PROJECTIVE RECTIFICATION OF IMAGE TRIPLETS FROM THE FUNDAMENTAL MATRIX

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ABSTRACT

This paper describes a method for image rectification of a trinocular setup. The rectification method used is an extension of a recent approach based on the fundamental matrix to generate the correcting homographies in the case of a stereo pair. The extended method uses the fact that the triplet of images can be treated as two pairs and that homographies are simply projections of the different images planes onto new planes. Rectification thus becomes a matter of deciding which plane will be the common one and what transformation or homography is to be applied to each image.

1. INTRODUCTION

Feature matching, among other applications, is greatly improved by image rectification. As a matter of fact, the set of epipolar lines corresponding to a set of matches is transformed after rectification into a set of lines that are either vertical or horizontal. In the case of matching, the problem is therefore reduced to a simple line scan.

Many methods exist and have been implemented to solve the rectification problem for stereo and trinocular vision. Hartley worked on finding the rectifying transformation from the fundamental matrix with a strong mathematical justification [1]. Loop et al. developed a method to find the rectifying homographies and added some constraints to reduce the distortion introduced by rectification [2]. These methods are applicable to uncalibrated cameras and, in the case of two views, are close in theory to an algorithm presented by Mallon and Whelan [3]. The method they proposed follows Hartley's in principle but has its own original distortion reduction procedure. This approach is the one that is extended in this paper.

In the case of three views, Luping et al. presented a technique to rectify a triangular triplet of images using the perspective projection matrices (PPM) [4]. This technique uses camera calibration and is therefore not suitable for uncalibrated environments. Zhang et al. proposed a method to obtain the rectification homographies using the fundamental matrices, minimizing the distortion by adjusting 6 free parameters [5]. This method uses a set of three constraints on the triplet of images which allow the recovery of the three rectifying homographies in a closed form. Sun presents three methods that compute the projection matrices for the three images also using pair wise fundamental matrices [6]. The projection matrix of the reference image is a composition of 4 transformations; the other two are derived from the latter. In all these cases, the algorithm is designed for three views and uses three views constraints to achieve its goals.

The method presented here is close to the one used in [5, 6] but is based on the method presented in [3]. This latter method uses the fundamental matrix in a similar way as in [1] but the novel aspect is the distortion reduction. It is a method developed for stereo. In this paper, we mainly describe how this 2-view algorithm was adapted to the three view case in conjunction with intermediate plane transfer by homography. In other words, we show that considering pair wise constraints and combining the resulting homographies leads us to satisfying rectification results on different sets of images.

2. RECTIFYING TRINOCULAR IMAGES

2.1. Notations

The images will be referred as image i with i varying from 1 to 3. For triplet of images, the indexing order will correspond to : left, middle and right for a horizontal triplet and up, down and right for an L-shaped triplet.

The epipoles will be noted e_{ij} which stands for the projection of the camera center of the image j onto image plane i. e_{ij} corresponds to $(e_{ij}x, e_{ij}y, 1)^T$ in the image plane. A plane containing two rectified images i and j will be noted $\mathcal{P}_{ij}$. H_i will be used to designate the rectifying homography of image i.

2.2. Projective rectification from the fundamental matrix

The algorithm presented in this paper is an extension of the method defined by Mallon and Whelan [3]. Their goal was to obtain homographies that will simply be applied to each image to obtain its rectified counterpart. The main element and starting point of the rectification process is the fundamental matrix between a pair of stereo images as mentioned in [3, 1]. We use the 7 point algorithm presented in [7]. The rectification process, which somewhat follows Hartley's blueprint, can be summed up as follows:

2.2.1. Fundamental matrix

Recover the fundamental matrix F as mentioned above. Any other method of fundamental matrix computation could be suitable as long as the result is fairly accurate. Eight or more matches are enough to compute the fundamental matrix. The Projective Vision Toolkit (PVT [8]) developed by Whitehead and Roth could be used to find automatically matches for a pair (and also for a triplet) of images.

2.2.2. Epipoles

Recover the epipoles e_{12} (in left image) and e_{21} (in right image) from an SVD decomposition of F. This is justified by the fact that $Fe_{12} = 0$ and $e_{12}^T F = 0$ [9].

2.2.3. Left homography

From the epipoles, compute the rectifying homography H_1 on the left image by forcing the corresponding epipole to infinity in the horizontal direction (from $e_{12} = (e_{12}x, e_{12}y, 1)^T$ to $e_{12} = (1, 0, 0)^T$ in projective space) :

$$H_1 = \begin{pmatrix} 1 & 0 & 0 \\ -e_{12}y/e_{12}x & 1 & 0 \\ -1/e_{12}x & 0 & 1 \end{pmatrix} \tag{1}$$

2.2.4. Right homography

Once H_1 is found, an additional constraint on the problem mentioned in [3] is used to solve for H_2. As a matter of fact, the fundamental matrix of the original setup being F, the resulting rectified fundamental matrix should equal to the trivial matrix F_h

$$F_h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \tag{2}$$

This leads to the mathematical constraint:

$$H_2^T F_h H_1 = \alpha F \tag{3}$$

We know F, H_1 and F_h. We want to solve for H_2 and α with :

$$H_2 = \begin{pmatrix} 1 & 0 & 0 \\ h_1 & h_2 & h_3 \\ h_4 & h_5 & h_6 \end{pmatrix} \tag{4}$$

Equation (3) is transformed to a system of the type $AX = 0$ where X stands for elements h_i of the homography H_2 in a column with α as its last element . The system is then solve by using the SVD of A and extracting its right singular vector of least singular value. H_2 is then normalized. Steps 2.2.1 to 2.2.4 produce satisfying rectifying homographies. The last step is the distortion reduction introduced by [3] and summarized in the next section.

