ENHANCEMENT OF TEXTURED IMAGES USING COMPLEX DIFFUSION INCORPORATING SCHRÖDINGER'S POTENTIAL

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ABSTRACT

The complex diffusion process, recently introduced in image processing and computer vision by combining the linear diffusion equation and the 'free-particle' Schrödinger equation, is further generalized by incorporating the Schrödinger potential. We show that this generalized complex diffusion equation is inherently endowed with processing properties suitable for dealing with textures in a naturally coupled manner. The schrödinger potential is self adopting to the specific properties of an image at hand in that it implements an image specific wavelet shrinkage algorithm. Results indicate that the generalized complex diffusion processing scheme not only preserves textures better than demonstrated by previously reported results, but can even enhance textures by adjusting the coefficient that determines the magnitude of the potential, relative to the potentialless complex diffusion. This is in a way analogous to the enhancement of edges by the Forward-and-Backward diffusion process.

1. INTRODUCTION

The scale space is a well established and productive multiresolution approach to image processing and analysis (e.g. [10]). Considering a given image as the initial condition, I_0, the information distribution over all scales is obtained by the solution $I(x, y, t)$ of the linear diffusion equation. To overcome the compromise in the quality of detail information in the form of edges, Perona and Malik [9] introduced a nonlinear adaptive diffusion process, wherein the diffusion coefficient is diminished in its value as the process approaches large gradients, i.e. edges. This adaptive process has been further generalized to incorporate forward-and-backward (FAB) diffusion [4]. These authors have also generalized the diffusion image processing scheme by combining the inherently real-valued diffusion equation with the imaginary-valued 'free' (potentialless) Schrödinger equation, thereby combining properties of the forward and backward diffusion, in that the imaginary part is a smoothed second derivative scaled by time. Thus, the complex diffusion process combines both highpass and lowpass scale-spaces and exhibits in its discrete form both Gaussian and Laplacian-of-Gaussian pyramids [1].

Both the real-valued and complex-valued linear and nonlinear diffusion processes have been shown to be effective under the assumption that images are primarily composed of smooth areas and edges, adopted from computer vision. Consequently, the quality of textures is compromised in most previously published versions of the real and complex diffusion processes. The purpose of the present study is to deal with the textural attributes of images in a coherent manner as part of a general processing scheme that is capable of achieving it, in addition to denoising and enhancement of edges exhibited by real- and complex-valued diffusion processes.

It is well known from physics that the Schrödinger equation with potential generates an oscillatory response of a particle to the conditions imposed by a specific structure of the potential. This behavior of the Schrödinger equation highlights the idea that incorporating the potential into the complex diffusion equation, may introduce some kind of 'dynamic boundary conditions' that may serve as a filter or even enhancing mechanism for textures. We therefore incorporate a potential that is devised as a function of the textural properties and the structure of the processed image. This is accomplished by implementing a potential comprised of a nonlinear wavelet transform of the initial (given) noisy image. It is shown that such a potential indeed preserves or even enhances the texture that is highly structured as compared to the white Gaussian noise.

2. ANISOTROPIC DIFFUSION

The anisotropic diffusion process is defined by

$$I_t = \nabla \cdot (c(|\nabla I|)\nabla I)\,; \quad I(t)|_{t=0} = I_0, \quad c(\cdot) > 0 \;, \quad (1)$$

with zero Neumann boundary-conditions, where I_0 is the initial image and $c(\cdot)$ is a positive monotonically decreasing weight function. Choosing I_0 to be a noisy image, which consists of white Gaussian noise $\mathcal{N}$ with variance σ^2 added

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to the original noiseless image,

$$I_0 = I_{original} + \mathcal{N} \quad , \tag{2}$$

this process yields an effective denoising algorithm without compromising on the quality of sharp edges. The edges can be even enhanced by the so-called Forward-and-Backward diffusion [4].

