AN AUTOMATIC 3D CITY MODEL : A BAYESIAN APPROACH USING SATELLITE IMAGES

Florent Lafarge[1,2], *Xavier Descombes*[1], *Josiane Zerubia*[1]

Marc-Pierrot Deseilligny[2]

[1]Ariana Reseach Group - INRIA/I3S
2004, routes des Lucioles, BP93
06902 Sophia Antipolis, Cedex France
E-mail=Name.Lastname@inria.fr

[2]Institut Géographique National
2/4 avenue Pasteur
94165 Saint-Mandé, Cedex France
E-mail=Name.Lastname@ign.fr

ABSTRACT

We propose a parametric model for automatic 3D reconstruction of urban areas from high resolution satellite data. An automatic building extraction method based on marked point processes is used to provide rectangular building footprints. Based on a parametric model with rectangular ground footprint, the proposed method is developed using a Bayesian approach : we search for the best configuration of parametric models with respect to both a priori knowledge of models and their interactions, and a likelihood which fits models to the DEM. A simulated annealing is used to find the configuration which maximizes the a posteriori density of the Bayesian expression.

1. INTRODUCTION

For the last decade, the automatic 3D reconstruction of urban areas has become a topic of interest. Faced with the urbanization development, the use of 3D-models with connected planar facets is used in various applications such as the computing of electromagnetic wave propagation or the creation of virtual realities. Several automatic methods giving satisfactory results, such as perceptual organization [1], parametric models [2] or structural approach [3], have been developed using aerial images.

Nowadays, this problem is tackled by another kind of data : the submetric satellite images. The main advantages of satellite data compared to aerial images are a high swath width and ground coverage. However, such data have a "relatively low" resolution and a "low" signal to noise ratio to deal with 3D reconstruction problems. Those drawbacks do not allow to use standard methods developed for the aerial image case and lead us to propose a new method based on an important prior knowledge concerning urban structures.

An automatic building extraction method [4] based on marked point processes is used to provide rectangular building footprints. It consists in extracting the building outlines through a configuration of rectangles from Digital Elevation Models (DEM) which are altimetric descriptions of urban areas. Figure 1 shows the result using a DEM provided from multiple stereo pairs of PLEIADES simulations (0.5 meter resolution - B/H=0.2) by the French Geographic Institute (IGN) and computed by an algorithm based on [5].

Our goal is then to construct a 3D city model from the DEM and associated rectangular building footprints. To do so, a new parametric method for automatic 3D building reconstruction based on a Bayesian approach is developed. A parametric model with rectangular ground footprint is preferred since it is less complex, more robust to satellite data and more adapted to rectangular building footprints. The method is based on the definition of a density which contains both a priori knowledge on the buildings, taking into account the interactions existing between neighboring models, and a data term which fits the models to the DEM. A simulated annealing scheme is used to find the configuration which maximizes this density.

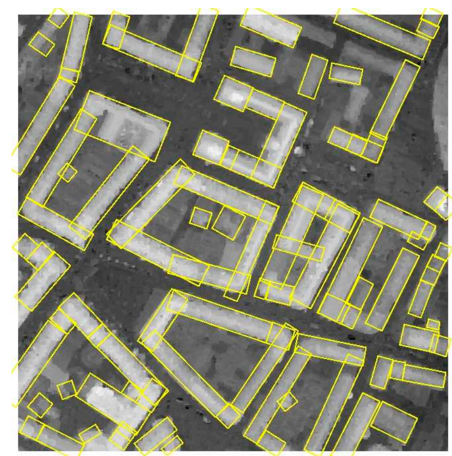


Fig. 1. Building extraction result from a Digital Elevation Model

2. PARAMETRIC MODEL

2.1. Choice of the parametric model

Using parametric models with rectangular ground footprint is an advantage because such models are not complex and have few parameters. Figure 2 shows the proposed parametric model which allows to represent a large majority of roof form. Roof top surface is constrained to be null since a non-null surface is improbable. Therefore, the roof top is a point or a segment. It is a general parametric model of buildings defined by 6 parameters (without taking into account rectangular base parameters) : H_g, the height of the getter of roof, H_c, the roof top height and (a, b, c, d), parameters describing the roof form. More details are available in [6].

