VECTOR FIELDS MODELIZATION USING BASIS OF POLYNOMIALS: APPLICATION TO THE ANALYSIS OF SIMPLE FACE MOVEMENTS

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ABSTRACT

In this communication we present an original and general model for the approximation of vector fields and especially displacement vector fields. The proposed method uses the orthonormal multivariate polynomials framework to approximate vector fields as combinations of these particular functions. Then we demonstrate the noise robustness of our model. And finally we show that the model can efficiently be used for the recognition of simple face movement in a webcam acquired sequence.

1. INTRODUCTION

There is a wealth of work on movement extraction [1], [2], [3], [4] and among them comparatives between various methods [5] and denoising algorithms [3].

But only a few articles deals with movement analysis, a still recent research subject. Moreover these articles generally focuses on a very specific application field. For example some works study only the behavior of the human face or the human body [6].

Here we present a method which purpose is to characterize all types of movement as a linear combination of orthonormal polynomials [7]. Contrary to the articles previously cited it wants to be most general as possible.

Section 2 details the theoretical part of the developed method. In section 3 we test the noise robustness of the suggested model. In section 4 we present a simple face movement recognition process based on the proposed vector field model. Finally we give our conclusions and perspectives in section 5.

2. MODELING OF VECTOR FIELDS

Let $\mathcal{U} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ and $\mathcal{V} : \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ respectively be the maps corresponding to the displacement of the pixel $(x_1, x_2) \in \Omega$ according to the Cartesian axis. A vector field can therefore be defined by the following map:

$$\begin{array}{rccc} \mathcal{F}: & \Omega \in \mathbb{R}^2 & \rightarrow & \mathbb{R}^2 \\ & (x_1, x_2) & \mapsto & (\mathcal{U}(x_1, x_2), \mathcal{V}(x_1, x_2)) \end{array}$$

We seek to study movement in an analytical way. For this reason our method consists in approximating the function $\mathcal{F}$ by a linear combination of polynomials. After having defined a functional vector space, we use an inner product to create an orthonormal polynomial basis. Finally we define in this space operations allowing to obtain the required analytical expression.

2.1. Definition of the vectorial space

We denote $\mathcal{E}_p$ as the vector space of functions from $\Omega \subset \mathbb{R}^2$ to $\mathbb{R}$ which contains the functions $\mathcal{U}$ and $\mathcal{V}$. Let ϕ be the set of elements of $\mathcal{E}_p$ composed of bivariate polynomials defined as follows:

$$P(x_1, x_2) = \sum_{i=0}^{I} \sum_{j=0}^{J} a_{i,j} \, (x_1)^i \, (x_2)^j \tag{1}$$

where $I \in \mathbb{N}^+$ is the maximal x_1 degree, $J \in \mathbb{N}^+$ the maximal x_2 degree and $\{a_{i,j}\}_{i \in [0;I]}^{j \in [0;J]} \in \mathbb{R}^{I*J}$ are the coefficients. The global degree of the polynomial is I+J.

2.2. Definition of an orthonormal base

To get an orthonormal base we provide $\mathcal{E}_p$ with the inner product of two functionals F_1 and F_2:

$$\langle F_1 \mid F_2 \rangle = \int_a^b \int_a^b F_1 \, F_2 \, \omega(x_1, x_2) \quad dx_1 dx_2 \tag{2}$$

where $\omega(x_1, x_2)$ is a weighting function that eventually be chosen according to a given problem. We notice that with this inner product we can define the distance between two functions:

$$\|F_1 - F_2\| = \sqrt{\langle F_1 - F_2 \mid F_1 - F_2 \rangle} \tag{3}$$

Providing the vectorial space with the inner product allows to define a polynomial basis $\mathcal{B}$. To normalize this basis, all its elements $\{P_1, P_2, \ldots, P_n\}$ must verify $\langle P_i \mid P_j \rangle = \delta^{ij}$. Thus we seek to construct a set of orthonormal polynomials using the Gram-Schmidt's orthogonalization procedure to get an orthonormal base.