An improvement of this process was achieved using a complex diffusion coefficient with small imaginary component [6], by replacing $c(\cdot)$ with $\widetilde{c}(\cdot) = e^{i\theta}c(\cdot)$, using a small θ. The resulting imaginary part is a smooth approximation of the Laplacian of the real image, while the real part is the linear diffusion scale-space of the initial image. By nonlinearly coupling the real and imaginary parts of the complex diffusion, the ramp preserving diffusion (RPD) process emerges [6]:

$$I_t = \nabla \cdot (c(\mathrm{Im}(I)) \nabla I), \quad c(\mathrm{Im}(I)) = \frac{e^{i\theta}}{1 + \left(\frac{\mathrm{Im}(I)}{k\theta}\right)^2}, \tag{3}$$

where k is a soft threshold which needs to be determined. By using the imaginary part as an edge detector, diffusion within ramp type edges is enabled.

3. WAVELET SHRINKAGE

Let $\mathcal{W}(I_0) = \{w_{j,k,l}\}$ denote the wavelet transform of the noisy image I_0. Examining (2) in the wavelet coefficient domain, the wavelet coefficients of I_0, w, can be separated into those corresponding to $I_{original}$ and $\mathcal{N}$, denoted by θ and z respectively:

$$w_{j,k,l} = \theta_{j,k,l} + z_{j,k,l} \quad , \tag{4}$$

where j and k are the translation indices, and l is the scale index. We use an orthonormal basis of compactly supported wavelets, with which the noise, $z_{j,k,l}$, remains white also in the wavelet coefficient domain [2]. A subset of thresholded wavelet coefficients provides a sparse representation of natural images, which are usually spatially inhomogeneous functions [8]. Combining the last two statements, it is concluded that a few of the w coefficients contribute primarily signal and contain most of the energy, whereas the rest of the coefficients are small in absolute value and consist mostly of noise z. Due to the latter, soft thresholding of the detail coefficients (which contain texture information),

$$\eta_\lambda(w_{j,k,l}) = sign(w_{j,k,l}) \max(|w_{j,k,l}| - \lambda, 0) \quad , \tag{5}$$

and subsequently applying inverse wavelet transform, results in denoising of the image. The approximation coefficients are not thresholded as they are not sparse and mainly consist of signal rather than noise.

4. DIFFUSION WITH POTENTIAL

There are different texture handling approaches [5], but we restrict the discussion to generalized complex diffusion. The complex diffusion PDE is very similar to the free particle Schrödinger equation [6]

$$i\bar{h}\frac{\partial\psi}{\partial t} = -\frac{\bar{h}^2}{2m}\Delta\psi + V(x)\psi \tag{6}$$

with $V(x) \equiv 0$. Let us examine therefore the general diffusion equation with potential:

$$I_t = c\Delta I + V(x) I; \quad I(t)|_{t=0} = I_0, \quad c > 0 \tag{7}$$

where $V(x)$ is a spatially varying potential. Using operator form this can be written as

$$I_t = HI; \quad I(t)|_{t=0} = I_0, \quad c > 0 \quad , \tag{8}$$

where $H = c\Delta + V(x)$. The solution is

$$I = e^{Ht} I_0 = e^{(c\Delta + V(x))t} I_0 \quad , \tag{9}$$

where e^{Ht} denotes the operator which consists of the power series of the exponent. With this derivation, the stability of the solution can be examined and the potential family for which a solution exists can be determined. Without getting into this detailed analysis which is beyond the scope of this paper, we examine an analogue to the physical potential, in the complex diffusion process. The motivation is to add a force which preserves texture while the image is selectively smoothed and denoised:

$$I_t = \nabla \cdot (c(\mathrm{Im}(I)) \nabla I) + \alpha V I \tag{10}$$

As wavelet coefficients (under the model described in section 3) represent the extent of local oscillations in a given scale [8], they seem a natural choice from which to derive the potential. The potential V is obtained by applying soft wavelet shrinkage to the initial noisy image, but reconstructing only the detail coefficients as they represent the textural information lost in (3). Multiplying it by a factor α is required in order to balance the potential's influence on the denoising process and that of the complex diffusion term. Let $\mathcal{T}$ be the nonlinear operator of thresholding and setting on zero the approximation coefficients, then

$$V = \mathcal{W}^{-1}(\mathcal{T}\{\mathcal{W}(I_0)\}) \quad . \tag{11}$$

The wavelet transform can be designed or selected to best fit the image at hand. To accommodate orientation specific texture components, nonseparable wavelets are preferred. In order for α to be as general as possible $\alpha = \alpha_0 / \max|V|$, where α_0 is determined empirically. Further insight into the best choice of α is gained by the stability and consistency analysis in section 5.

