REALIZATION OF 2-D SEISMIC MIGRATION FIR DIGITAL FILTERS FOR 3-D SEISMIC VOLUMES VIA SINGULAR VALUE DECOMPOSITION

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ABSTRACT

In this paper, we propose a new scheme for realizing predesigned two-dimensional (2-D) complex-valued seismic migration FIR digital filters, which are used for migrating three-dimensional (3-D) seismic volumes. The realization is based on Singular Value Decomposition (SVD) of such quadrantal symmetrical 2-D FIR filters. In order to simplify the SVD computations for such impulse response structure, we apply a special matrix transformation on the migration filter impulse responses where we guarantee the retention of their wavenumber phase response. Unlike the existing realization methods which are used for this geophysical application, this realization via SVD results in perfect circularly symmetrical magnitude and phase wavenumber responses. It also saves 23.08% of the number of multiplications per output sample as well as 61.54% of the number of additions per output sample when compared to direct implementation with symmetry via true 2-D convolution.

1. INTRODUCTION

One of the most important processing steps in seismic data processing, known as migration, is to correct for the incorrectly positioned appearing layers in seismic acquired sections and to obtain a depth seismic image by the use of so called *Seismic Migration filters* [1, 2]. The frequency-space (or frequency-inline-crossline) $(\omega - x - y)$ migration method is considered to be one of the most attractive techniques for performing seismic migration [1, 3]. The most important feature of such a migration technique is that it can be used for migration of one-way wavefields accurately through heterogeneous media. Also, they result in stable migration images due to the new improvements in the design of these filters like the ones reported in [1, 2].

1.1. 2-D Seismic Migration for 3-D Seismic Data Volumes

Migration for 3-D seismic data sets is performed one angular frequency (ω_o) at a time using two-dimensional (2-D) migration filters:

$$H_d(e^{jk_x}, e^{jk_y}) = \exp[jb\sqrt{k_{c_p}^2 - [k_x^2 + k_y^2]}] \quad (1)$$

where Δx and Δy are the in-line and cross-line spatial sampling intervals, respectively, k_x and k_y stand for the in-line and the cross-line wavenumbers, Δz is the migration depth step size, Δt is the time sampling interval, c_o is the velocity of the geological material, $b = \Delta z / \Delta x$ and, finally, $k_{c_p} = \frac{\Delta x}{\Delta t} \frac{\omega_o}{c_o}$ is the cut-off wavenumber. Clearly, the desired 2-D wavenumber response has a circular symmetry in the magnitude as well as the phase response (which is a non-linear function). The $\omega - x - y$ migration of a spatially sampled seismic wavefield $u(x_i, y_j, e^{j\omega_o}, z_k)$ from depth say z_k to $z_{k+1} = z_k + \Delta z$ is performed independently for each frequency ω_o, by a direct 2-D spatial convolution with a designed 2-D non-causal quadrantally symmetrical $N \times N$ (N is odd) complex-valued migration filter impulse response $h[n_1, n_2]$ using [1]:

$$u(x_i, y_j, e^{j\omega_l}, z_{k+1}) = \sum_{n_1=(-N+1)/2}^{(N-1)/2} \sum_{n_2=(-N+1)/2}^{(N-1)/2} h[n_1, n_2] \times u(x_{i-n_1}, y_{j-n_2}, e^{j\omega_l}, z_k). \quad (2)$$

In this case, the migration (filtering) process is carried over all frequencies ω_l, where $l = 0, \cdots, M-1$ and M is the number of frequency samples. A typical $\omega - x - y$ migration process for seismic signals sampled at $\Delta t = 4$ msec requires 1000 2-D filters that are designed and stored to migrate the seismic section $u(x_i, y_j, e^{j\omega_l}, z_k)$ to $u(x_i, y_j, e^{j\omega_l}, z_{k+1})$. This results in performing 1000 2-D convolution processes to get only one slice of the final 3-D migrated image (wavefiled) $u(x, y, z)$. So if one needs 500 depth slices, $500,000$ 2-D convolutions are required. Using direct convolution of these 2-D complex-valued $N \times N$ impulse responses, the computational complexity per output sample will be $500,000 \times N^2$, where N is the FIR filter length in the n_1 or n_2 directions. In this application, even by taking advantage of the quadrantally symmetric property of such 2-D impulse responses, the computational complexity will still be high [1].