This article is limited to the study of Legendre's polynomials. They are defined by a recursive process:

$$\begin{cases} P_{0,0} = 1 \\ P_{1,0} = x_1 \\ P_{0,1} = x_2 \\ P_{i+1,j} = \dfrac{2i+1}{i+1}\, x_1\, P_{i,j} \quad - \quad \dfrac{i}{i+1}\, P_{i-1,j} \\ P_{i,j+1} = \dfrac{2j+1}{j+1}\, x_2\, P_{i,j} \quad - \quad \dfrac{j}{j+1}\, P_{i,j-1} \end{cases} \tag{4}$$

By definition the weighting function associated with these polynomials is $\omega(x_1, x_2) = 1$ and the domain is $[-1; 1]$. Of course, before any operation the pixel coordinates of $(i, j) \in \mathbb{N}^{+2}$ of a MxN image must be converted in this domain by an affine transformation.

The *degree of the basis* is the higher degree of its polynomials. For example a basis of degree 2 contains six polynomials:

	$(x_2)^0$	$(x_2)^1$	$(x_2)^2$
$(x_1)^0$	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$
$(x_1)^1$	$P_{1,0}$	$P_{1,1}$	$-$
$(x_1)^2$	$P_{2,0}$	$-$	$-$

(5)

The degree of the basis is directly related to the complexity of the movement, as indeed a lower degree basis can not modelize a complex movement.

2.3. Projection of a vector field onto the basis

Projection of a vector field onto a basis of degree D is obtained by computing the inner product between the two functions $\mathcal{U}$ and $\mathcal{V}$ associated with the field and each polynomial $P_{i,j}$ of the basis. The obtained scalars correspond to the coefficients of two polynomials $P_{\mathcal{U}}$ and $P_{\mathcal{V}}$. Thereafter we call them *characteristic polynomials*:

$$\begin{cases} P_{\mathcal{U}} = \displaystyle\sum_{i=0}^{D} \sum_{j=0}^{D-i} \alpha_{i,j}\, (x_1)^i\, (x_2)^j \quad \text{with} \alpha_{i,j} = \langle\, \mathcal{U} \mid P_{i,j}\, \rangle \\ P_{\mathcal{V}} = \displaystyle\sum_{i=0}^{D} \sum_{j=0}^{D-i} \beta_{i,j}\, (x_1)^i\, (x_2)^j \quad \text{with} \beta_{i,j} = \langle\, \mathcal{V} \mid P_{i,j}\, \rangle \end{cases} \tag{6}$$

2.4. Computing a vector field from characteristic polynomials

To approximate a vector field from its two characteristic polynomials $P_{\mathcal{U}}$ and $P_{\mathcal{V}}$, we compute the two components of each vector by fixing the two variables of each polynomial according to the position of the vector in the field:

$$\forall i, \forall j \in [a, b] \begin{cases} \mathcal{U}(i,j) = P_{\mathcal{U}}(i,j) \\ \mathcal{V}(i,j) = P_{\mathcal{V}}(i,j) \end{cases} \tag{7}$$

An example of movement generated from a linear combination of different polynomials is shown figure Fig. 1.

3. NOISE ROBUSTNESS

To have a reliable movement recognition method, the developed process must be robust to noise. That is why we present here two experiments demonstrating the noise robustness of our model.

$$\begin{cases} P_{\mathcal{U}} = 3\, P_{0,2} + 2\, P_{1,1} \\ P_{\mathcal{V}} = 2\, P_{0,0} - 2\, P_{1,0} \end{cases}$$

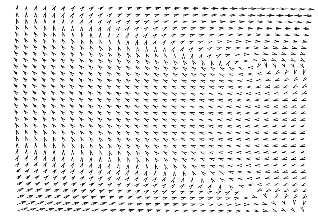


Fig. 1. Example of a polynomial linear combination.

