REGION-BASED SUPER-RESOLUTION USING MULTIPLE BLURRED AND NOISY UNDERSAMPLED IMAGES

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ABSTRACT

Super-resolution is the process of combining multiple low-resolution images to produce a higher resolution image. Because the super-resolution problem is an ill-posed one, many regularized algorithms have been proposed. Those algorithms usually use the same regularization term for a whole image. However, since an image generally contains various regions of different characteristics, simple regularization is not good enough for the prospective result. In this paper, we propose a region-based super-resolution algorithm to apply a suitable regularization term for each region. In the algorithm, the image is divided into homogeneous and inhomogeneous regions. According to the type of region, we apply different filters for regularization. Regularization parameters are also adaptively determined during the iteration. Simulation results show that the proposed algorithm is superior to the conventional algorithm in terms of objective quality as well as subjective one.

1. INTRODUCTION

Since the super-resolution problem had been addressed [1], researchers proposed many algorithms such as the iterative back-projection, POCS, ML, and MAP algorithms [2-5]. Generally, the super-resolution problem is an ill-posed problem due to the insufficient number of low-resolution (LR) images and an ill-conditioned blur operator. Moreover, inaccurate registration and noise make the super-resolution process unstable. Therefore, regularization is incorporated with super-resolution, to take care of the ill-posedness of the super-resolution problem and provide a meaningful solution. One of the most widely used regularization cost is the Tikhonov cost [6]. Although regularization makes the algorithm more stable and robust against errors, it tends to blur edges. To alleviate this problem, several algorithms have been developed. For example, one algorithm performs regularization based on a bilateral prior [7] and others use an anisotropic nonlinear diffusion as regularization [8].

The main reason of edge blurring is that the regularization term based on the Laplacian operator, which works well for flat pixels but is not proper for edge pixels. This means that different regularization terms should be used according to the characteristic of pixels. In this paper, we propose a region-based super-resolution algorithm for suitable regularization depending on pixel characteristic. In the algorithm, we first divide an image into homogeneous and inhomogeneous regions, and apply the different filter depending on the pixel characteristic in each region. In addition, the regularization parameter is adaptively determined during the iteration.

The rest of this paper is organized as follows. In Section 2, we introduce the observation model and the conventional super-resolution algorithm using regularization. In Section 3, we describe the motivation of adopting a region-based approach. Then, we explain the proposed algorithm in detail in Section 4. Experimental results and conclusion are given in Sections 5 and 6.

2. PRELIMINARIES

2.1. Observation Model

The observation system can be modeled by imitating an image acquisition system, which consists of the warping, blurring, and subsampling procedures, and the addition of noise. Therefore, we can represent the observation model as

$$\mathbf{L}_k = \mathbf{W}_k\mathbf{H} + \mathbf{E}_k, \quad 1 \leq k \leq p, \tag{1}$$

where p is the number of LR images, $\mathbf{L}_k$ is an $M_xM_y\times1$ vector representing the k-th $M_x\times M_y$ LR image, $\mathbf{H}$ is the $q^2M_xM_y\times1$ vector representing a $qM_x\times qM_y$ HR image; q is the magnification factor; $\mathbf{E}_k$ is the independent identically distributed Gaussian noise; and $\mathbf{W}_k$ is the k-th $M_xM_y\times q^2M_xM_y$ matrix representing warping, blurring, and subsampling operators. Note here that images are

$$\begin{bmatrix}\mathbf{L}_1\\ \vdots\\ \mathbf{L}_p\end{bmatrix} = \begin{bmatrix}\mathbf{W}_1\\ \vdots\\ \mathbf{W}_p\end{bmatrix}\mathbf{H} + \begin{bmatrix}\mathbf{E}_1\\ \vdots\\ \mathbf{E}_p\end{bmatrix} \Rightarrow \mathbf{L} = \mathbf{W}\mathbf{H} + \mathbf{E} \tag{2}$$

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In this paper, we assume that the warping denotes a global translational shift, the optic distortion is not considered, and the PSF has a box-shape. Then, the weights corresponding to q^2 HR pixels covering a LR pixel would be set to $1/q^2$ and the other weights would be set to zero.

