OPTIMIZATION AND INTERPOLATION FOR DISTORTED CONTOUR ESTIMATION

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ABSTRACT

Distorted curves retrieval is faced for robotic way screening, particle trajectory characterization, aerial and satellite image analysis. This image processing problem has been transposed to an array processing problem by adopting specific conventions. Some solutions for wavefront distortions canceling have already been proposed. In this paper we aim at improving an existing method for distorted curves retrieval, making use of a global optimization algorithm. We show that it is possible to combine an optimization method to an interpolation method in order to obtain a reliable and fast algorithm.

1. INTRODUCTION

One crucial problem encountered in the field of array processing is the distortion of antennas and wavefronts. These distortions must be taken into account in order to retrieve efficiently the characteristics of the sources. Some methods have already been proposed : in [1] and [2], the distortions of the array are canceled ; in [3], distortions of both antenna and wavefront are taken into account. In this paper we aim at improving the existing method for distorted contour retrieval in images. For this specific image processing problem the snakes algorithms provided valuable results [6]. Their performances depends on a manual initialization. A specific formalism and a signal generation method [4] adapt array processing methods to image processing. Precisely, a straight line in an image is assimilated to a wavefront. In [5], this problem is extended to the case of distorted contours by means of a local optimization method. Our novel methods are based on the global optimization "DIRECT" algorithm [7], and interpolation methods. In section 2, we remind the problem of distorted wavefront retrieval in the frame of array processing. In section 3, we propose an optimization method based on "DIRECT" algorithm, that faces the difficult case of noisy images. In section 4, we show how to combine optimization and interpolation methods, and we emphasize on the advantages of spline type interpolation. Section 5 exposes some simulation results that illustrate the performances of our algorithms. Concluding section 6 summarizes the interest of our paper.

2. DISTORTED WAVEFRONTS CHARACTERIZATION

Characterization of distorted wavefronts is a crucial issue in the field of array processing. Indeed, a plane wavefront, without distortions, is difficult to guarantee in practice because the sensors can move from their original positions during the experimentation. This is the case with an elastic sensor array. The distortion can also be due to inhomogeneity of the propagation medium. It is necessary to take into account the phase shifts introduced by the distortions of the wavefront or the antenna in order to resolve the acoustic sources, i.e. estimate their direction of arrival. Fig. 1 represents distorted wavefronts impinging on a distorted antenna (sensors 1,2,. . .,N). A method proposed by Bourennane et al. in [2] employs an extension of a High Resolution method in order to retrieve the antenna shape when a plane wave is impinging. This leads to valuable results, especially when antennas are composed of a small number of sensors. Saidi et al. considered the case when both impinging wavefronts and antenna are distorted [3]. They make use of a global optimization algorithm called "DIRECT" [7]. They combine the well-known High Resolution method "MUSIC" to "DIRECT" in order to retrieve not only the direction of arrival of the sources but also a deformation of the antenna. "DIRECT" algorithm can form part of a novel method for distorted contour estimation in images by means of array processing and optimization methods.

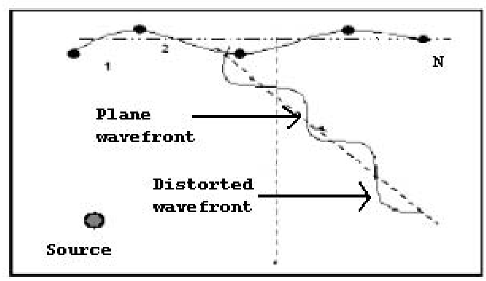


Fig. 1. Origins of phase distortions : distorted antenna or perturbed wavefronts.