3.1. First experiment

The test process is as follows: in a basis $\mathcal{B}$, two characteristic polynomials $P_{\mathcal{U}_o}$ and $P_{\mathcal{V}_o}$ are generated by a linear combination of basis polynomials. The vector field $\mathcal{F}_o$ associated with these two polynomials is computed. A Gaussian noise $\mathcal{G}$ is added to this field $\mathcal{F}_o$ to obtain a noisy field $\mathcal{F}_n$. This field $\mathcal{F}_n$ is projected onto the basis $\mathcal{B}$ to obtain the two characteristic polynomials $P_{\mathcal{U}_n}$ and $P_{\mathcal{V}_n}$ corresponding to this field $\mathcal{F}_n$. Finally the result field $\mathcal{F}_r$ is computed from polynomials $P_{\mathcal{U}_n}$ and $P_{\mathcal{V}_n}$. Then noise robustness is measured by comparing the initial vector field $\mathcal{F}_o$ and the previously computed field $\mathcal{F}_r$.

Tests are made with 320x240 vector field size. Initial fields are randomly generated (coefficients of characteristic polynomials $P_{\mathcal{U}_o}$ and $P_{\mathcal{V}_o}$ are taken according to an uniform distribution).

We use a normal distributed noise of standard deviation $\sigma_{\mathcal{G}}$ which is determined by the noise quantity we want to add: $\sigma_{\mathcal{G}} = \sqrt{std(\mathcal{F}_o)/SNR}$. Here the signal-to-noise ratio (SNR) move between 0.1 and 2.0 by stage of 0.1.

Measurement used to compare vector fields is the mean square error (MSE) between two vector fields.

Figure Fig. 2 represents the evolution of the MSE between fields $\mathcal{F}_o$ and $\mathcal{F}_n$ (bright curve) and fields $\mathcal{F}_o$ and $\mathcal{F}_r$ (dark curve), according to the SNR and the basis degree here from 0 to 6.

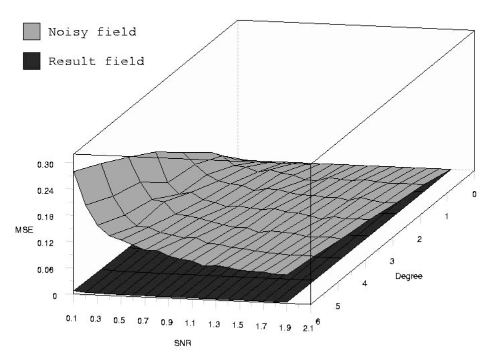
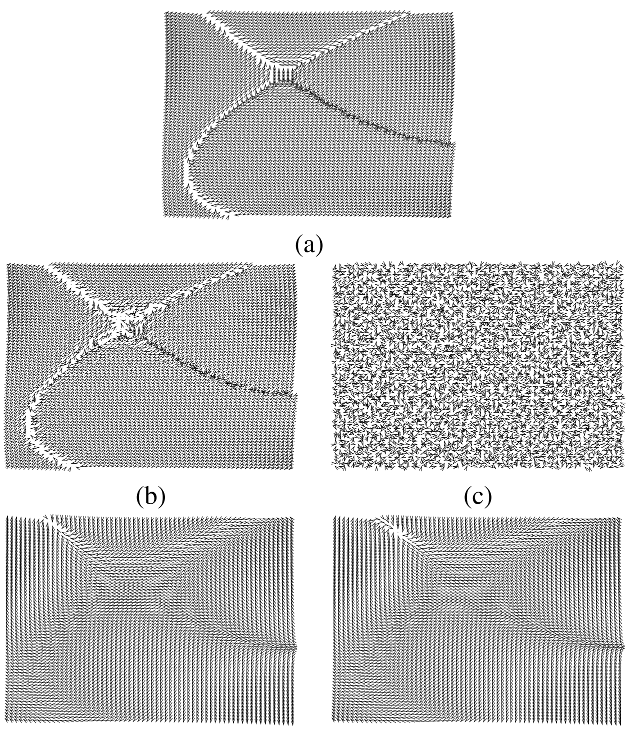


Fig. 2. Noise influence on the system.

Even though the added noise is significant we note that, considering the MSE, the reconstructed field with this method is very close to the original field. Moreover it is also true whatever the degree of the basis.