2.2. Super-Resolution with Regularization

Since the super-resolution problem is an ill-posed one, regularization is used to stabilize the ill-posed problem. The regularized super-resolution problem can be considered a constraint least squares (CLS) problem. And CLS is formulated by choosing **H** minimizing the following equation.

$$\mathbf{H} = \arg\min_{\mathbf{H}} \left\{ \|\mathbf{L} - \mathbf{W}\mathbf{H}\|_2^2 + \lambda \|\mathbf{C}\mathbf{H}\|_2^2 \right\}, \tag{3}$$

where **C** is a high-pass filter and λ denotes the regularization parameter. Since the cost function in Eq. (3) is convex and differentiable, this equation can be solved by setting the derivative of the cost with respect to **H** to zero. Then, the solution becomes

$$\mathbf{H} = (\mathbf{W}^T\mathbf{W}+\lambda\mathbf{C}^T\mathbf{C})^{-1}\mathbf{W}^T\mathbf{L}. \tag{4}$$

To get the final result, we need to invert the system matrix, $\mathbf{W}^T\mathbf{W}+\lambda\mathbf{C}^T\mathbf{C}$, of size $M_xM_y\times q^2M_xM_y$. However, this is not practical because the matrix size is too large. Therefore, iterative methods have been considered such as gradient descent, conjugate gradient (CG), preconditioned CG, and error relaxation methods. In this paper, we adopt the gradient descent method and this leads Eq. (3) to the following iteration scheme.

$$\mathbf{H}^{n+1} = \mathbf{H}^n + \alpha^n\{\mathbf{W}^T(\mathbf{L}-\mathbf{W}\mathbf{H}^n) - \lambda^n\mathbf{C}^T\mathbf{C}\mathbf{H}^n\}, \tag{4}$$

where α^n represents the updating step size at the nth iteration step. It is automatically determined as follows [9].

$$\alpha^n = \frac{\left\|\mathbf{W}^T(\mathbf{L}-\mathbf{W}\mathbf{H}^n) - \lambda^n\mathbf{C}^T\mathbf{C}\mathbf{H}^n\right\|^2}{\left\|\mathbf{W}\left(\mathbf{W}^T(\mathbf{L}-\mathbf{W}\mathbf{H}^n) - \lambda^n\mathbf{C}^T\mathbf{C}\mathbf{H}^n\right)\right\|^2 + \lambda^n\left\|\mathbf{C}\left(\mathbf{W}^T(\mathbf{L}-\mathbf{W}\mathbf{H}^n) - \lambda^n\mathbf{C}^T\mathbf{C}\mathbf{H}^n\right)\right\|^2}, \tag{5}$$

3. MOTIVATION

Fig. 1 shows simulation results of super-resolution when the noise variance is 25. We can notice that the image obtained with regularization contains less noise but more blurred edges than the one without regulation. This blurring effect is due to the Laplacian operator adopted for regulation. The operator is useful for applying the constraint for flat pixels but is not suitable for edge pixels. To prevent edges from blurring, the regularization term and parameters are to be changed according to characteristic of the pixel. This is one of the motivations why we adopt a region based approach.

PSNR curves given in Fig. 2 provide the other motivation to a region based approach. In the figure, the PSNR for homogeneous regions increases until the 3rd iteration, then starts to decrease. This phenomenon is not desirable for the convergence of the iteration procedure. The main reason of this tendency is that α^n and λ^n in Eq. (4) are determined for the whole image including various regions of different characteristics.

(a) (b)

Fig. 1. Simulation results of super-resolution for the noise variance of 25. (a) Without regularization (27.0dB) and (b) with regularization (32.2dB)

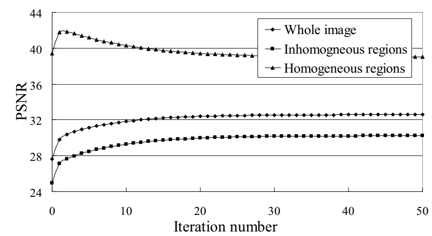

Fig. 2. PSNR graphs for a whole image, inhomogeneous regions, and homogeneous regions.

4. THE PROPOSED ALGORITHM

We now propose a region-based super-resolution algorithm. The algorithm consists of three parts, region segmentation, super-resolution for homogeneous regions, and super-resolution for inhomogeneous regions.

4.1. Region Segmentation

For a region-based approach, image segmentation has to be performed in advance. For segmentation, we first generate LR edge maps from LR images through the eigenvalue decomposition of the structure matrix of every pixel. The eigenvalue decomposition is achieved as

$$\mathbf{J} = [\mathbf{v}_1 \quad \mathbf{v}_2] \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix}, \tag{6}$$

where $\mathbf{v}_i$ is an eigenvector of the pixel and μ_i ($\mu_1 \le \mu_2$) is an eigenvalue. Here, $\mathbf{v}_1$ represents an edge direction at the pixel and $\mathbf{v}_2$ represents the direction perpendicular to $\mathbf{v}_1$. If $\mu_1 \le \mu_2 + T$, we can decide the pixel as an edge pixel. Otherwise, we decide it as a flat pixel. By applying this decomposition process to all the pixels, we can obtain LR edge maps. Then, we back-project the LR edge maps into the HR domain by using $\mathbf{W}^T$ in Eq. (2). Since the obtained HR edge map contains some undesired pixels, the refinement step is needed. Undesired pixels are mainly due to noise and they occur randomly. If the size of an edge region is smaller than the threshold value, the region is regarded as a flat region. Thereby, the final segmented HR image is obtained.

