IMAGE DENOISING IN THE TRANSFORMED DOMAIN USING NON LOCAL NEIGHBORHOODS

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ABSTRACT

In this paper we address a denoising technique based on calculation of non local means through neighborhoods. Non local neighborhoods are computed in a transformed domain, namely the wavelet domain. A noisy image is transformed using a lifting scheme. The wavelet coefficients in each subband image are modelized by a Generalized Gaussian Distribution (GGD) whose parameters (scale and shape parameters) are estimated using an appropriate technique. The estimated parameters are used to define a generalized non local mean which allows us to restore the original image. Processing in the wavelet domain is suitable since image are often available in a compressed domain, beside, processing smaller images allows us to reduce the computational cost.

1. INTRODUCTION

Image denoising is an attractive field of image processing. Many techniques for image denoising are based on "local smoothing" [7][8]. These techniques, when performed in the spatial domain as well as in the frequency domain fail to properly restore the fine structures of the image. This is due to the fact that fine structures are functionally considered as noise. To go through this difficulty, some methods were introduced where the assumption of regularity of the image is relaxed. These alternative methods consider another property of natural images which is redundancy. The Non Local Mean (NLM) technique introduced in [6] restores the original image by considering non local neighborhoods of a given pixel. The concept of non local neighborhoods is very useful in natural as well as in textured images. In fact, it exploits the redundancy and allows a better contribution of different image structures to denoise similar ones. In this article, we first present an overview of the NLM as introduced in [6] then we estimate the parameters of distribution of the orthogonal wavelet coefficients, finally, using the latter estimated parameters we introduce a non local technique inspired from NLM that we apply on wavelet sub-band images.

2. PROBLEM STATEMENT

Let $\mathbf{g}$ be the noisy image of support I and $\mathbf{f}$ the original one. Let p be a fixed pixel on image $\mathbf{g}$. The estimated value of the pixel p in the original image is computed as a weighted average of all the pixels in the image:

$$\hat{f}(p) = \sum_{q \in I} w(p,q)\mathbf{g}(q) \tag{1}$$

where $w(p,q)$ is a weight associated to p which depends on the similarity between the current pixel p and the pixel q.

$$w(p,q) = \frac{1}{Z(p)} exp(-\frac{\|\mathbf{g}_p - \mathbf{g}_q\|_2^2}{h^2}) \tag{2}$$

where $\mathbf{g}_p$ and $\mathbf{g}_q$ denote the neighborhoods of pixels p and q respectively and h is a "control" constant that controls the decay of the exponential function. Z is a normalizing factor which is, for each pixel, the sum of all exponential terms.

3. THE PROPOSED NON LOCAL METHOD

In this section, we propose to deal with the noisy image in a transformed, compressed domain, namely the wavelet domain. Nowadays, digital images are often transmitted, received and processed in a compressed domain, so it makes sense to proceed denoising in the wavelet domain which is widely used in compression standards. So, in the following, we suppose we decompose the image using the second generation of wavelets based on lifting scheme [4, 5]. Namely, $\mathbf{g}^a, \mathbf{g}^h, \mathbf{g}^v, \mathbf{g}^d$ stand for approximation, horizontal, vertical and diagonal sub-images corresponding to four wavelet subbands in a given scale. More precisely, we use here a lifting scheme when computing sub-images. In [3], it is proved that wavelet coefficients in the detail sub-bands can be modelized by means of a generalized gaussian. More precisely, wavelet coefficients of detail sub-bands follow a generalized gaussian distribution which can be expressed as follows:

$$\forall x \in \mathbb{R} \quad f_X(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{-(|x|/\alpha)^\beta} \tag{3}$$

where $\Gamma(\cdot)$ is the Gamma function, α is the scale parameter and β is the shape parameter. The value of β determines the decay rate of the pdf. Our purpose then, is to estimate the scale and shape parameters.
In equation (1), the weights $w(p,q)$ act as a similarity measure between the neighborhood of pixel p and pixel q. In fact, when $w(p,q)$ is large, the corresponding neighborhood strongly contributes to the denoising of pixel p and vice versa. Here, as we proceed in the wavelet domain, we propose to evaluate the similarity between two image windows using the following formula:

$$w(p,q) = \frac{1}{Z(p)} exp(-\frac{\|\mathbf{g}_p - \mathbf{g}_q\|^{\beta}}{(h\alpha)^{\beta}}) \tag{4}$$

where the GGD replaces the gaussian measure used in (2) as similarity measure. Therefore, we introduce a generalized expression for the Non Local Mean defined by:

$$\hat{f}(p) = \sum_{q \in I} w(p,q)\mathbf{g}(q) \tag{5}$$

where $w(p,q)$ are given by equation (4).