3.2. Second experiment

The figure Fig. 3 shows two examples of field reconstruction. The original field (a) is randomly generated from a basis of degree 3, (b) represents the slightly disturbed original field ($\sigma_{\mathcal{G}} = 0.01$) and (c) the strongly disturbed original field ($\sigma_{\mathcal{G}} = 1.0$). (d) and (e) show the two fields reconstructed from the two previous disturbed fields. We can visually observe the denoising qualities of our model: information about movement is preserved, even though the noise is important.



(a) (b) (c) (d) (e)

Fig. 3. Vector field reconstruction.

These two experiments demonstrate the good noise robustness of this method.

4. APPLICATION

We have just seen how to approximate a vector field using two characteristic polynomials. To study the movement in a sequence, that is to say a set of vector fields, we will study the evolution according to the time parameter of the coefficients of these characteristic polynomials.

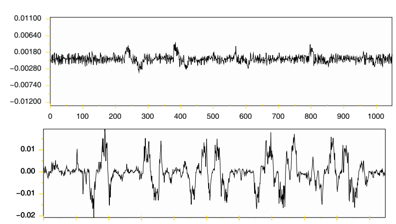
The example presented here shows the face of a person. This one turns his head on the left, on the right and to the bottom in a random way (i.e. without preset sequence). The video, coming from SERIBEL's project[1], is acquired using a webcam. It contains 1050 images of size 320x240. The figure Fig. 4 shows two frames of the sequence.

(a) (b)

Fig. 4. (a): The first frame, (b): The position after a movement towards the left.

All vector fields are extracted from the original sequence [8]. For each field, the characteristic polynomials are computed by projections onto the basis. Then movement is given by studying time variations of the coefficients of these polynomials.

The degree of this basis is 2. For each images pair twelve coefficients are obtained: six for $P_{\mathcal{U}}$ and six for $P_{\mathcal{V}}$ (cf. Eq. 5). To determinate the correlation of these coefficients, a principal component analysis (PCA) is made. Here it shows that for $P_{\mathcal{U}}$ nearly 83 % of the information is carried by only one factorial axis $\vec{E}(P_{\mathcal{U}})$ and for $P_{\mathcal{V}}$ more than 94 % of the information is also contained in only one axis $\vec{E}(P_{\mathcal{V}})$. Consequently studying the movement of this sequence corresponds to study the evolution of the coefficients of $P_{\mathcal{U}}$ after projection onto $\vec{E}(P_{\mathcal{U}})$ and those of $P_{\mathcal{V}}$ after projection onto $\vec{E}(P_{\mathcal{V}})$ (cf. Fig. 5).



(a)

(b)

Fig. 5. (a): Projection of the $P_{\mathcal{U}}$ coefficients onto $\vec{E}(P_{\mathcal{U}})$, (b): projection of the $P_{\mathcal{V}}$ coefficients onto $\vec{E}(P_{\mathcal{V}})$.

The physical meaning of these curves is obtained by studying the values of the two eigenvectors $\vec{v_1}(P_{\mathcal{U}})$ and $\vec{v_1}(P_{\mathcal{V}})$ obtained during the PCA. The two vector fields $\mathcal{F}_{P_{\mathcal{U}}}$ and $\mathcal{F}_{P_{\mathcal{V}}}$ (cf. Fig. 6), computed by a linear combination of the polynomials P of the basis weighted by the values of the two eigenvectors $\vec{v_1}(P_{\mathcal{U}})$ and $\vec{v_1}(P_{\mathcal{V}})$, allow interpretation of these results.

We can see than lots of vectors of the field $\mathcal{F}_{P_{\mathcal{U}}}$ are turned vertically upwards. Then an increase of the curve means a movement upwards and a decrease of the coefficients means a

[1]Strategies Expertes de Recherche d'Informations Bibliographiques En Ligne sponsored by TCAN CNRS.