4.2. Super-Resolution for Homogeneous Regions

Since homogeneous regions do not include details, we focus on noise reduction in these regions. To effectively reduce the noise, the proper selection of regularization term and regularization parameter is important. Since a major prior knowledge for homogeneous regions is smoothness, a high-pass filter is suitable for the regularization term. Thus, we choose the 2D Laplacian operator. The necessity of adaptive regularization parameter had already been introduced [10]. To properly select the adaptive parameter in homogeneous regions, we modify the scheme previously suggested [10]. A regularization parameter has to satisfy following properties; i) λ^n is proportional to $\|\mathbf{L}^n-\mathbf{WH}^n\|^2$, ii) λ^n is inversely proportional to $\|\mathbf{CH}^n\|^2$, iii) λ^n is larger than zero, and iv) λ^n should make the gradients of $\|\mathbf{L}^n-\mathbf{WH}^n\|^2$ and $\|\mathbf{CH}^n\|^2$ with respect to $\mathbf{H}$ small. While properties i), ii), and iii) were already considered in the previous scheme, property iv) is newly introduced. If we assume that $\|\mathbf{L}^n-\mathbf{WH}^n\|^2$ has a small value and $\|\mathbf{CH}^n\|^2$ has a large value near the optimal solution, the three properties i), ii), and iii) may produce the regularization parameter of a small value, which is not desirable. To alleviate this problem, we add property iv), and the regularization parameter is defined as

$$\lambda^n = \ln\left(\frac{\left(\|\mathbf{L}-\mathbf{WH}^n\|^2\right)^2}{\|\mathbf{W}^T(\mathbf{L}-\mathbf{WH}^n)\|^2} \cdot \frac{\|\mathbf{C}^T\mathbf{CH}^n\|^2}{\left(\|\mathbf{CH}^n\|^2\right)^2} + 1 \right). \tag{7}$$

4.3. Super-Resolution for Inhomogeneous Regions

Inhomogeneous regions contain edge pixels as well as flat pixels. Therefore, it may be desirable to use a different regularization term according the pixel characteristic. Using the eigenvalue decomposition, we first classify pixels in inhomogeneous regions into flat and edge pixels. Then, for flat pixels, we can adopt an omni-directional filtering for regularization and, for edge pixels, adopt directional filtering along the edge direction. Thereby, we can preserve edge sharpness. To implement this idea to the regularization term, we use the anisotropic nonlinear diffusion model [11],

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \cdot \mathbf{D}\nabla \mathbf{H}. \tag{8}$$

Here, $\mathbf{H}$ denotes the HR image and $\mathbf{D}$ denotes the diffusion tensor. $\mathbf{D}$ is determined by modifying μ_i in Eq. (6). Then, we can obtain $\mathbf{D}$ as

$$\mathbf{D} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix}, \tag{9}$$

where

$$\eta_1 = 1$$
$$\eta_2 = \begin{cases} 0 & \text{for edge pixels,} \\ 1 & \text{for flat pixels.} \end{cases} \tag{10}$$

Since η_1 and η_2 represent the smoothing strength along the edge direction and its normal direction, respectively, η_1 is set to 1 and η_2 is set to different values according to pixel characteristic. If we represent $\mathbf{D}$ as

$$\mathbf{D} = \begin{bmatrix} \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{D}_{21} & \mathbf{D}_{22} \end{bmatrix}, \tag{11}$$

Eq. (8) can be rewritten as

$$\frac{\partial \mathbf{H}}{\partial t} = \left(\mathbf{G}_x\mathbf{D}_{11}\mathbf{G}_x + \mathbf{G}_x\mathbf{D}_{12}\mathbf{G}_y + \mathbf{G}_y\mathbf{D}_{21}\mathbf{G}_x + \mathbf{G}_y\mathbf{D}_{22}\mathbf{G}_y\right)\mathbf{H} = \mathbf{RH}, \tag{12}$$

where $\mathbf{G}_x$ and $\mathbf{G}_y$ denote the matrices of gradients along x and y. Using Eqs. (5) and (11), we can obtain an iterative super-resolution procedure for inhomogeneous regions, i.e.,

$$\mathbf{H}^{n+1} = \mathbf{H}^n + \alpha^n\left\{\mathbf{W}^T\left(\mathbf{L}-\mathbf{WH}^n\right) - \lambda^n\mathbf{R}^T\mathbf{RH}^n\right\}. \tag{13}$$

Here, we determine the regularization parameter λ^n by incorporating the following properties: i) λ^n is larger than zero and ii) λ^n makes the normalized gradient of $\|\mathbf{L}^n-\mathbf{WH}^n\|^2$ and $\|\mathbf{RH}^n\|^2$ small.

$$\lambda^n = \ln\left(\frac{\|\mathbf{L}-\mathbf{WH}^n\|^2}{\|\mathbf{W}^T(\mathbf{L}-\mathbf{WH}^n)\|^2} \cdot \frac{\|\mathbf{C}^T\mathbf{CH}^n\|^2}{\|\mathbf{CH}^n\|^2} + 1 \right). \tag{14}$$

Note here that compared to Eq. (7), only 2 properties are used to derive Eq. (14).

