DISTRIBUTED SOURCE AND JOINT SOURCE-CHANNEL CODING: FROM THEORY TO PRACTICE

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ABSTRACT

This paper reviews the theory behind distributed source coding and distributed source-channel coding and surveys the progress made in code design during the last years.

1. INTRODUCTION

Distributed source coding (DSC) refers to the compression of the outputs of two or more physically separated (but correlated) sources that do not communicate with each other [1]. These sources send their compressed outputs to a central point for joint decoding. The case in which the sources are transmitted to the receiver through noisy channel(s) is known as distributed joint source-channel coding (DJSCC). The goal then is to minimize the energy required by the sources to achieve reliable communications. In both cases, the existence of correlation leads to theoretical limits that are substantially better than those obtained in its absence.

Driven by a host of emerging applications (e.g., wireless video and distributed sensor networks), during the last years there has been a flurry of activities in practical designs [2] for DSC – more than 30 years after Slepian and Wolf laid its theoretical foundation [1]. Although some designs have their roots in the source coding community (see [3] and references therein), most of the recent works are based on considering DSC as a problem of channel coding with side information [4]. In this context, the first practical schemes for source coding of correlated sources appeared in [5, 6]. However, it is only recently with the introduction of iterative techniques based on turbo-like codes [7, 8] that the theoretical limits have been practically achieved.

2. SLEPIAN-WOLF CODING (SWC)

2.1. Theory

Let $\{(X_i, Y_i)\}_{i=1}^{\infty}$ be a sequence of pairs of correlated, discrete random variables $(X, Y) \in (\mathcal{X}, \mathcal{Y})$. Then for lossless compression of X and Y, a rate of $R_X + R_Y = H(X, Y)$ is sufficient if they are encoded jointly. However, the rate $R_X + R_Y = H(X, Y)$ is sufficient even for separate encoding (with joint decoding) of X and Y. This result, proved by Slepian and Wolf [1] for i.i.d. pairs and extended by Cover [9] for jointly ergodic sources, is quite surprising since it states that there is no loss of coding efficiency with separate encoding when compared to joint encoding as long as joint decoding is performed.

2.2. Practice

The proof of the Slepian-Wolf theorem [1] is based on random binning, which is non-constructive; in practice one has to resort to either pseudo-random binning [8] or to algebraic binning [10], i.e., a structured binning/grouping scheme with algebraic operations. To approach the corner points in the Slepian-Wolf rate region, Wyner first suggested the use of parity-check codes in his 1974 paper [4]. The basic idea is to generate the bins as the cosets of some "good" parity-check code. In compressing, a sequence of n input bits is mapped into its corresponding $(n - k)$ $syndrome$ bits, achieving a compression ratio of $n : (n - k)$. In general, if the correlation between the source and side information can be modeled with a "virtual" correlation channel, then a good code for this channel will provide a good Slepian-Wolf code by using coset codes as bins. Thus, the seemingly source coding problem of SWC is actually a channel coding one, and near-capacity channel codes such as turbo and LDPC codes can be used to approach the Slepian-Wolf limit. Practical designs for SWC using turbo-like codes, including the cases of non-binary sources and correlation with memory, have appeared in [8, 11, 12, 13, 14, 15, 16, 17, 18].

3. WYNER-ZIV CODING (WZC) AND MULTITERMINAL (MT) SOURCE CODING

3.1. Theory

The problem of rate distortion with side information at the decoder, or the Wyner Ziv problem [19], asks the question of how many bits are needed to encode X under the constraint that the average distortion between X and its reconstruction $\hat{X}$ is $E\{d(X, \hat{X})\} \leq D$, assuming side information Y is available at the decoder but not at the encoder. It generalizes the Slepian-Wolf setup [1] in that coding of

Garcia-Frias's work was supported in part by NSF grant CCR-0311014 and Xiong's work by NSF grant CCR-0430720.