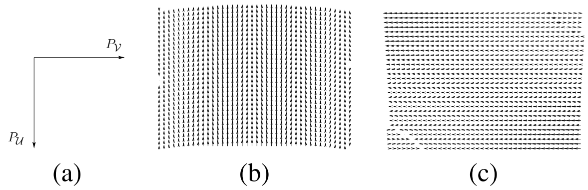


Fig. 6. Reference fields used to interpret results. (a): coordinates system used, (b): $\mathcal{F}_{P_u}$ field, (c): $\mathcal{F}_{P_v}$ field.

movement of head downwards. The same idea can be applied to the field $\mathcal{F}_{P_v}$: an increase means a movement towards the left and a decrease means a movement towards the right.

These curves represent movements but not the position of the head compared to a position of reference. As in kinematics the velocity vector is the derivative of the position vector, to study the position of the head in time equals calculating the integral of the two previous curves. Then the curves represented figure Fig. 7 are obtained.

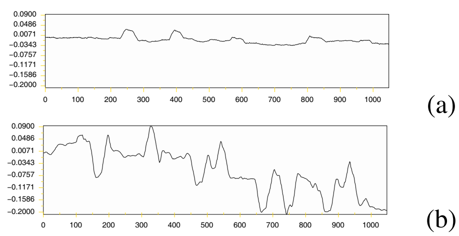


Fig. 7. (a): Evolution of the vertical position of the head, (b): Evolution of the horizontal position of the head both compared to the initial position.

Figure Fig. 8 shows the sequence at a given time. The two curves are presented to recognize the position more easily. For example when the right curve makes a variation towards the left, that means that the head of the person is positioned towards the left. Thanks to these two curves all positions can be given and so typical positions in the sequence can be recognized.

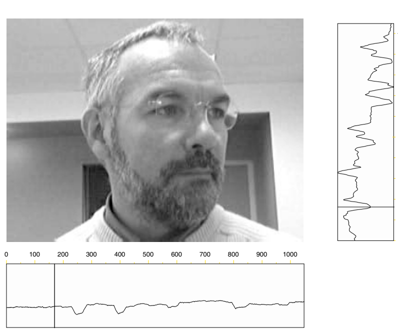


Fig. 8. Example of variations of the weights on a movement towards the right.

5. CONCLUSION

This work presents a first step towards a movement recognition method. We have proposed an original and general method for the modelization of vector fields. We have shown its good noise robustness and its ability for the analysis of simple movements.

Thereafter we plan, on the one hand, to test the method on more complex movements such as rotations, zooms, panoramics... and, on the other hand, to use other families of orthogonal polynomials to generate the basis.

Finally, as shown in the previous application movements can be characterized with very few coefficients. Consequently it could be interesting to use this method a video compression framework.

6. REFERENCES

[1] B.K.P. Horn and B.G. Schunck, "Determining optical flow," *Artificial IntelligenceI*, vol. 17, no. 1-3, pp. 185–203, August 1981.

[2] B.D. Lucas and T. Kanade, "An iterative image registration technique with an application to stereo vision," in *DARPA81*, 1981, pp. 121–130.

[3] H. Scharr and H. Spies, "Accurate optical flow in noisy image sequences using flow adapted anisotropic diffusion," vol. 20, no. 6, pp. 537–553, July 2005.

[4] F. Lauze, P. Kornprobst, and E. Mmin, "A coarse to fine multiscale approach for linear least squares optical flow estimation," *British Machine Vision Conference*, vol. 2, pp. 767–776, 2004.

[5] J.L. Barron, D.J. Fleet, and S.S. Beauchemin, "Performance of Optical Flow Techniques," *International Journal of Computer Vision*, vol. 12, no. 1, pp. 43–77, 1994.

[6] C. Cedras and M. Shah, "Motion-based recognition: A survey," *IVC*, vol. 13, no. 2, pp. 129–155, March 1995.

[7] Charles F. Dunkl and Yuan Xu, *Orthogonal polynomials of several variables*, Cambridge University Press, 2001.

[8] B. Augereau, B. Tremblais, and F. Fernandez-Maloigne, "Vectorial computation of the optical flow in color image sequences," in *13th Color Imaging Conference*, november 2005, (to appear).