A NON-ISOTROPIC PARAMETRIC MODEL FOR IMAGE SPECTRA

Ryan S. Prendergast and Truong Q. Nguyen

Department of Electrical and Computer Engineering
University of California, San Diego
La Jolla, CA 92093 USA
E-mail: rprender@ucsd.edu, nguyent@ece.ucsd.edu

ABSTRACT

Image restoration and enhancement problems are often considered using a solution based on a model for the image's power spectral density (PSD). This paper considers a new PSD model for images through modification of a previously considered isotropic model. The new model and an algorithm for estimating its parameters are presented. Several simulations considering PSD estimation for images corrupted by additive noise, distortion, and resolution reduction are presented to demonstrate the improvements made with this new modelling approach.

1. INTRODUCTION

A classic approach to the problems of image restoration and enhancement considers modelling and processing in the Fourier domain. One standard Fourier domain technique is the Wiener filtering solution, which determines a linear filter serving to minimize the mean-squared error (MSE) of the processed image as compared to the ideal case. This filter is found as a function of some linear degradation process as well as the power spectral density (PSD) functions of the input process and any additively interfering processes. The accuracy of the Wiener solution is naturally dependent on the accuracy of the functions modelling the degradation and PSDs, which are typically unknown in the practical case. Various common techniques for estimating these unknown functions are discussed in [1], as applied to the problem of image restoration.

The focus of this paper is on PSD modelling of images. Specific motivation came from recent work by these authors [2], which presented a generalized structure for linear MMSE super-resolution of a high resolution (HR) image from multiple degraded low resolution (LR) versions. The solution to this super-resolution problem requires knowledge of the desired HR image's PSD, which is unknown in the practical case and must be estimated from the degraded LR images. Similar motivation is also found in recent work of Aly and Dubois [3], which considered single-image resolution enhancement based on an underlying HR spectral model. While the presence of distortion and noise interferes with spectrum estimation in the classic restoration scenario, in these resolution enhancement scenarios the estimation process is even further hampered due to aliasing.

One of the primary goals of this paper is thus to determine an effective approach for estimating an image's PSD in the presence of noise, distortion, and aliasing. Although this is an ill-posed problem, a reasonable estimate can be determined based on some assumptions of the general structure of typical image spectra. For this purpose, a general parametric PSD model is constructed based on observations of test images. A numerical technique for model estimation is also presented. While the focus of this paper is on the applications of restoration and enhancement, the presented spectral modelling technique is of use for other Fourier-based approaches to image processing problems. For example, an accurate parametric model may be of use in certain encoding applications since it provides a representation of an image's statistics through a small number of bits.

There is not a large amount of previous work on image-specific spectral modelling. Several previous approaches have considered modification of standard 1D signal approaches to a 2D framework (some survey of various types of 2D PSD models are presented in portions of the books [4, 5]). However, this paper contends that these sort of approaches (e.g., a 2D ARMA model) typically do not represent images well in comparison to models designed specifically for images. Another popular modelling approach is based on an isotropic (that is, rotationally invariant) decaying exponential correlation function [6, 7]. This model is well-suited for a general variety of images and can be represented using only two parameters (one decay and one scaling coefficient). One drawback to an isotropic model is that the correlation and spectral functions of a typical image are not rotationally invariant. Because of this, a non-isotropic spectral model is presented in this paper. An earlier approach by these authors used a different technique that introduced non-isotropic features over the standard isotropic model [8], however this method was of use for a smaller class of images containing straight edge features.

The paper discusses the characteristics of image spectra in Section 2, and presents the parametric model and a numerical algorithm for its estimation in Section 3. Simulation results are then presented in Section 4. Finally, Section 5 provides concluding remarks and highlights future work.

2. IMAGE SPECTRAL CHARACTERISTICS

The presented parametric model will be constructed from an image's non-parametric PSD function. The non-parametric model is commonly obtained directly from the image by squaring the magnitude of its 2D discrete Fourier transform (DFT). Assuming an $M \times N$ pixel image $c[m, n]$, this is

$$S[k, l] = \left| \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} c[m, n] W_M^{km} W_N^{ln} \right|^2, \qquad (1)$$

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where $W_M = e^{-j(2\pi/M)}$. This is a sampled version of the normalized periodic spectrum $S_{xx}(e^{j\omega_x}, e^{j\omega_y})$. Note that (1) is an unscaled version of the standard 2D periodogram.