2.2.5. Distortion reduction

The distortion mentioned here is not related to lens distortion. It is introduced by the homographies after rectification. The reduction step is not mandatory but it makes the images look more natural. Essentially, these transformations are $K_i = A_i H_i$ with i = 1,2. As a matter of fact A_1 and A_2 will be applied to H_1 and H_2 to achieve this goal and these transformations are of the form

$$A_i = \begin{pmatrix} a_{i1} & a_{i2} & a_{i3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

The values of a_1 and a_2 are found by simplex minimization (Nelder-Mead or amoeba algorithm [10]) of the function

$$f(a_1, a_2) = \sum_{i=1}^{n} [(\sigma_1(\mathbf{J}(\mathbf{K}, \mathbf{p_i})) - \mathbf{1})^\mathbf{2} + (\sigma_\mathbf{2}(\mathbf{J}(\mathbf{K}, \mathbf{p_i})) - \mathbf{1})^\mathbf{2}] \tag{6}$$

Where J is the jacobian of the transformation $K_i = A_i H_i$ at a point p_i contained in a grid over the image plane and σ_i are its singular values. Interestingly enough the jacobian describes "the creation and loss of pixels as a result of the application of K" [3]. The value of a_3 is left to the user for flexibility in centering the resulting image.

2.3. Extension to a "vertical" pair

This section describes one of the extensions we added to the method presented in [3]. For a vertical pair, the process is very similar. The differences in each step are justified by the difference of configuration. The fundamental matrix F_v for such a configuration is given by [9]:

$$F_v = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

This changes the steps but the idea remains the same:

a. Recover the fundamental matrix

b. Recover the epipoles

c. Recover the homography H_1 corresponding to the top image. Applying this transformation to the image sends the epipole e_{12} to infinity in the vertical direction : from $e_{12} = (e_{12}x, e_{12}y, 1)^T$ to $e_{12} = (0, 1, 0)^T$ in projective space). Thus :

$$H_1 = \begin{pmatrix} 1 & -e_{12}x/e_{12}y & 0 \\ 0 & 1 & 0 \\ 0 & -1/e_{12}y & 1 \end{pmatrix} \tag{7}$$

d. Using the same type of constraint as in the horizontal case (2.2.4), we obtain a linear system that is solved the same way using the same formulas but with H_1 and F_h replaced by H_1 of Equation (7) and F_v. This allows us to recover H_2 which in this case is of the form :

$$H_2 = \begin{pmatrix} h_1 & h_2 & h_3 \\ 0 & 1 & 0 \\ h_4 & h_5 & h_6 \end{pmatrix} \tag{8}$$

e. The distortion reduction step is exactly the same except the transformations are of the type A_i:

$$A_i = \begin{pmatrix} 1 & 0 & 0 \\ a_{i1} & a_{i2} & a_{i3} \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

This reflects the fact that the distortion and centering steps will affect the vertical coordinate and the user-defined value of a_3 corresponds to a translation along the vertical axis of the rectified image.

3. RECTIFICATION OF 3 VIEWS

The extension to triplets of images is different conceptually but uses the horizontal and vertical rectification at different stages. The concept is illustrated in Figure 1. The triplet is processed pair by pair. The images are denoted 1,2 and 3. For 1 and 2, the rectification without the distortion reduction step gives us H_1 and H_2. Similarly, for images 2 and 3 the rectification without distortion reduction gives us H_2' and H_3. The rectification does not include the distortion since we want to stay consistent on the type of images we are working on : they are all affected by the same type of effects. The distortion reduction will therefore be the last phase of this process.

3.1. Horizontal triplets

The middle image 2 is common to the two pairs so we have H_2 and H_2'. Each of the computed homographies 'sends' the image plane 2 on two different planes containing respectively the rectified image 1 i.e $\mathcal{P}_{12}$ and the rectified image 3 i.e $\mathcal{P}_{23}$ (see Figure 1).

