.

Full schematics of the modulator can be requested to the authors.

4. Assignment Goals

Taking into account ΣΔ converter concepts and given the hardware described, students have to perform measurements on the board.

Pursued objectives are:

- Understand a ΣΔ modulator, calculating and simulating its transfer function $H_N(z)$
- Perform FFT Spectrum Analyzer measurements
- Visualize oversampling effects
- Analyze Switched Capacitor Integrators
- Measure differential outputs
- See Noise Shaping in action
- Design an analog reconstruction filter (instead of the digital decimation filter)
- Compare results with different inputs.

Students are lectured before the assignment. They also know how to work with oscilloscopes and spectrum analyzers. Therefore, they can begin the task right away. They have a week to do their job, and afterwards they present a report which is graded.

5. Student's Work

As a result of many reports received, we wanted to show how they reach the objectives we have proposed.

Figure 8 is a MathCad [13] plot of frequency responses calculated for both noise transfer function HN(s) and signal transfer function Hx(s)

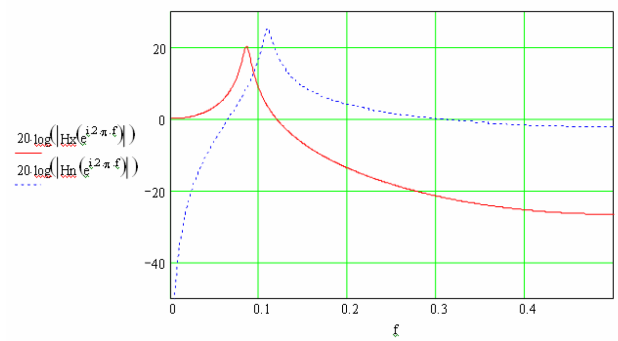

Figure 8: ΣΔ Lowpass & Highpass Filter Responses

A key motivation in this assignment is to obtain quite similar results at the output of the system, when an actual signal goes through the modulator. Noise spectral density after the second order ΣΔ loop can be seen in Figure 9

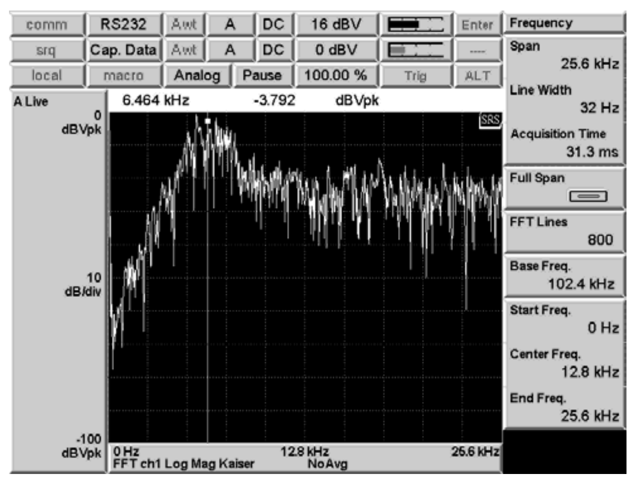

Figure 9: Actual ΣΔ Highpass Filter Response

They also use the oscilloscope to measure the integrators output. Subtracting both channels they see how a differential integrator works.

As they can change the clock frequency, students play with the idea of low oversampling, seeing how noise "invades" signal bandwidth.

They also design the reconstruction filter, analyzing the constraints it should have, therefore understanding the decimation filter role. They can, in addition, apply an averaging filter with the oscilloscope and turn a one bit digital signal into an analog one as in Figure 10.

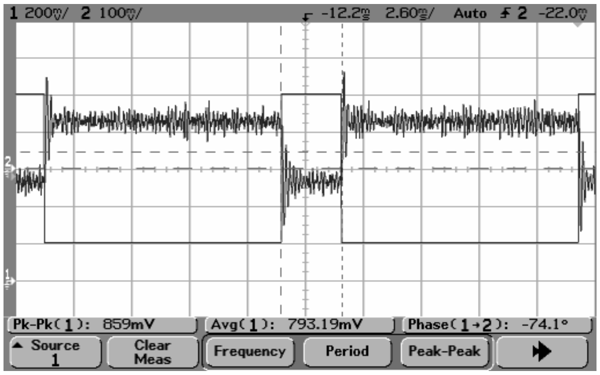

Figure 10: ΣΔ Input and Averaged Output Signals

In Figure 11, this pulse train signal spectrum has a wider bandwidth than fB/2, but it is clear that noise shaping has changed converter performance significantly. Characteristic Sinc Spectrum is still visible as noise grows at higher frequencies. At this point, students can change oversampling rate, input signal frequency and the FFT spectrum analyzer settings to fully understand the effect.

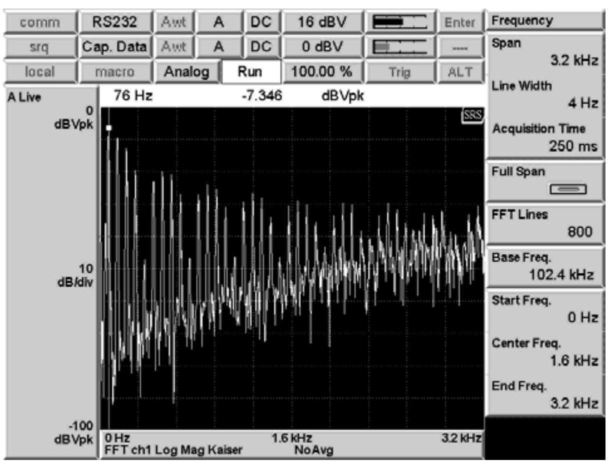

Figure 11: Pulse train Output Spectrum

6. Results

A key point of this work is to measure results, which can be done in several ways:

- Grades
- Time consumed
- Acceptance

To grade students we ask for written reports. They compile measurements, theory involved, diagrams and conclusions. We evaluate their accomplishments. As the task takes a few hours for experimented students, we don't give the assignment considerable weight in the overall grade. It has to be proportional to time consumed and to relevance in SP. Nevertheless they grasp the idea of multirate systems, switched capacitors behavior, quantization noise effects, spectrum measurements with FFTs and digital FIR filtering. The combination of such a wide line of subjects in a hands-on assignment turns the learning process into an efficient integration of concepts.

The level of acceptance becomes from the grades. Students acknowledge they were able to "see" what they were taught. Reports are almost always good, and final exams show that when they are questioned in ΣΔ subjects they answer with conviction.

7. Conclusions

This paper has described a SP assignment for an undergraduate course based on SD converters. Although the subject is seldom part of such syllabus, we showed that concepts involved and laboratory assignment results worth its introduction. Acceptance is high and actual measurements of abstract fundamentals help students learn more efficiently.

SP subjects are increasing over the years. An introductory course in the area must keep its contents moving toward new developments. A lab assignment based on SD Modulator can contribute in that direction.

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