Since the DFT magnitude is circular-shift invariant, spatial inconsistencies across the image borders (left and right or top and bottom) create horizonal and vertical edge effects that lead to false increases of magnitude along the ω_x and ω_y axes of $S(e^{j\omega_x}, e^{j\omega_y})$. To mitigate the effect of these false edges, the image is first subjected to a spatially-variant blurring operation along the borders to blend the features of opposing sides into a smoother transition without altering the majority of the image. This operation is performed as an alternative to windowing, which corresponds to convolving the spectrum by the window's squared magnitude response. As the spectra of typical images are highly dominated by the low frequency terms, convolution by the window response can often cause more harm than good. For similar reasons sliding window methods are avoided (it is important to emphasize that while a PSD model is being used, standard images are not stationary random processes). Herein, all mention of $S(e^{j\omega_x}, e^{j\omega_y})$, which is also referred to as the "true spectrum," assumes its samples are obtained directly through the squared magnitude of the DFT of the border-blended image.

The typical characteristics of image spectra are as follows. The PSD decays quickly from its peak component at the origin of the normalized spectrum, typically dropping several orders of magnitude within a very short frequency range. This is true extending radially from the origin in all directions, although the trend of decay often varies with the angular direction. Because of this it is in many ways simpler to consider the spectrum in polar coordinates, although due to spatial-domain sampling it is necessary to bandlimit the polar representation such that it falls within a single period of the Fourier space.

3. PARAMETRIC MODELLING

One commonly considered image spectral model [6, 7] is found from the continuous isotropic autocorrelation function

$$R(x, y) = \exp[-\alpha(x^2 + y^2)^{1/2}], \text{ for } \alpha > 0 \qquad (2)$$

The Fourier transform yields the PSD function

$$S(\Omega_x, \Omega_y) = 2\pi\alpha(\alpha^2 + \Omega_x^2 + \Omega_y^2)^{-3/2}, \qquad (3)$$

which can be appropriately bandlimited for use with sampled data. Transformed to polar coordinates, this isotropic spectral model is clearly independent of the angular coordinate, allowing

$$S(\Omega_r) = 2\pi\alpha(\alpha^2 + \Omega_r^2)^{-3/2}. \qquad (4)$$

This model can provide a reasonably accurate description of the image statistics with two parameters (α and a scaling constant).

As was noted in the previous section, a rotationally invariant spectrum is not appropriate for all images. However, previous results using (4) have found that its radially decaying shape can generally serve as a good model for images. This paper therefore seeks to improve upon this existing model by allowing its shape to change dependent on angular coordinate. In this non-isotropic modification the parameter α becomes a function of the angular coordinate Ω_θ, resulting in

$$S(\Omega_r, \Omega_\theta) = 2\pi\alpha(\Omega_\theta)(\alpha(\Omega_\theta)^2 + \Omega_r^2)^{-3/2}. \qquad (5)$$

The rectangularly bandlimited version is defined through

$$S_{\mathrm{BL}}(\Omega_r, \Omega_\theta) = \begin{cases} S(\Omega_r, \Omega_\theta) & \Omega_r < \frac{\pi}{\min(|\cos(\Omega_\theta)|, |\sin(\Omega_\theta)|)} \\ 0 & \text{otherwise.} \end{cases} \qquad (6)$$

Since $\alpha(\Omega_\theta)$ varies as a continuous function of the angular coordinate in (5), some form of approximation is required for its parametric representation. In this paper, $\alpha(\Omega_\theta)$ is identified only at regularly spaced samples over [0,π). (PSD symmetry allows complete representation using only half the typical interval of Ω_θ.) The complete $\alpha(\Omega_\theta)$ is then found through a cubic spline interpolation. The periodic nature of $\alpha(\Omega_\theta)$ can be exploited in computing this interpolation. While an improvement can certainly be found through allowing nonuniform sampling of $\alpha(\Omega_\theta)$, this will require additional encoding of specific angular information. For typical images $\alpha(\Omega_\theta)$ should not vary too much over Ω_θ, opening the possibility for improved coding efficiency, e.g., through a differential-based parameter coding. Finally, note that even a continuous $\alpha(\Omega_\theta)$ leads to a discontinuity at $\Omega_r = 0$ in (5) and (6). However, in practice these PSD models are sampled in frequency and this component can be parameterized separately or computed as a function of the $\alpha(\Omega_\theta)$ parameters.