In the explicit numerical scheme

$$I^{n+1} = I^n + dt\nabla \cdot (c(\mathrm{Im}(I))\nabla I) + dt\alpha V I \quad , \tag{12}$$

where n is the time index, the potential is added with each iteration to the result of the RPD term multiplied by dt, the time step. This motivates the implementation of the following PDE rather than (10)

$$I_t = \nabla \cdot (c(\mathrm{Im}(I))\nabla I) + \alpha V \tag{13}$$

as RPD evolves eventually to the cartoon model [11] of the image, to which we would like to add the textural information contained in V. This indeed yields better results.

5. STABILITY AND CONSISTENCY

Let us determine whether (12) is consistent with (10). Let Δt be the time interval and Δx the spatial interval. Then, the numerical scheme operator is

$$L\left(I_{i,j}^n\right) = I_{i,j}^n + \Delta t\left[\frac{c}{\Delta x^2}\left(I_{i+1,j}^n - 2I_{i,j}^n + I_{i-1,j}^n + I_{i,j+1}^n - 2I_{i,j}^n + I_{i,j-1}^n\right) + \alpha V_{i,j} I_{i,j}^n\right] - I_{i,j}^{n+1} \tag{14}$$

Substituting the Taylor expansions of $I_{i\pm1,j}$ and $I_{i,j}^{n+1}$ into (14) results in the local truncation error

$$L(I) = I + \Delta t\left[\frac{c}{\Delta x^2}\left(\Delta x^2 I_{xx} + \frac{\Delta x^4}{12}I_{xxxx} + \Delta x^2 I_{yy} + \frac{\Delta x^4}{12}I_{yyyy}\right) + \alpha V I\right] - \left[I + \Delta t I_t + \frac{\Delta t^2}{2}I_{tt}\right] + O\left(\Delta t \Delta x^4\right) + O\left(\Delta t^3\right) \tag{15}$$

Substituting (10) into (15) we get

$$L(I) = \frac{\Delta t c \Delta x^2}{12}\left(I_{xxxx} + I_{yyyy}\right) - \frac{\Delta t^2}{2}I_{tt} + O\left(\Delta t \Delta x^4\right) + O\left(\Delta t^3\right) \tag{16}$$

Hence $L(I)$ tends to zero as $(\Delta x, \Delta t) \to (0,0)$ and the numerical scheme is consistent with the PDE (10).

Using both the matrix and von Neumann's methods we arrive at the same stability condition

$$2(\Delta t c - 1) \leq \alpha \Delta t V \leq 0, \quad \Delta t c \leq 1. \tag{17}$$

6. RESULTS

In the simulations we used the wavelet shrinkage function from the Matlab wavelet toolbox with the ideal threshold For RPD we used Gilboa's Matlab code [3], with the ideal soft threshold k and stopping time for RPD. We simulated (13) with the scheme described in [6], except for the additional potential term dtV which is augmented with each iteration. After processing several images we noticed that $\alpha_0 = 0.25 \cdot range$ works fairly good for most images, where $range$ is the maximal value for an image pixel, although for few images smaller α_0 was needed. Results of processing a noisy image (SNR=10dB) are compared in Fig. 1. RPD as expected yields good denoising of smooth areas and preserves edges. It does result however in partial smoothing of textural regions (trousers). The wavelet shrinkage doesn't cope as well with the smooth areas, but there is almost no loss of the texture. With potential, texture is preserved and ,interestingly, it denoises smooth regions better than RPD. This is accomplished due to the fact that preserving texture during the process enables longer evolution and, thus, stronger denoising of smooth regions. As far as SNR, RPD reaches 13.8 dB, wavelet shrinkage 13.9 dB and our scheme manages to enjoy the advantages of both and yields 14.2 dB. In other cases similar improvement was obtained such as 0.5dB and 0.6 dB with the lenna and peppers images respectively.