1.2. State of the Art

Different approaches have been proposed to mitigate such expensive 3-D migration process which heavily relies on direct convolution of 2-D complex-valued FIR filter impulse responses like the splitting method where the migration is performed by splitting the process to alternatively migrate along the in-line and cross-line directions, independently [1], i.e., assuming that the 2-D migration filters are separable. Other realization examples used for such application are based on the McClellan transformations [3] and its improved version reported in [1]. All of these techniques result in stable migration images, need one-dimensional (1-D) filters, and possess, in general, cheap computations where the number of multiplications per output sample required are proportional to N. However, all of these realization methods do not yield circularly symmetric migration filters especially for $k_x \approx k_y \gg 0$ and, therefore, they lead to significant migration errors [1].

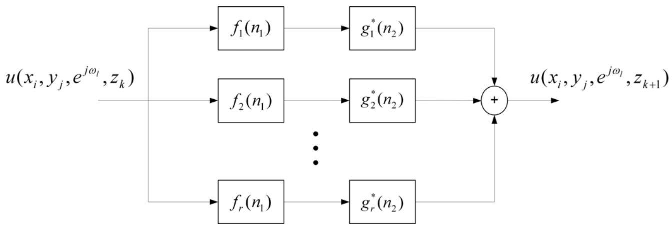


Fig. 1. SVD based realization structure of the predesigned 2-D seismic migration FIR digital filters.

1.3. Problem Definition: Revisited

In view of the above discussion, and as a result of the advances in storage and processing power for computer systems, there is a need for migrating 3-D seismic data sets with true 2-D seismic migration filters that are cheap to implement, result in stable migrated images, and possess perfectly circular symmetry with respect to their wavenumber responses. Digital FIR filter realization techniques based on Singular Value Decomposition (SVD) have been proposed for the realization of 2-D zero-phase FIR digital filters [4] and, more recently, for 2-D linear-phase FIR digital filters [5]. In both papers, the predesigned 2-D FIR filters where realized with their proposed SVD techniques for general FIR filters, including symmetrical and anti-symmetrical ones. In general, the SVD realization structure has several attractive advantages. It is suitable for parallel processing [4, 5] such as the case for $\omega - x - y$ migration. Also, it is flexible in the sense that we can select the number of realization parallel sections that correspond to the most significant singular values. Hence, this results in computational complexity savings in trade-off introduction of small errors in the wavenumber response. Finally, depending on the number of parallel sections used in the realization, its computational complexity is proportional to N.

1.4. Paper Contributions

In this work, the mathematical development of realizing 2-D complex-valued quadrantal symmetrical seismic migration FIR filters which are used for the $\omega - x - y$ 3-D migration using SVD, is shown where we exploit the existence of insignificant singular values and discard them while we still retain the phase response of these migration filters. In order to simplify the SVD computations for such impulse response structure (i.e., quadrantal symmetry), we apply a special matrix transformation on the migration filter impulse responses where we guarantee the retention of their wavenumber phase response. As a result, our proposed realization method for such geophysical application overcomes the problems of other reported realization schemes in terms of computational complexity, stable migrated images, and circularly symmetrical wavenumber response.

2. SVD REALIZATION FOR 2-D COMPLEX-VALUED SEISMIC MIGRATION FIR IMPULSE RESPONSE

Let $h[n_1, n_1]$ be an already designed $N \times N$ quadrantally symmetrical 2-D seismic migration FIR impulse response where $h[n_1, n_1] \in \mathbb{C}^{N\times N}$ for $n_1, n_2 = -(N-1)/2, \cdots, (N-1)/2$ and N is an odd number. Define $\mathbf{A}$ to be an $N \times N$ matrix whose elements are representing the quadrantally symmetrical 2-D seismic migration FIR impulse response as given by:

$$\mathbf{A} = \{h[n_1, n_1]\}, \text{ for } |n_1, n_2| \leq (N-1)/2. \tag{3}$$

2.1. Singular Value Decomposition & FIR Realization

In general, the SVD of $\mathbf{A}$ can be written as

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^* \tag{4}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, $*$ denotes the complex conjugate transpose, and $\mathbf{\Sigma}$ is a diagonal matrix whose diagonal elements represent the singular values of $\mathbf{A}$, i.e.,