3.1. Numerical Parameter Estimation

The main focus of this parameter estimation is selection of $\alpha(\Omega_\theta)$ at specific pre-assigned samples on Ω_θ. A numerical optimization approach is used to tune the parameters from the non-parametric PSD, which is taken directly from the image. Since the given images will often be degraded (noisy, blurred, and of reduced resolution) it will be necessary for the parameter estimation process to maintain a degree of robustness against these forms of degradation.

The given bandlimited non-parametric PSD, $S_{\mathrm{BL}}(\Omega_r, \Omega_\theta)$, is divided into K contiguous angular zones, each spanning π/K radians. The information in each zone is used to determine one sample of $\alpha(\Omega_\theta)$ corresponding to a Ω_θ in the center of the zone (the standard isotropic PSD is equivalent to selecting only a single zone). Within each zone, the trend of radial decay is found by numerically integrating over Ω_θ, leaving a function of Ω_r. As typical images' spectra decay so significantly from the peak at DC, an unweighed numerical fitting will typically conform to the lower frequency components while sacrificing accuracy at higher frequencies. To compensate for this the PSD is first subjected to a base 10 logarithmic scaling, providing the function

$$\bar{S}(\Omega_r) = \int_{b_l}^{b_u} \log_{10}\left(S_{\mathrm{BL}}(\Omega_r, \Omega_\theta)\right) d\Omega_\theta \qquad (7)$$

where b_l and b_u are the lower and upper angular boundaries of the zone. For a numerical solution to this integration, a sampled representation of $S_{\mathrm{BL}}(\Omega_r, \Omega_\theta)$ is first found by interpolating its value at the points of a separable polar mesh from the given rectangularly sampling of $S(e^{j\omega_x}, e^{j\omega_y})$. The integration of (7) is then found through simple averaging. In this paper the logarithmic scaling is performed prior to numerical integration. Note that while the spectrum is rectangularly periodic, the integration (7) is calculated only for $\Omega_r \leq \pi$ so that each zone is examined over the same radial span. This essentially discards a large span of radial information in some zones, but this is not especially critical since the decay in PSD magnitude tends to level off at these high frequencies.

The final model parameters are determined from the zones' average radial decay functions (7). The PSD is first considered isotropic (that is, as a single zone) to numerically determine an initial decay α_i and scaling constant. The scaling constant is locked and used in each zone, where the zone-specific samples of $\alpha(\Omega_\theta)$ are numerically computed using α_i as an initial guess. Both the initial parameter selection and subsequent zone-specific parameter selection phases use the same iterative numerical optimization algorithm.

This algorithm seeks to minimize the mean-squared difference between the logarithm of the model in (4) and the measurement of (7). That is, between $\log_{10}(S(\Omega_r))$ and $\bar{S}(\Omega_r)$. This is done for samples of Ω_r in $(0, \pi]$, specifically ignoring the component at $\Omega_r = 0$, the inclusion of which was found to decrease the model accuracy. All angular zones are considered independently and the parameters are selected using the MATLAB Optimization Toolbox function `fmincon`, which seeks a constrained minimum to a function of several variables. For the initial isotropic optimization, α_i and the scaling constant are determined under the constraint that they are positive. After locking the scaling constant, the secondary zone-specific optimizations only require minimization over a single variable. Optimizing the decay of each zone individually offers a reduction in computational complexity, through computing the mean-squared differences of 1D functions instead of the complete 2D surface and through minimizing several functions of one variable instead of a more complicated function of several variables.

4. SIMULATION RESULTS

To examine the performance of the proposed PSD model, it will be used along with models for noise and distortion (the later two of which are assumed known) to design a linear MMSE restoration and enhancement filter. Without resolution change, this is simply the standard Wiener restoration problem [1, 4, 5]. The addition of resolution enhancement to the problem requires modification which can be found in [2]. This modification factors the effects of aliasing into the MMSE solution.

For full examination of the spectral model, the solution will be found for three cases: using the original image's non-parametric PSD obtained via (1), using the parametric PSD obtained through the proposed numerical fitting of the original non-parametric PSD, and using the parametric PSD obtained through the proposed numerical fitting of the degraded image's non-parametric PSD. This last case is representative of the semi-blind problem where the true statistical model remains unknown. The 512×512 boat and Lena images of Fig. 1 are used for simulation.