3.1. Parameter estimation

Several approaches have been investigated to estimate α and β [1, 2]. The most commonly used method exploits the following expression of the ratio of standard deviation σ to the mean absolute value $E[\mid X \mid]$:

$$\frac{\sigma}{E[\mid X \mid]} = \frac{\sqrt{\Gamma(1/\beta)\Gamma(3/\beta)}}{\Gamma(2\beta)} = \mathcal{F}(\beta). \tag{6}$$

More precisely, from a sample set $(x_1, \ldots, x_N)$, the estimates $\hat{m} = \frac{1}{N}\sum_{n=1}^{N} |x_n|$ and $\hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N}(x_n - \hat{m})^2$ are computed. Then, the solution $\hat{\beta} = \mathcal{F}^{-1}(\frac{\hat{\sigma}}{\hat{m}_1})$ is found by using interpolation and a look-up table. Recently, M.N. Do and M. Vetterli proposed a maximum-likelihood estimation: the shape parameter is the solution of a transcendental equation which is solved by an efficient Newton-Raphson algorithm [2]. These parameters are, then, used in the NLM expression in order to restore the original image. Once, β is estimated, the estimate of α is given by:

$$\hat{\alpha} = (\frac{\beta}{L}\sum_{i=1} L\,|x_i|^{\beta})^{\frac{1}{\beta}}$$

3.2. Choice of the local neighborhoods

In the classical NLM method the whole image is spanned and all its blocks contribute to denoise each pixel with a certain multiplicative weight. Here we propose to select only "significant" neighborhoods. That is to mean that, to denoise a fixed pixel p, we only consider image blocks that are similar, according to a certain distance, to the neighborhood of p. In order to select those blocks and to eliminate the other, we compare each neighboring pixels to p, with the considered block using an SVD-based metric. Let consider P the direct neighborhood of pixel p. It consists in $n \times n$ image block centered in pixel p. Let consider Q another $n \times n$ image block. Our aim is to compare P and Q in order to make a decision weather Q must participate to the denoising of p. Let consider P and Q as a matrices and compute their respective singular values, s_i and $\hat{s}_i$ respectively for $i = 1 \ldots n$. It is shown in [10] that comparing the singular values gives us a precise idea about the similarity between two images. So we consider the following metric:

$$d_{p,q} = \sqrt{\sum_{i=1}^{n} |s_i - \hat{s}_i|^2}$$

If $d_(p,q)$ is smaller than a fixed threshold, then, Q will participate to denoising P otherwise it won't.

3.3. Generalized NLM (GNLM) algorithm

In the GNLM, we propose to denoise the different sub-bands of the noisy image differently. It is also possible not to perform a denoising procedure on all sub-bands but uniquely the detail ones. Let consider the case where we denoise all sub-bands. In the detail ones, we have already mentioned that the wavelet coefficients follow a GGD. For this reason, we compute the NLM estimate using the generalized gaussian weighted norm proposed in (5).

In this equation, α models the width of the pdf peak (standard deviation), while β is inversely proportional to the decreasing rate of the peak. As α gives us an idea about the distribution of the wavelet coefficients around their mean value, we can state that when α is "small", the wavelet coefficients are concentrated around the mean value, thus, we can consider that we must only consider neighborhoods situated in a small area around the considered pixel. We propose, here, to take α as "radius" for neighborhoods. Inversely, when α is "large", this means that in the considered sub-band there are not many similar structures, so, that we can consider large "radius" for neighborhoods.

Concerning the approximation sub-image, we can denoise it using the classical NLM method. We also can denoise it using a simpler local smoothing technique without significant loss of performance. Indeed, the fine image structures are preserved as treated in the detail sub-bands using the proposed GNLM algorithm. Table 1 summarizes the GNLM algorithm and the neighborhood selection procedure.

1. Decompose the noisy image using lifting scheme.
2. For each image sub-band, estimate the scale parameter α and the shape parameter β.
3. For each pixel p of the sub-image, compute

$$d_{p,q} = \sqrt{\sum_{i=1}^{n} |s_i - \hat{s}_i|^2}$$

.
4. If $d_{p,q} < \tau$ retain the pixel q otherwise do not retain it.
5. For each pixel p of the considered sub-image, compute

$$Z(p) = \sum_{q \in I, q \neq p} exp(-\frac{\|\mathbf{g}_p - \mathbf{g}_q\|^\beta}{(\alpha h)^\beta})$$

by considering only the retained pixels q in the sum.
6. For each pixel p, compute the corresponding denoised pixel: $\hat{f}(p) = \sum_{q \in I} w(p,q)\mathbf{g}_q$

Table 1: The generalized NLM algorithm.