$$\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_N) \tag{5}$$

and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_N \geq 0$ [6]. Let the rank of $\mathbf{A}$ be $r \leq N$. Hence, $\sigma_{r+1} = \sigma_{r+2} = \cdots = \sigma_N = 0$ and (4) can be rewritten as:

$$\mathbf{A} = \sum_{k=1}^{r} \sigma_k \mathbf{u}_k \mathbf{v}_k^* = \sum_{k=1}^{r} \mathbf{f}_k \mathbf{g}_k^* \tag{6}$$

where $\mathbf{u}_k$ and $\mathbf{v}_k$ represent the kth column vectors of $\mathbf{U}$ and $\mathbf{V}$, respectively, and $\mathbf{f}_k = \sqrt{\sigma_k}\mathbf{u}_k$ and $\mathbf{g}_k = \sqrt{\sigma_k}\mathbf{v}_k$. Equation (6) suggests that the 2-D seismic migration FIR digital filter can be realized using r parallel 2-D subfilters where each 2-D subfilter is composed of a cascade of two N-length 1-D complex-valued seismic migration FIR digital filters. These 1-D filters have impulse responses given by $f_k(n_1)$ and $g_k(n_2)$. Fig. 1 demonstrates the SVD based realization structure for the migration filtering process where the implementation complexity will depend on the value of r, which is equal to $(N+1)/2$ in the case of quadrantally symmetrical impulse responses.

2.2. SVD Realization of Migration FIR Filters

For the analysis given below, we will follow [5]. Define $\mathbf{J}$ to be an $(N-1)/2 \times (N-1)/2$ contra-identity matrix where the contra-diagonal elements are equal to 1 and the remaining elements are zeros. Since $\mathbf{A} \in \mathbb{C}^{N\times N}$ possesses quadrantal symmetry and N is odd, $\mathbf{A}$ can be written as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{a}_1 & \mathbf{A}_1\mathbf{J} \\ \mathbf{a}_2^* & c & \mathbf{a}_2^*\mathbf{J} \\ \mathbf{J}\mathbf{A}_1 & \mathbf{J}\mathbf{a}_1 & \mathbf{J}\mathbf{A}_1\mathbf{J} \end{bmatrix} \tag{7}$$

where $\mathbf{A}_1$ is an $(N-1)/2 \times (N-1)/2$ matrix, $\mathbf{a}_1$ and $\mathbf{a}_2$ are $(N-1)/2$-dimensional column vectors, and c is a complex scalar. We can easily show that the matrix $\mathbf{Q} \in \mathbb{C}^{N\times N}$, which is defined as

$$\mathbf{Q} = \frac{1}{2}\begin{bmatrix} \mathbf{I} + j\mathbf{I} & \mathbf{0} & \mathbf{J} + j\mathbf{J} \\ \mathbf{0} & \sqrt{2} + j\sqrt{2} & \mathbf{0} \\ \mathbf{I} + j\mathbf{I} & \mathbf{0} & -\mathbf{J} + j\mathbf{J} \end{bmatrix} \tag{8}$$

is a unitary matrix. Then, we obtain a unitary matrix $\mathbf{B} \in \mathbb{C}^{N\times N}$ similar to $\mathbf{A}$ by the relation:

$$\begin{aligned} \mathbf{B} &= \mathbf{Q}\mathbf{A}\mathbf{Q}^* \\ &= \begin{bmatrix} \mathbf{A}_1 + \mathbf{J}\mathbf{A}_1\mathbf{J} & \frac{\sqrt{2}}{2}\mathbf{a}_1 + \frac{\sqrt{2}}{2}\mathbf{J}\mathbf{a}_1 & \mathbf{0} \\ \sqrt{2}\mathbf{a}_2^T & 2c & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \end{aligned} \tag{9}$$