3. GLOBAL OPTIMIZATION METHOD

By means of a formalism proposed in [4], some signals can be generated out of an image. Array processing methods such as TLS-ESPRIT [4] can be employed to retrieve the overall orientation of the curves. A novel method for distorted contours estimation has been proposed in [5]. This algorithm is based on array processing and optimization methods. A signal vector $\mathbf{z}_{input}$ is generated out of the image. For this purpose a propagation constant μ is introduced, and a "propagation scheme" is set in order to transpose the content of each line of the image onto the components of the signal vector $\mathbf{z}_{input}$; an initialization step is applied in order to find the parameters angle and offset [5] of an initialization straight line that fits the distorted curve to be retrieved. Finally the fixed step gradient method is applied in order to estimate the shift values between the pixels of initialization straight line and distorted curve. In this section, we set the inverse problem that must be solved in order to retrieve the distortions of the curve with respect to the initialization straight line. Then we focus on the advantages provided by DIRECT algorithm, an optimization method which improves the solving of the inverse problem.

3.1. Setting of an inverse problem

The principle of the proposed method is to minimize the squared difference between $\mathbf{z}_{input}$ and a model signal which is recursively modified. We initialize the procedure with a signal vector corresponding to the initialization straight line [5]. Let $\mathbf{X}_0 = [X_0(1), \ldots, X_0(N)]$ and $\mathbf{X}_{input} = [X_{input}(1), \ldots, X_{input}(N)]$ be the vectors containing the coordinates of the pixels of the initialization straight line and the distorted curve respectively. Corresponding signal vectors are defined respectively by :

$$\mathbf{z}_0 = [e^{-j\mu X_0(1)}, \ldots, e^{-j\mu X_0(N)}] \tag{1}$$

$$\mathbf{z}_{input} = [e^{-j\mu X_{input}(1)}, \ldots, e^{-j\mu X_{input}(N)}] + \mathbf{n} \tag{2}$$

Where $\mathbf{n}$ is the noise vector that appears in the case of a noisy image. At step l of the recursive procedure, a coordinate vector $\mathbf{X}_l$ is obtained, and we aim at minimizing the criteria J defined by :

$$J(\mathbf{X}_l) = |\mathbf{z}_{input} - \mathbf{z}_{estimated\ for\ \mathbf{X}_l}|^2 \tag{3}$$

where $|.|$ denotes the L_2 norm for a vector of complex elements. The series converges $(l \rightarrow +\infty)$ towards a vector $\widehat{\mathbf{X}}$ such that

$$\mathbf{z}_{\widehat{\mathbf{X}}} = \mathbf{z}_{input} \tag{4}$$

That is to say $\widehat{\mathbf{X}}$ is the global minimum argument of J. In order to find this minimum, a fixed step gradient method can be employed [5]. A more efficient optimization method leads reliably to the global minimum, namely the DIRECT algorithm.

3.2. DIRECT algorithm

With a view to finding the minimum argument of J, a global method is used. Contrary to the fixed step gradient method, that may converge towards a local minimum, DIRECT -DIviding RECTangles- algorithm performs global optimization [7]. An iterative procedure is run : the value of each unknown at a given step does not depend directly on the gradient of the function obtained at the previous step. At each iteration step, the space which is searched is divided into blocks, and a subset of optimal blocks is selected. The major drawback of this method is that it leads to large computational times, when the number of unknowns (the size of the unknown vector) is elevated. This is the case in practice when it is applied to our image processing method, because the size N of one side of an image is also the number of unknowns that shall be retrieved. We propose in the following to associate an interpolation method to DIRECT algorithm. This will permit to reduce drastically the number of unknowns of our optimization problem.

4. INTERPOLATION METHOD

This section reminds first the principles of an interpolation by splines functions. Then we show how to optimize the choice of the points of the distorted curve that shall be interpolated in order to retrieve the whole curve.

4.1. Principles of our interpolation method

Let $\{M_i, i = 1, \ldots, O\}$ be a set of O points $(1 < O < N)$ that belong to the distorted curve. Let us first suppose that the coordinates $\{My_i, Mx_i, i = 1, \ldots, O\}$ of these points are known. The basic idea of our method is to interpolate the curve that fits the best this set of points. For this purpose we chose a spline type interpolation. Cubic splines, especially, provide a good compromise between variations of local curvature and computational load. Interpolation is performed with piecewise third order polynomials [8]. A cubic spline f interpolating on $\{M_i, i = 1, \ldots, O\}$ is a function such that $\{f(My_i) = Mx_i, i = 1, \ldots, O\}$. It is a piecewise polynomial function consisting of $O-1$ cubic polynomials f_i defined on $[My_i, My_{i+1}]$. The criteria that is chosen to build the interpolated curve is an energy. More precisely, The function f that gives the coordinates of the points of the interpolated curve along the horizontal X axis is the one that minimizes the integral E of equation (5).