2.2. Notations

Let us consider:

- S, a set of sites and $I = \{x(s)/s \in S\}$, a set of intensities defined for a given DEM.
- $\mathcal{R}$, the object space of a rectangle which is defined by five

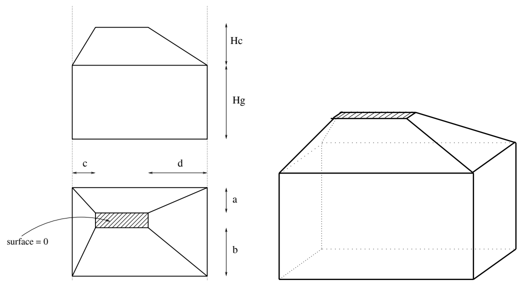


Fig. 2. Parametric model of buildings

parameters : its center (x_c, y_c) and its length, width and orientation (L, l, ϕ).

- $\mathcal{C} \in \mathcal{R}^N$, the rectangle configuration representing the building footprints associated with I and computed by the method described in [4] (N represents the number of rectangles).
- S_i, the subset of S whose sites are inside the rectangle $i \in \mathcal{C}$.
- $\mathcal{D} = \{x(s) \in I / s \in S_i \text{ , } i \in \mathcal{C}\}$, the set of data
- $\mathcal{T}$, the state space and $\theta = (\theta_i)_{i \in \mathcal{C}} \in \mathcal{T}$, a configuration of models.
- f_{θ_i}, the function from S_i to $\mathbb{R}$ which associates the roof altitude of the model defined by θ_i to each site of S_i.

Let us consider $\eta > 0$. A model θ_i will be said η-weakly symmetric if $|a - b| < \eta$ **or** $|c - d| < \eta$. θ_i will be said η-strongly symmetric if $|a - b| < \eta$ **and** $|c - d| < \eta$.

3. DENSITY FORMULATION

let us consider the measurable space $(\mathcal{T}, \mathcal{B}(\mathcal{T}), \mu(.))$ associated to the Lebesgue measure $\mu(.)$ on $\mathbb{R}^{6N}$. We consider the random variable Θ distributed in $\mathcal{T}$ which follows an unnormalized density $h(.)$ against $\mu(.)$. $h(.)$ is actually the posterior density of a configuration θ of models, given $\mathcal{D}$. In a Bayesian framework, this density can be obtained as follows :

$$h(\theta) = h(\theta/\mathcal{D}) \propto h_p(\theta)\mathcal{L}(\mathcal{D}/\theta) \qquad (1)$$

A requirement is to be able to build both a prior density $h_p(\theta)$ and a likelihood $\mathcal{L}(\mathcal{D}/\theta)$. In the following, these two terms are detailed.

3.1. Likelihood

Let us consider $\mathcal{D}_i$, the partial data of rectangle i defined as $\mathcal{D} = \bigcup_{i \in \mathcal{C}} \mathcal{D}_i$. $\mathcal{L}(\mathcal{D}_i/\theta_i)$ represents the probability of observing $\mathcal{D}_i$ knowing the object θ_i. By considering the hypothesis of conditional independence (it means we disregard the overlapping of rectangles), the likelihood can be expressed as:

$$\mathcal{L}(\mathcal{D}/\theta) = \prod_{i \in \mathcal{C}} \mathcal{L}(\mathcal{D}_i/\theta_i) = \prod_{i \in \mathcal{C}} \exp(-\|f_{\theta_i} - x\|_i) \qquad (2)$$

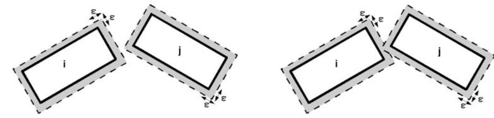
$\|.\|_i$ is the norm defined from the function space of S_i to $\mathbb{R}$ by :

$$\|f\|_i = \frac{1}{card(S_i)} \sum_{s \in S_i} |f(s)| \qquad (3)$$

So, the likelihood is linked to the Z-error of the L_1 norm between the DEM and the parametric modeling defined by the configuration θ. The L_1 norm is preferred to the L_2 norm since the DEM is neither exact nor accurate. The L_2 norm is too sensitive to the DEM errors.