where $\mathbf{B}_1$ is an $(N+1)/2 \times (N+1)/2$ matrix. Note that $\mathbf{A}^*\mathbf{A}$ is also unitary and similar to $\mathbf{B}^*\mathbf{B}$ with respect to $\mathbf{Q}$. This implies that $\mathbf{A}^*\mathbf{A}$ and $\mathbf{B}^*\mathbf{B}$ both have the same eigenvalues and, consequently, the same singular values, i.e., the matrices $\mathbf{A}$ and $\mathbf{B}$ are unitary equivalent [6]. Now, let the SVD of $\mathbf{B}$ be given as:

$$\mathbf{B} = \mathbf{U_B}\mathbf{\Sigma_B}\mathbf{V_B^*} \tag{10}$$

where $\mathbf{U_B}$ and $\mathbf{V_B}$ are unitary and $\mathbf{\Sigma_B}$ is a diagonal matrix with singular values in decreasing order. Based on (9), one can rewrite (10) as:

$$\mathbf{B} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{11}$$

where this implies that we can determine the SVD of $\mathbf{B}$, by only computing the SVD of

$$\begin{aligned} \mathbf{B}_1 &= \mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1 \\ &= \begin{bmatrix} \mathbf{U}_{11} & \mathbf{b}_1 \\ \mathbf{b}_2^T & U_0 \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_{11} & \mathbf{0} \\ \mathbf{0} & \Sigma_0 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{11} & \mathbf{c}_1 \\ \mathbf{c}_2^T & V_0 \end{bmatrix}. \end{aligned} \tag{12}$$

Thus, $\mathbf{A}$ can be expressed as:

$$\begin{aligned} \mathbf{A} &= \mathbf{Q}^*\mathbf{B}\mathbf{Q} \\ &= \mathbf{Q}^*\mathbf{U_B}\mathbf{\Sigma_B}\mathbf{V_B^*}\mathbf{Q} \\ &= \hat{\mathbf{U}}\mathbf{\Sigma_B}\hat{\mathbf{V}}^* \end{aligned} \tag{13}$$

where

$$\begin{aligned} \hat{\mathbf{U}} &= \mathbf{Q}^*\mathbf{U_B} \\ &= \frac{1}{2}\begin{bmatrix} (\mathbf{I}-j\mathbf{I})\mathbf{U}_{11} & (\mathbf{I}-j\mathbf{I})\mathbf{b}_1 & \mathbf{0} \\ (\sqrt{2}-j\sqrt{2})\mathbf{b}_2^* & (\sqrt{2}-j\sqrt{2})U_0 & \mathbf{0} \\ (\mathbf{J}-j\mathbf{J})\mathbf{U}_{11} & (\mathbf{J}-j\mathbf{J})\mathbf{b}_1 & \mathbf{0} \end{bmatrix} \end{aligned} \tag{14}$$

and

$$\begin{aligned} \hat{\mathbf{V}} &= \mathbf{Q}^*\mathbf{V_B} \\ &= \frac{1}{2}\begin{bmatrix} (\mathbf{I}-j\mathbf{I})\mathbf{V}_{11} & (\mathbf{I}-j\mathbf{I})\mathbf{c}_1 & \mathbf{0} \\ (\sqrt{2}-j\sqrt{2})\mathbf{c}_2^* & (\sqrt{2}-j\sqrt{2})V_0 & \mathbf{0} \\ (\mathbf{J}-j\mathbf{J})\mathbf{V}_{11} & (\mathbf{J}-j\mathbf{J})\mathbf{c}_1 & \mathbf{0} \end{bmatrix}. \end{aligned} \tag{15}$$

As expected, only the first $(N+1)/2$ columns of both (14) and (15) are nonzero and they are symmetric. Since both $\hat{\mathbf{U}}$ and $\hat{\mathbf{V}}$ are unitary and both $\mathbf{A}$ and $\mathbf{B}$ have identical singular values, (13) gives an SVD of $\mathbf{A}$. Finally, the SVD of $\mathbf{A}$ can be represented based on (13), where the $\mathbf{u}_k$'s and $\mathbf{v}_k$'s are replaced respectively with the first $(N+1)/2$ columns of $\hat{\mathbf{U}}$ ($\hat{\mathbf{u}}_k$'s) and $\hat{\mathbf{V}}$ ($\hat{\mathbf{v}}_k$'s), respectively.