$$E = \int_{u=My_1}^{u=My_O} \frac{f''(u)^2}{(1+f'(u)^2)^{5/2}} du \tag{5}$$

This criteria is such that we obtain the curve passing through the points $\{M_i, i = 1, \ldots, O\}$ with minimum curvature variations. The curve having length N which is interpolated from the O points $\{M_i, i = 1, \ldots, O\}$ is denoted $spline(\{My_i, Mx_i, i = 1, \ldots, O\}, N)$.

4.2. Final method

The application of an interpolation method requires the knowledge of some node points. In this subsection we will show how to combine an optimization method to an interpolation method and how to make use of the straight line fitting procedure to initialize the optimization algorithm that will lead to the correct node points coordinates $\{Mxi, i = 1, \ldots, O\}$. The coordinates of the points of the initialization straight line provides us with an initial set of values $\{Mxi, i = 1, \ldots, O\}_0$ for the coordinates of the node points along the X axis. The coordinates $\{Myi, i = 1, \ldots, O\}_0$ of the node points along the Y axis can be chosen such that they are regularly distributed among the N lines of the image. The criteria that shall be minimized is modified respect to equation (3) so that it depends only on O variables : the values of the elements of the set $\{Mxi, i = 1, \ldots, O\}_l$. Its expression on iteration l is :

$$J(spline(\{My_i, Mx_i, i = 1, \ldots, O\}_l, N)) \qquad (6)$$

When this series converges ($l \rightarrow +\infty$), node points coordinates $\{Mx_i, i = 1, \ldots, O\}$ are chosen optimally with respect to criteria J. The initialization straight line is employed for giving the lower and upper bounds of the possible values for $\{Mx_i, i = 1, \ldots, O\}$, which are *a priori* assumed to be close to the coordinates of the extreme points of the initialization line. The final estimation of the coordinates of the points of the distorted curve along the X axis is obtained through the relation :

$$\widehat{\mathbf{X}} = spline(\{My_i, Mx_i, i = 1, \ldots, O\}, N) \qquad (7)$$

$\widehat{\mathbf{X}}$ is the vector of size N that characterizes the whole distorted curve.

4.3. Summary of the proposed algorithms

An outline of the proposed distorted contour estimation method is given as follows :

1. Derive artificial signals using Equation (2) ;
2. Estimate the parameters of a straight line that fits the distorted contour (beginning of section 3) ;
3. First method : Find the position $\widehat{\mathbf{X}}$ of the distorted curve pixels which satisfies to Equation (4) by using a global optimization method : the DIRECT algorithm ;
4. Final method : Estimate $\widehat{\mathbf{X}}$ by an interpolation method. The position of the interpolation nodes is found by DIRECT algorithm, introducing a spline interpolation function in the optimized criteria (Equation (6)).

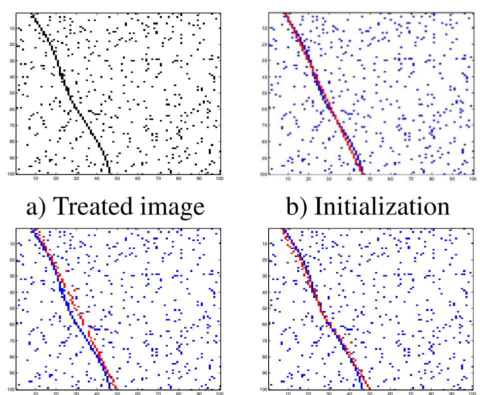
The final estimate of the distorted curve is found by interpolating the nodes that were obtained by DIRECT algorithm (Equation (7)).