3.2. Prior density

The prior term allows to favor some configurations and penalize other ones. Some interactions between objects are defined thanks to a neighborhood relationship ν (see figure 3). The existence of a



(a) (b)

Fig. 3. Neighborhood relationship ν - **(a)** : non neighboring rectangles **(b)** : neighboring rectangles

neighborhood is very important. It allows to consider the problem from a global point of view (i.e. by considering a building as a collection of rectangles instead of seeing it as a unique rectangle). ϵ defines the neighborhood width. It has been set up to one meter, that is a distance which tolerates small errors concerning the rectangle linking up and is smaller than the average width of a street.
The prior density derives from different Gibbs energies developed in the following. It is given by:

$$h_p(\theta) = \exp - [U_s(\theta) + U_h(\theta) + U_r(\theta)] \qquad (4)$$

3.2.1. Roof symmetry

In urban areas, a large majority of buildings have symmetric roofs. Models which are not at least weakly symmetric are improbable models (see figure 4). We aim at favoring the weakly and strongly symmetric models with respect to the other ones.
Let us consider $n_f(\theta)$ and $n_F(\theta)$, the numbers of objects of the configuration θ which are η-weakly and η-strongly symmetric respectively (η is a parameter having a sub-metric value). The constant negative potential ω_f and ω_F are associated with $n_f(\theta)$ and $n_F(\theta)$ respectively. The energy related to the symmetry is then given by:

$$U_s(\theta) = \omega_f n_f(\theta) + \omega_F n_F(\theta) \qquad (5)$$

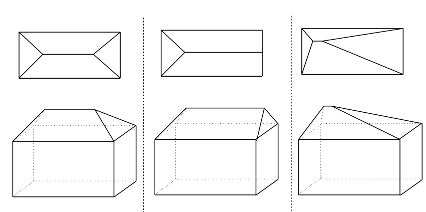


Fig. 4. **left** : Roof which is strongly symmetric, **center** : weakly symmetric roof, **right** : improbable model

3.2.2. Getter of roof height adjustment

The getter of roof heights of buildings are dependent of neighboring buildings. It is important to define an interaction term which favors the getter of roof height alignment between neighboring rectangles. This term has to be:

- attractive for similar getter of roof heights (i.e. with a difference lower than half a floor)

- repulsive for different getter of roof heights (i.e. with a difference between half a floor and one floor)
- neutral for distant getter of roof heights (i.e. with a difference higher than one floor)

To do so, we define the energy term U_h as follows:

$$U_h(\theta) = \sum_{i\nu j} f_h(|H_{g_i} - H_{g_j}|) \tag{6}$$

where f_h is a real value function specified in [6] (see figure 5), which depends on ω_h, a positive constant potential and H_e, a constant which represents half a floor height (in practice, we take $H_e = 1.5$ meter).

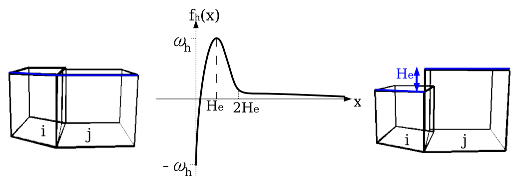


Fig. 5. **left** : favored configuration **center** :function f_h **right** : penalized configuration

3.2.3. Roof top linking up

It is important to develop an interaction which favors the continuity of roof tops between neighboring rectangles. More precisely, this term must favor roof top linking up. We propose an interaction which attracts the roof top extremities of neighboring buildings (see figure 6). The associated energy is modeled as follows:

$$U_r(\theta) = \sum_{i\nu j} \omega_r \, d(e_{\theta_i}, e_{\theta_j})^2 \tag{7}$$

where e_{θ_i} is the point (in $\mathbb{R}^3$) corresponding to the roof top extremity of the model θ_i and ω_r, a positive constant potential. $d(.,.)$ is the distance related to the $L2$-norm in $\mathbb{R}^3$.