We now want to discard insignificant singular values and, therefore, reduce the number of parallel sections required to realize our migration FIR filters. That is, we want to approximate $\mathbf{A}$ by:

$$\mathbf{A}_K = \sum_{k=1}^{K} \sigma_k \hat{\mathbf{u}}_k \hat{\mathbf{v}}_k^* \sum_{k=1}^{K} = \hat{\mathbf{f}}_k \hat{\mathbf{g}}_k^* \tag{16}$$

where $K < (N+1)/2$. In this case, the number of parallel sections in Fig. 1 are reduced and this results in significant savings in terms of the computational complexity for obtaining a final seismic image while according to (14) and (15) we guarantee the even symmetry of the 1-D constituent filters to result in an overall desired wavenumber response. Clearly, since the 1-D subfilters are of even symmetry, the number of multiplications per output sample required to realize the 2-D complex-valued seismic migration FIR filter using the SVD realization scheme is $K(N+1)$, where $K < (N+1)/2$. The number of multiplications per output sample in this case is much less than when compared to the direct convolution. Also, we will save in the number of multiplications per output sample even when compared to migration performed via direct convolution taking into consideration that such FIR filters are of quadrantal symmetry as far as

$$K(N+1) < \frac{(N+1)^2}{4}. \tag{17}$$

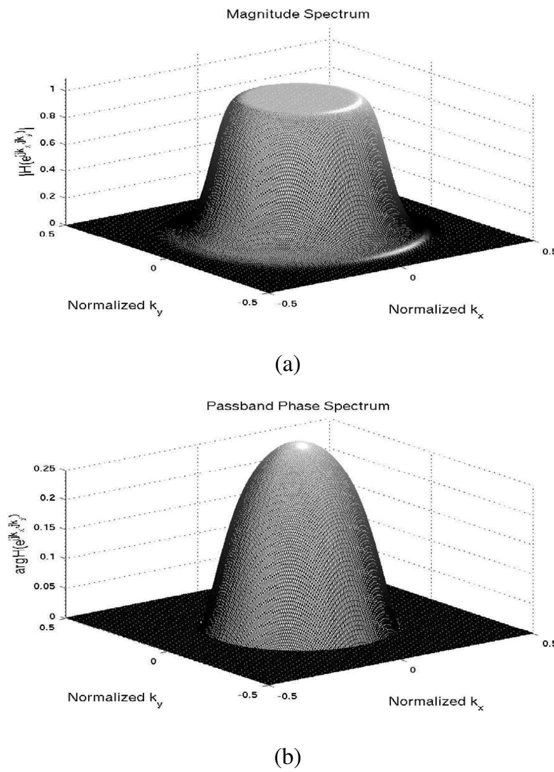


Fig. 2. A complex-valued 25×25 predesigned 2-D seismic migration FIR digital filter with a cut-off $k_{c_p} = 0.25$: (a) Magnitude spectrum, and (b) Phase spectrum.

3. SIMULATION RESULTS

A 25×25 complex-valued seismic migration FIR filter was designed using the Modified Projections onto Convex Sets (POCS) method described in [2] for $\Delta z = 2$ m, $\Delta x = \Delta y = 10$ m, $\Delta t = 0.004$ seconds, $\omega_o = 50\pi$ radians/sec, and a velocity $c_o = 1000$ m/s, to give a normalized cut-off wavenumber of $k_{c_p} = 0.25$ (see Fig.2 (a) for its magnitude response and (b) for its passband phase response). The 2-D FIR filter impulse response matrix is transformed to be in the form of (9) and then decomposed to give the resultant $\mathbf{B}_1$ matrix based on (11). The rank of the impulse response matrix of this filter is of full rank, i.e., $rank(\mathbf{B}_1) = 13$. That is, the number of parallel sections that should be used to correctly implement such filters is equal to 13 sections. However, Fig. 3 suggests that we can implement such a filter matrix with less number of parallel sections (see Fig. 1) by discarding the insignificant singular values according to (16) where we can see that 4 or 5 parallel sections are sufficient to realize our migration filter. Fig. 4 (a) and (b) shows the magnitude and