5. EXPERIMENTS

In order to demonstrate the performance of the proposed algorithm, several experiments are performed. In the experiments, we use the cameraman, Lena, baboon, and black images of 256x256. For each image, four LR images of 128x128 are generated by translating, blurring, and down-sampling it, and by adding noise. We assume that the noise has the variance σ^2 of 25. Fig. 3 shows simulation results for the "cameraman" image. Note in Fig. 3(c) that white and black regions represent inhomogeneous and homogeneous regions, respectively. Compared with the result from conventional algorithm in Fig. 3(b), the proposed algorithm significantly reduces the noise especially in the homogeneous regions and provides sharper edges as shown in Fig. 3(d). Here, the conventional algorithm uses Laplacian regularization with a constant regularization parameter [9,10]. Fig. 4 shows the graphs of PSNR versus the number of iterations. It is clearly seen from the graphs that the proposed algorithm provides higher PSNR with faster convergence. Fig. 5 provides the experiment results of the "black" image. We can easily note in the images that the proposed algorithm provides sharper edges. The overall performance is summarized in Table 1 for the four test images. The table shows that the average PSNR increase in the proposed algorithm is about 2dB. Also, the proposed algorithm converges faster than the conventional one.

6. CONCLUSIONS

In this paper, to improve the edge sharpness and reduce the noise in the super-resolved image, we propose a region-based super-resolution algorithm. In this algorithm, the two different filters of an omni-directional filter and a directional filter are used for regularization according to pixel characteristic. We also introduce a new adaptive regularization parameter. Experimental results show that the proposed algorithm provides high PSNR with sharper edges.

Fig. 3. Simulation results for cameraman; (a) original, (b) conventional algorithm with regularization (c) segmented result, (d) proposed algorithm

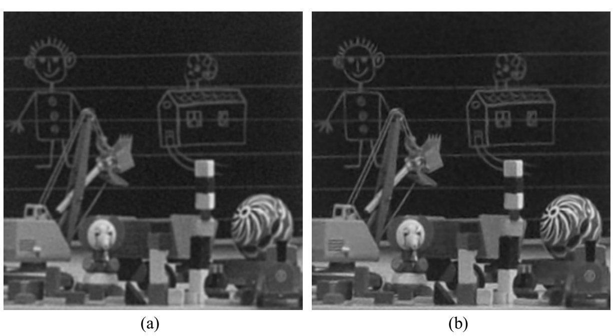

Fig. 5. Simulation results for the “black” image. (a) conventional algorithm, (b) proposed algorithm

Table 1. Comparison of PSNR and number of iterations between the conventional and proposed algorithms.

Image	Algorithm	PSNR (dB)	Number of iterations	
			Homogeneous regions	Inhomogeneous regions
Cameraman	Conventional	32.25	16	16
	Proposed	34.32	5	9
Lena	Conventional	34.16	10	10
	Proposed	36.37	6	9
Baboon	Conventional	28.01	14	14
	Proposed	29.78	5	10
Black	Conventional	35.31	12	12
	Proposed	37.50	14	10

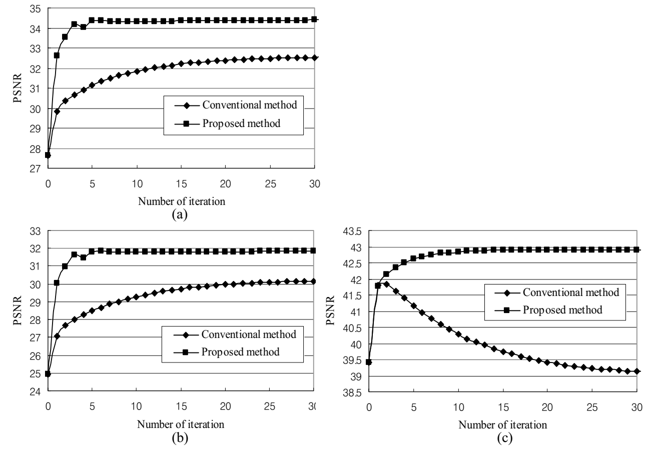

Fig. 4. PSNR versus the number of iterations for (a) the whole image, (b) inhomogeneous regions, and (c) homogeneous regions.

7. REFERENCES

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