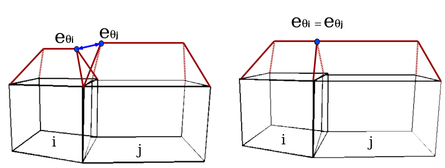


Fig. 6. **left** : penalized configuration **right** : favored configuration

4. OPTIMIZATION

We want to find the configuration which maximizes the density $h(.)$. We search for the Maximum A Posteriori estimator θ_{MAP}:

$$\theta_{MAP} = \arg\max_{\theta} h(\theta) \tag{8}$$

This is a non convex optimization problem in a very high dimension space $\mathcal{T}$. A simulated annealing, embedded into a MCMC sampler, based on [7] is well adapted to this problem. It consists in :

- proposing a state $\theta^\star$ (following a uniform distribution)
- accepting the perturbation $\theta \rightarrow \theta^\star$ with probability

$$\min\left(\left(\frac{h(\theta^\star)}{h(\theta)}\right)^{\frac{1}{D_t}}, 1\right) \tag{9}$$

where D_t is a sequence of temperatures which tends to zero as t tends to infinity. At the beginning of the algorithm (i.e. when the temperature is high), the process is not really selective : it allows to explore the density modes. When the temperature decreases, configurations which have a high density will be favored. More details about the optimization process are available in [6].

5. RESULTS

In most cases, using energy models implies parameter tuning. Those parameters correspond to weights of the various energy terms ω_f, ω_F, ω_h and ω_r, which are chosen by trial and error.
Figure 7-a shows the result obtained from Amiens downtown DEM (see figure 1). Figure 7-b is the associated 3D ground truth provided by the French Geographic Institute. The result is satisfactory with respect to the 3D ground truth. The main drawback is the presence of artefacts due to a non optimal rectangle overlapping and roof top linking up impossibility in some specific places (see figure 7-c).
Figure 7-d represents the associated error map which provides three pieces of information. First, it provides the not found areas of the building extraction (in black). They correspond to low flat buildings of inner courtyards that the building extraction method [4] cannot detect. Then, we can see the false alarms of building extraction (in white - rate : 12%), mainly located around the reference building footprint (due to a "drooling" DEM on the building contours which generates wider rectangles). Finally, it provides Z-errors between the reconstruction result and the 3D ground truth (red to yellow). The corresponding Root Mean Square Error (RMSE) of common building footprints is 3.2 meters. This value is satisfactory for a fully automatic method using 2.5 meter Z-resolution DEMs.
Figure 7-e presents a result on a blockhouse. We can see the importance of the prior knowledge through getter of roof height alignments and roof top linkings up.
The computing time is quite high. For example, 5 minutes are necessary to obtain the result of Amiens city hall (see figure 7-e) using a Pentium IV-3Ghz.

6. CONCLUSION

Results obtained by the new approach proposed in this paper show that the use of a parametric model is well adapted to deal with satellite data in an automatic context. The obtained 3D reconstructions, and especially the roof reconstructions, are satisfactory : a large majority of urban structures is close to reality. The proposed prior knowledge allows to compensate for the low quality of data.
In future works, two drawbacks should be corrected. At first, some artefacts, due to a non optimal rectangle overlapping and roof top linking up impossibility at some areas, should be eliminated by using post-processing based on improvements of rectangular building footprints. Moreover, the computing time should be reduced. To do so, the prior knowledge of roof symmetry will be used to define a new parametric model instead of being used in the Bayesian expression, in order to reduce the state space.