5. SIMULATION RESULTS

The squared images employed have either $N = 100$ or $N = 200$ rows. When our straight line fitting procedure is applied, the propagation parameters values [5] are $\mu = 1$ and $\alpha = 2.5 * 10^{-3}$ (parameter for the variable speed propagation scheme, employed for estimating the offset of the initialization straight line). Before applying our optimization procedure, we generate the signal $\mathbf{z}_{input}$ once again, with a parameter value $\mu = 10^{-2}$. This avoids any phase indetermination problem [5].

5.1. DIRECT algorithm

The results presented on Fig. 2 were obtained after running the fixed step gradient algorithm, and DIRECT algorithm, with 50 iterations. Fig. 2 shows that the initialization



a) Treated image b) Initialization

c) Gradient method d) DIRECT

Fig. 2. Distorted contours estimation on a noisy image, with an optimization algorithm.

step is efficiently performed : orientation value -21.7°, and offset value 9 characterize a straight line which fits the distorted contour. Then the optimization algorithm is run in order to retrieve the N shift values of the curve respect to the initialization straight line. When the fixed step gradient optimization method is employed, it tends to move the estimated distorted curve away from the initialization straight line : the algorithm minimizes the criteria J, by adding rather positive values to the pixel position values (Fig. 2c)). When this method is used the mean value of the error between the expected and estimated pixel positions values is 3.3 pixels. When the optimization algorithm DIRECT is run, part of the distortions of the curve are canceled, by adding either positive or negative shift values to the pixels of the initialization straight line (Fig. 2d)). When this method is employed the mean value of the error between the real and estimated pixel positions values is 1.9 pixels. Nevertheless, we note that curve continuity criteria is not fulfilled. Next presented results will exemplify the abilities of our interpolation method, as concerns in particular the curvature variations and continuity of the estimated contours.

5.2. Interpolation method

We expose in this subsection the way our interpolation method is working with 200×200 pixels images. Fig. 3a) and c) represent the initial image, Fig. 3b) and d) the result obtained after initializing and running DIRECT algorithm combined to an interpolation method. DIRECT algorithm was run with 12 iterations. We chose in both cases to perform an interpolation between 15 nodes, equally distributed along the 200 rows of the image. Figs. 3a) and b) concern the processing of a noisy image, similar to the image of Fig. 2. Figs. 3c) and d) exemplify the abilities of our method with a satellite photography representing a coast. The initialisation step is performed on the grey level image. A threshold applied after edge-enhancing leads to a binary image, which is employed to retrieve the distortions of the coast by means of our interpolation algorithm. Figs. 3b) and d) show

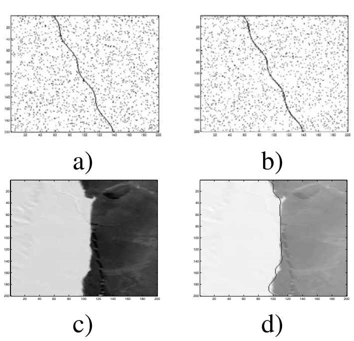


Fig. 3. Distorted contours estimation on a noisy image and a photography, by interpolation methods : a) and c) Initial images ; b) and d)Results obtained.

that the method that retrieves the distorted curve through an interpolation by spline functions leads to an estimated curve which is continuous and exhibits low curvature variations. For the noisy image the mean error value is 1.2 pixels. This method lasts about a minute whereas the application of DIRECT algorithm on all points of the curve lasts several dozens of minutes : As DIRECT algorithm computational complexity rapidly decreases with the number of variables, this method is much faster than the previous one, which does not include interpolation.

6. DISCUSSION AND CONCLUSION

This paper investigated the use of a global optimization method coming from array processing as a novel method for distorted curves retrieval in images. We reminded that the case of distorted wavefront characterization is faced in array processing. We adapted the "DIRECT" algorithm to the retrieval of a distorted curve, starting from an initialization straight line. Then we proposed to combine "DIRECT" to an interpolation method. Simulation results showed that our method works well with noisy images and grey level photographies. By employing a spline type interpolation method, we reduced the computational time, and obtained an estimation of the distorted curve with low curvature variations.

7. REFERENCES

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