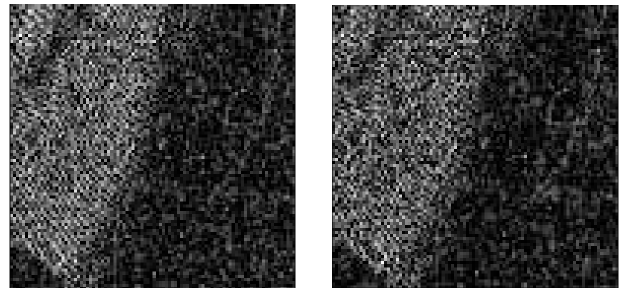

Fig. 2. Comparison of error images: RPD (left), result of processing with wavelet shrinkage (right). (For better quality of the detailed differences see [7].)

The choice of wavelet shrinkage as the corner stone of the potential is positively re-enforced when examining its final error image and the one of RPD (Fig. 2). It can be seen that with RPD the error energy is concentrated in the textural regions rather then smooth areas or edges, while with wavelet shrinkage quite the opposite can be seen. We simulated the scheme with a clean image as the initial condition and only with the potential term $I_t = \alpha V$. The result depicts texture enhancement (Fig. 3). The contrast of the textural regions is increased, producing clearer and prominent textures. Images, where their details and quality can be better compared, and Matlab codes can be found at [7].

7. CONCLUSIONS

Dealing with texture is essential for a complete image denoising. The proposed generalized complex diffusion, incorporating Schrödinger's potential, addresses this demand, while yielding same or even better results of edge-preserving-denoising afforded by other schemes. It is important to note that convergence of the process can be assurred when apply-

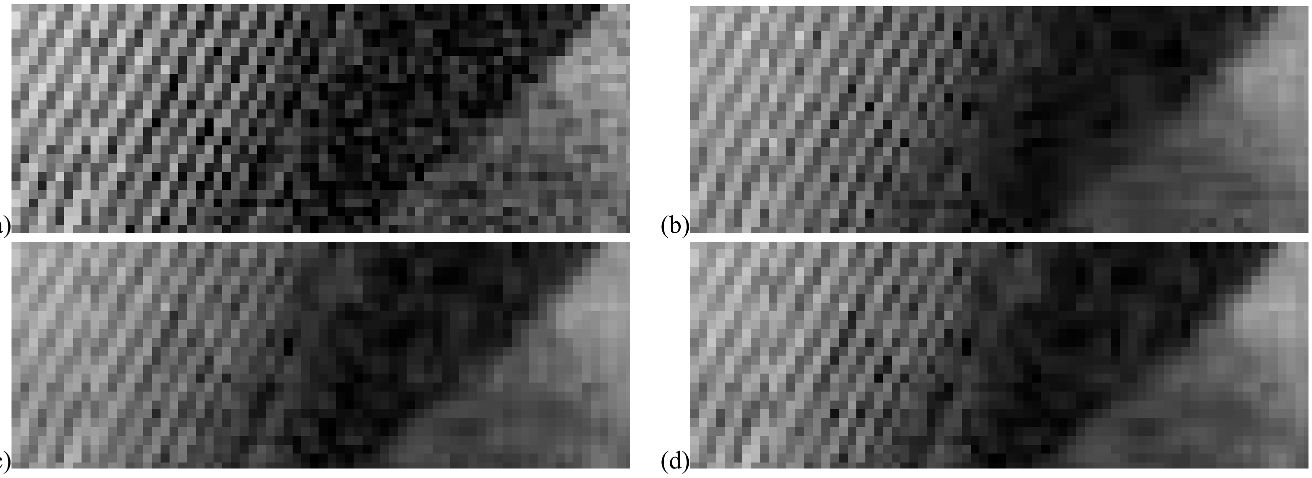

Fig. 1. Processing of a noisy image (SNR=10dB), the lower right corner of the full image shown in Fig. 3. (a) the noisy image, (b) soft wavelet shrinkage, (c) RPD, (d) with potential. (For better quality of the detailed differences see [7].)

Fig. 3. Processing the original image with the potential term, depicting enhancement of texture (right). Original image on left. (For better quality of the detailed differences see [7].)

ing the restrictions obtained from the consistency an stability analysis. We presently analyze in-depth the generalized complex diffusion equation. We further investigate an automated solution to the stopping time, and the effects of various parameters on image attributes.

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