4. EXPERIMENTAL RESULTS

We perform denoising on a 128×128 portions of different lena and cameraman images. First, we evaluate the performance of parameter estimation. Then, we study the effect of denoising the test images using the GNLM algorithm.

4.1. Parameter estimation performance

We decompose the Lena image using a lifting scheme to the first level of decomposition. For each detail sub-band, we plot the histogram of the wavelet coefficients as well as the estimated GGD using the α and β estimated parameters. The results are given on figure 3. We can clearly see that, for each sub-band, the estimated GGD fits well the wavelet coefficients histogram.

4.2. Denoising results

We propose, here, to denoise, only, the detail sub-bands using the genralized Non Local Mean algorithm proposed in equation (5) and to denoise the approximation sub-image using a simpler denoising method. Namely, we use here method developped and described in [11]. In figure 1, we compare the quality of the denoising using the NLM algorithm in all subbands and using the alternative method on the approximation subband. We can notice that the results are really similar. So, we apply the alternative denoising method to approximation sub-image in further simulations.
For the lena image, we compare the performance of the classical NLM algorithm to the GNLM algorithm. To do so, we use a similarity image quality measure (SSIM), developped in [12]. This measure evaluates the similarity between the original image and the denoised ones. Results are given on figure 2 and 4. We also studied the computational cost of the method compared with the classical one. As we perform denoising on sub-images whose size is the quarter the one of the original image, our method is faster than the classical one.

5. REFERENCES

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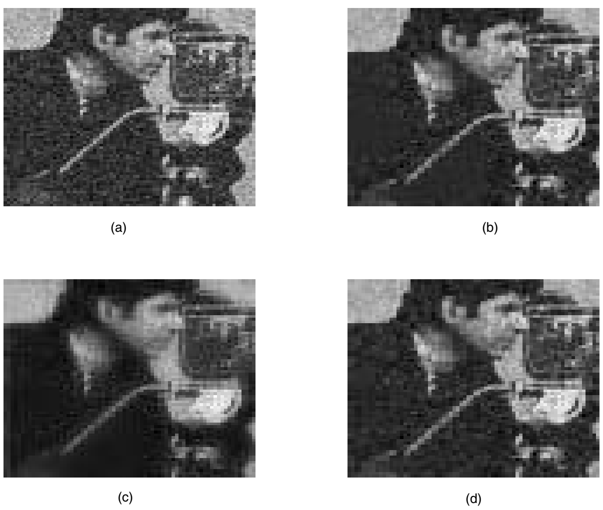

Fig. 1. (a): Noisy image, (b): Denoised image using the GNLM algorithm in all wavelet sub-bands, (c): Denoised image using GNLM in detail sub-bands and a simpler methof in approximation sub-band, (d): Denoised image using the noisy approximation sub-image, and denoised detail sub-bands by means of GNLM algorithm.

Fig. 2. From left to right: Denoised image using GNLM algorithm on a one level wavelet decomposition, Noisy image, original image

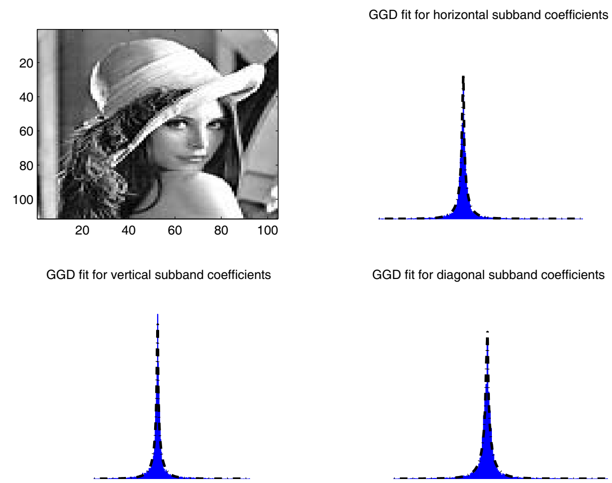

Fig. 3. Evaluation of the quality of parameter estimation on different detail sub-bands.

Fig. 4. From leeft to right: Noisy image, NLM denoised image (SSIM = 89%), GNLM denoised image (SSIM=92%)