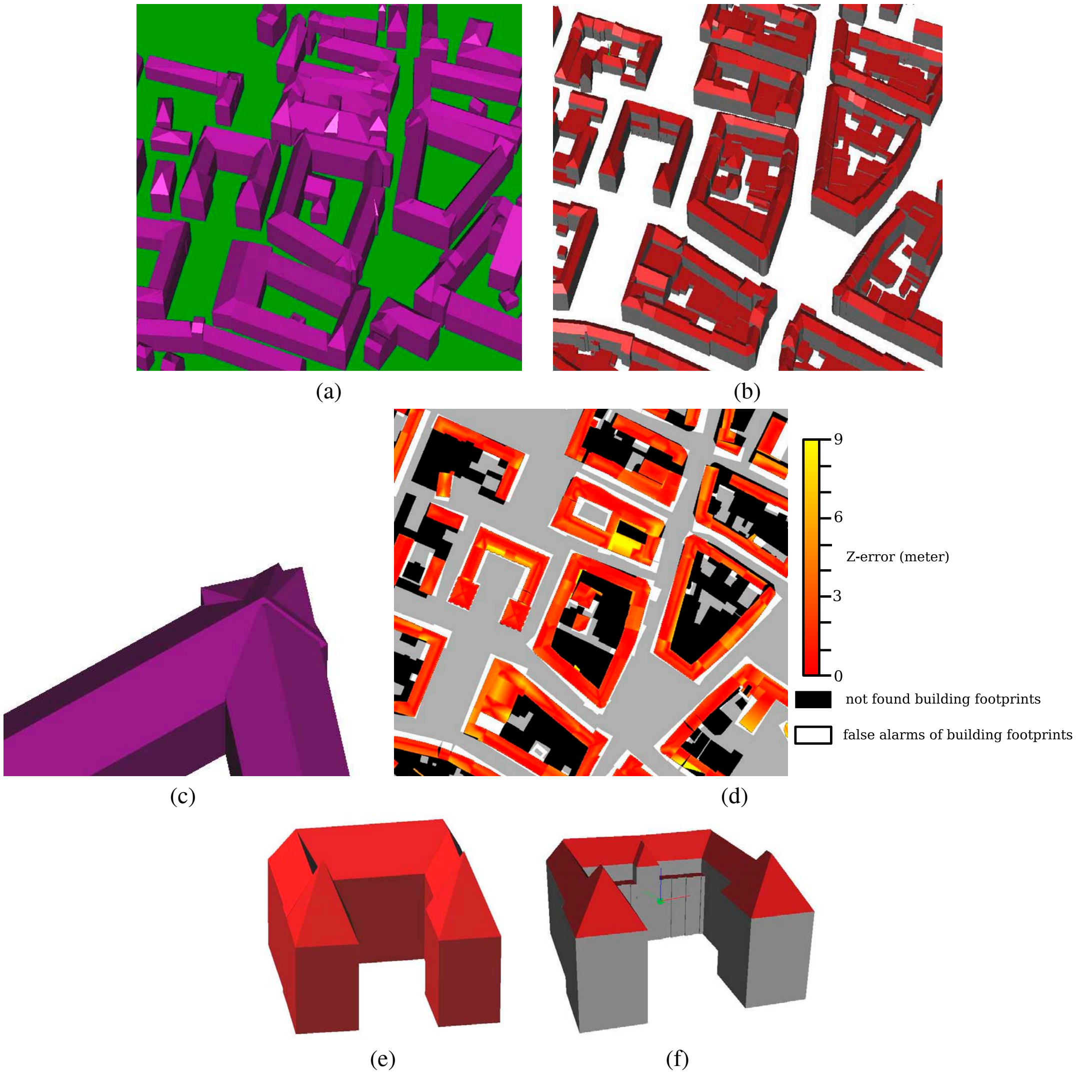


Fig. 7. **(a)** : result of Amiens downtown (France) from the DEM of the figure 1 **(b)** : the associated 3D ground truth ©IGN **(c)** : example of artefacts **(d)** : evaluation map **(e-f)** : example of a reconstructed building (e) and its associated 3D ground truth (f) ©IGN

7. ACKNOWLEDGMENTS

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8. REFERENCES

[1] R. Nevatia and K. Price, “Automatic and interactive modeling of buildings in urban environments from aerial images,” in *ICIP*, New York, 2002, pp. 525–528.

[2] U. Weidner and W. Forstner, “Fowards Automatic Building Reconstruction from High Resolution Digital Elevation Models,” *Journal of Photogrammetry and Remote Sensing*, vol. 50(4), pp. 38–49, 1995.

[3] F. Fuchs and H. Le Men, “Efficient subgraph isomorphism with a-priori knowledge. application to building reconstruction for cartography,” *Lecture Notes in Computer Science*, , no. 1876, Springer, 2000.

[4] M. Ortner, X. Descombes, and J. Zerubia, “Building extraction from Digital Elevation Model.,” in *ICASSP*, Honk Kong, Apr 2003.

[5] S. Roy and I.J. Cox, “A maximum-flow formulation of the n-camera stereo correspondence problem,” in *ICCV*, Bombay, Jan 1998.

[6] F. Lafarge, X. Descombes, J. Zerubia, and M.P. Deseilligny, “A parametric model for automatic 3d building reconstruction from high resolution satellite images,” Research Report 5687, INRIA, september 2005.

[7] M. Metropolis, A.W. Rosenbluth, A.H. Teller, and E. Teller, “Equation of state calculations by fast computing machines,” *Journal of Chemical Physics*, vol. 21, pp. 1087–1092, 1953.