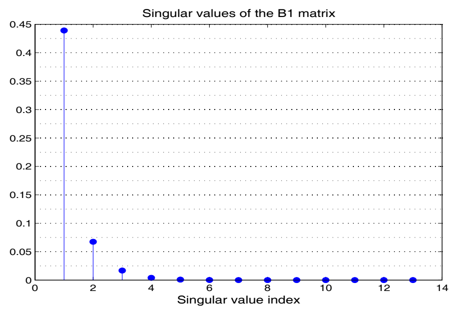
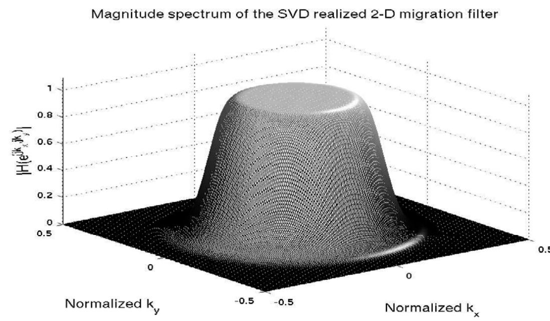


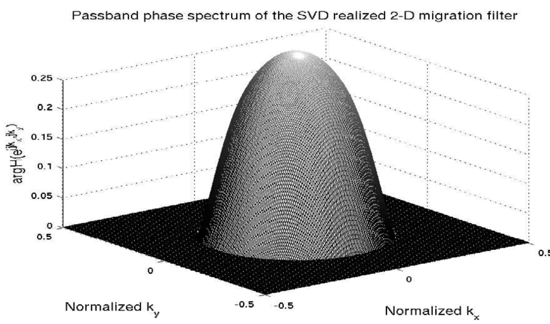
Fig. 3. Singular values of the matrix $\mathbf{B}_1$ of the complex-valued predesigned 25×25 2-D seismic migration FIR digital filter.

Table 1. Comparison of the number of multiplications and additions per output sample required to realize 2-D complex-valued FIR seismic migration filter $N = 25$.

Method	Multiplications	Additions
2-D convolution	$(\frac{N+1}{2})^2 = 169$	$N^2 - 1 = 624$
McClellan	$5\frac{N-1}{2} + 1 = 61$	$9(\frac{N-1}{2}) - 2 = 106$
Improved McClellan	$8\frac{N-1}{2} + 1 = 97$	$12(\frac{N-1}{2}) - 2 = 142$
SVD ($K = 5$)	$K(N+1) = 130$	$2K(N-1) = 240$



(a)



(b)

Fig. 4. A complex-valued 25×25 2-D seismic migration FIR digital filter with a cut-off $k_{c_p} = 0.25$ realized using our proposed SVD method with $K = 5$: (a) Magnitude spectrum, and (b) Phase spectrum.

the phase spectrums of the SVD realized version of the predesigned 2-D FIR filter with 5 parallel sections. Finally, Table. 1 compares the number of multiplications and additions per output sample required to realize the 2-D complex-valued seismic migration FIR filter using the direct convolution considering the quadrantal symmetry of the impulse response, the original McClellan transformation, the improved McClellan transformation and our suggested SVD technique for $N = 25$. The original McClellan transformation has the lowest number of multiplications per output sample among all of these schemes. However, the proposed SVD technique is more economical than the direct 2-D convolution with symmetry. Notice that the SVD realization produces perfect circularly symmetrical magnitude and phase (retained phase) responses and, therefore, it would be expected to result in superior 3-D migrated seismic volumes when compared to 3-D migration based on both McClellan transformations. This point will be investigated and confirmed in future work.

4. CONCLUSION

We presented a novel application of singular value decomposition (SVD) for realizing 2-D quadrantally symmetrical complex-valued seismic migration FIR digital filters which are used for the expensive application of 3-D migration. Both wavenumber magnitude and phase responses possess circular symmetry unlike migration FIR filters realized with the previously reported McClellan and the improved McClellan transformations for such geophysical applications. This SVD realization saved 23.08% of the number of multiplications per output sample and 61.54% of the number of additions per output sample when compared to direct implementation with symmetry via true 2-D convolution. We are currently deriving the required formulas to quantify the approximation error. This gives us the error bounds for this new proposed SVD scheme.

5. ACKNOWLEDGMENT

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