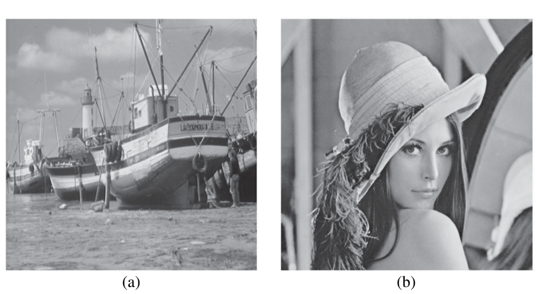

Fig. 1. Original images used for testing proposed model.

The effects of using the presented non-isotropic model and parameter selection algorithm are shown in Fig. 2. The boat image was corrupted by additive white Gaussian noise (AWGN) with a variance of 200. The model (6) was then numerically fit to the non-isotropic PSD for a varying number of angular zones. PSNR performance curves are shown for the parametric model determined from both the original and noisy PSDs. For comparison, the PSNR of the noisy image and its reconstruction using the non-parametric spectrum are shown. Both models show improvement as the number of zones increases, but the improvement steadily decreases beyond a low number of zones. Interestingly, the model obtained from the noisy image performs better than that obtained from the original. This is caused by the parameter selection algorithm, which has superior tuning performance with the addition of some noise. This example highlights interest in improving the algorithm. Based on this result, all subsequent results in this paper are obtained using 15 distinct angular zones.

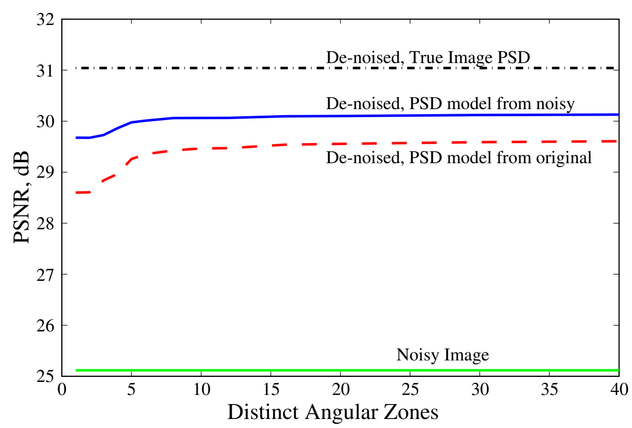

Fig. 2. PSNR of denoised boat image for varying number of angular zones. Original image corrupted by AWGN with $\sigma^2 = 200$.

Fig. 3 shows the performance for MMSE reconstruction from a decimated but otherwise uncorrupted boat image. Estimation from the low resolution image fits the parameters using the aliased spectrum, but constructs the model (6) based on the desired output resolution. There are very minor differences between the performances of the two parametric models, demonstrating the estimation approach is robust against aliasing. For all cases the model consistently performs about 1dB below the ideal case, and 3dB above a bilinear interpolation.

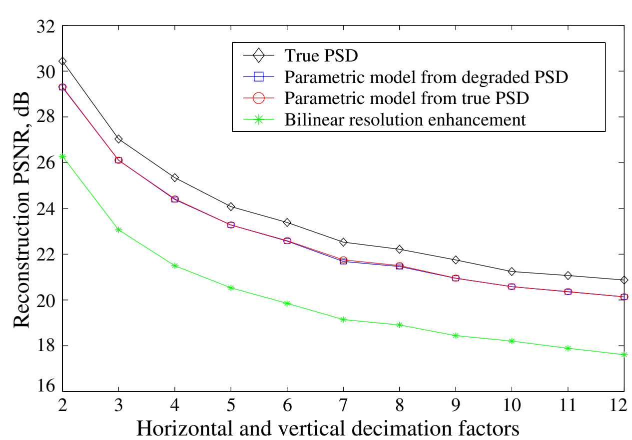

Fig. 3. Reconstruction performance for image resolution enhancement from decimated original.

In the remaining simulations, the image is first subjected to Gaussian blur, then combined with AWGN, and finally reduced in size by a factor of four through $(2, 2)$-fold decimation. Figs. 4 and 5 consider the boat image and examine the effects of varying the amount of blurring and noise, respectively. Fig. 4 shows the performance of the model obtained from numerical estimation using the degraded image remains about 1dB below the ideal, while the parametric model obtained from the original image moves closer to optimal performance as the amount of blurring increases. The performance for both cases moves closer to that of bilinear reconstruction as the blurring increases. The initial increase in performance at low levels of blurring is a result of the blurring function acting as an anti-aliasing filter prior to decimation. Fig. 5 shows consistent performance with increasing noise and concurs with the result of Fig. 2 in demonstrating a slight improvement in estimation performance in the presence of noise.