Our goal here is to find a way to transfer the plane $\mathcal{P}_{23}$ to $\mathcal{P}_{12}$; as a matter of fact we want to find the homography $\mathbf{h}$ between these two planes. This is done as follows:

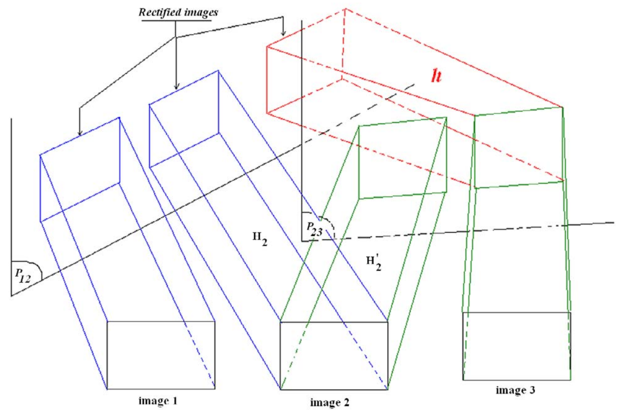

Fig. 1. Rectification principle for a triplet of images.

- Image 2 is transferred to plane $\mathcal{P}_{12}$ with H_2
- Image 2 is transferred to plane $\mathcal{P}_{23}$ with H_2'
- Image 3 is transferred to plane $\mathcal{P}_{23}$ with H_3
- $\mathbf{h}$ between $\mathcal{P}_{12}$ and $\mathcal{P}_{23}$ is therefore given by $\mathbf{h} = H_2 H_2'^{-1}$
- Image 3 is therefore transferred to plane $\mathcal{P}_{12}$ with H_3' given by :

$$H_3' = \mathbf{h} H_3 = H_2 H_2'^{-1} H_3 \tag{10}$$

Finally, distortion reduction for the horizontal configuration is applied to each homography H_1, H_2 and H_3'. An example of result is displayed in Figure 2.

3.2. "L-triplets"

The case of 'L'-shaped triplets is a combination of a vertical pair and a horizontal pair. All steps in the horizontal triplet procedure are repeated except for what follows:

- The pair 1, 2 is rectified using the vertical pair approach without the distortion reduction procedure (Section 2.3).
- The distortion rectification step uses the vertical distortion reduction approach for the rectified images 1 and 2. For image 3, the distortion reduction is also applied with the vertical approach described in section 2.2.5 to level the images 2 and 3 along the vertical axis.

3.3. Results and Observations

Examples of results are displayed in figures 2 and 3. All epipolar lines associated to the rectified triplets are either vertical or horizontal. The rectification was also effective on others sets of images not shown here. An important observation that is also mentioned in [3] is the fact that the rectification is ineffective for images where the epipoles appear in the image plane.

Another observation, that is rather obvious, is that a pair or triplet of images has to be taken close to the ideal configuration before using the corresponding rectification algorithm: i.e. it is impossible to rectify a vertical stereo pair of images with the horizontal stereo rectification approach.

An important source of error is clearly the fundamental matrix approximation. It is therefore a very important step that should be handled with care and carried out following one of the many existing techniques. For a set of algorithms, we suggest the reader to refer to [7].

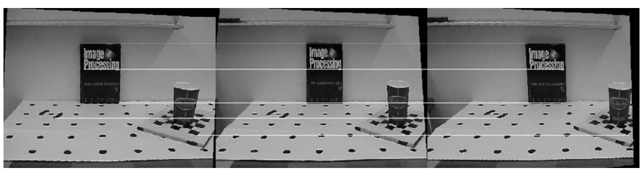

Fig. 2. Rectified triplet of images : horizontal configuration

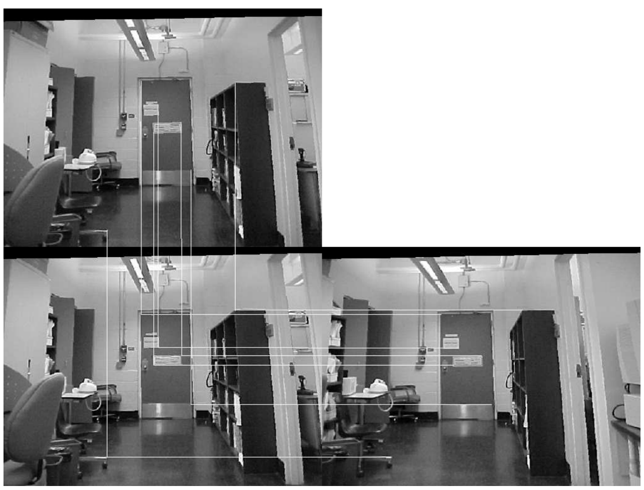

Fig. 3. Rectified triplet of images : L configuration

4. CONCLUSION

This paper presented an extension to a recent image pair rectification method based on fundamental matrix. This method has the advantage of being suitable for uncalibrated environments as well as producing a pair of rectifying homographies with a low distortion effect. The fact that it is based on the fundamental matrix justifies our choice of this method since the fundamental matrix approximation is well documented.

Extensions to different three-view configurations were introduced in Section 3. Our approach uses homography composition in order to rectify all images on a common plane with the constraint of epipoles to infinity in the destination image plane. Good results were obtained on different sets of images and these are further visually improved when the proper distortion reduction is applied as the final step.

5. ACKNOWLEDGEMENT

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6. REFERENCES

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