Finally, the Lena image is considered in Fig. 6 with noise, blurring, and varying levels of decimation. As this image is smoother than the boat, the performance of the model is closer to the ideal performance. Bilinear reconstruction naturally performs quite poorly.

5. CONCLUSIONS AND FUTURE WORK

The presented modification to a common isotropic image PSD was shown to improve reconstruction performance. A numerical algorithm for the model estimation was presented, along with simulations demonstrating performance of the model, and the robustness of the estimation process under degraded image conditions.

Some results indicated that while the estimation algorithm did perform well, improvement is possible and should be a subject for future work. To better evaluate the performance of the model for restoration and enhancement applications, future results will also be examined using subjective or alternate (to PSNR) evaluation metrics. An additional topic for future work involves extending the model to video. Finally, in super-resolution there are multiple degraded source images available which can be considered collectively to improve the spectral estimation.

6. REFERENCES

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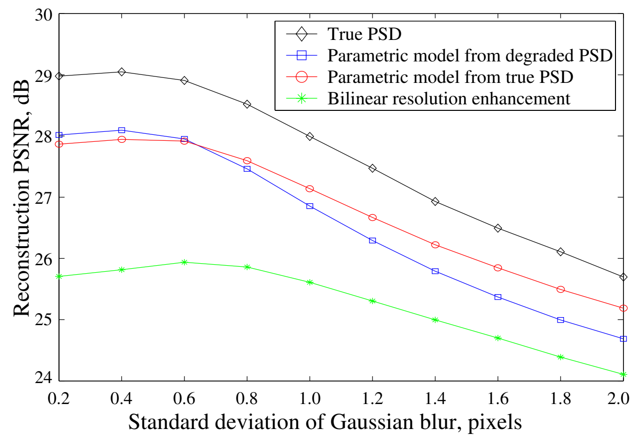

Fig. 4. Reconstruction of boat image with varying Gaussian blur, AWGN of $\sigma^2 = 50$, and $(2, 2)$-fold decimation.

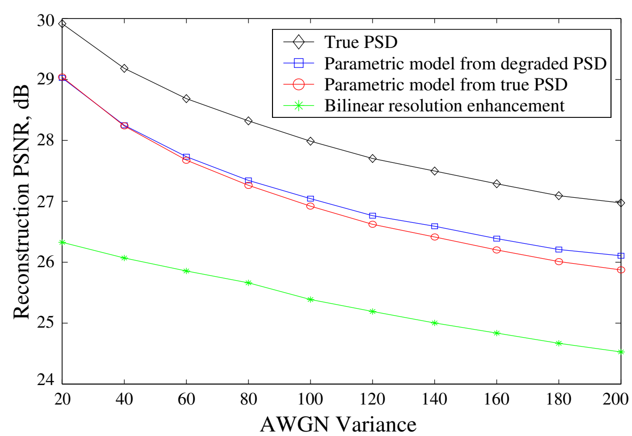

Fig. 5. Reconstruction of boat image with Gaussian blur of $\sigma = 0.6$, AWGN of varying σ^2, and $(2, 2)$-fold decimation.

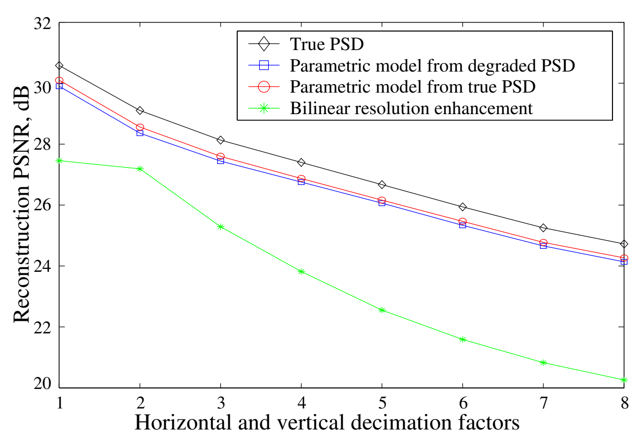

Fig. 6. Reconstruction of Lena image with Gaussian blur of $\sigma = 1.5$, AWGN $\sigma^2 = 50$, and varying decimation.