X is with respect to a fidelity criteria, rather than lossless. For both discrete and continuous alphabet cases of X and Y with a general distortion metric $d(\cdot, \cdot)$, Wyner and Ziv gave the rate-distortion function $R^*_{WZ}(D)$ for this problem as $R^*_{WZ}(D) = \inf I(X; Z|Y)$, where the infimum is taken over all random variables Z such that $Y \to X \to Z$ is a Markov chain and there exists a function $\hat{X} = \hat{X}(Z, Y)$ satisfying $E\{d(X, \hat{X})\} \leq D$.

An important result of [19] is that in general there is rate loss in the Wyner-Ziv problem when compared to joint encoding of X and Y. However, for zero mean and jointly Gaussian X and Y, WZC of X suffers no rate loss when the distortion metric is mean square error. Pradhan *et al.* [20] extended this no rate loss result to the more general case with $X = Y + Z$, where only the innovations Z need to be Gaussian (Y can follow arbitrary distribution). In the information-theoretical tutorial paper [10], the authors proposed the use of nested lattice codes for WZC. However, algebraic binning via nested lattice codes only facilitates high-dimensional analysis.

Another extension of SWC is to the case when one allows some distortion in reconstructions of the sources. This gives rise to MT source coding [21], which has gained increased interest recently due to its application in distributed sensor networks. There are two classes of MT problems. If each sensor observes directly one of the sources, we have direct MT source coding [21, 22]. On the other hand, if each sensor cannot observe directly the source which is to be reconstructed at the decoder, but is rather provided only with one of its noisy versions, then we speak of indirect (remote) MT source coding [23, 24].

The MT problem consists of determining the achievable rate region; that is, the rates at which sources (or, noisy observations) can be separately compressed, so that at the central unit they can be recovered jointly within a target distortion. Though intense research efforts have been conducted in solving MT problems, achievable rate regions, in general, are still unknown; only inner and outer bounds have been provided so far [21][1]. The quadratic Gaussian case was considered in [21, 22] and [23, 24] for the direct and indirect MT problem, respectively. In contrast to the direct problem, where even in this simple case the inner and outer bounds do not fully coincide, the Berger-Tung achievable region [21, 25] is shown to be tight for the indirect (or Gaussian CEO) problem [23].

3.2. Practice

Because distortion is introduced to the source in WZC, source coding (or quantization) is needed to quantize X. Usually there is still correlation remaining in the quantized version of X and the side information Y, and SWC should be employed to exploit this correlation to reduce the rate. Since SWC is based on channel coding, WZC is a source-channel coding problem. There are quantization loss due to source coding and binning loss due to channel coding. To reach the Wyner-Ziv limit in practice, one needs to employ both source codes (e.g., TCQ) that can achieve the granular gain and channel codes (e.g., turbo and LDPC codes) that can approach the Slepian-Wolf limit. In addition, if X is continuous, the side information Y should be used in jointly decoding and estimating $\hat{X}$.

Since the first major work on practical Wyner-Ziv code design called DISCUS [5] appeared in 1999, several groups have worked on this problem. A paradigm called Slepian-Wolf coded quantization (SWCQ) for practical WZC was proposed in [2]. The essence of SWCQ is to combine source coding (quantization) and channel coding (SWC) for efficient algebraic binning, where source codewords are grouped into coset channel codes (or bins) so that one index is sent for each bin to achieve compression.

For the quadratic Gaussian Wyner-Ziv problem, the performance gap of high-rate SWCQ to the Wyner-Ziv distortion-rate function $D^*_{WZ}(R)$ is exactly the same as that of high-rate classic source coding to the distortion-rate function $D(R)$ [2]. In a practical design example [2] of SWCQ with 2-D TCVQ, irregular LDPC code based Slepian-Wolf coding, and optimal estimation at the decoder, the performance gap to the Wyner-Ziv distortion-rate function is only 0.66 dB at 1.0 b/s and 0.47 dB at 3.3 b/s. These results are much closer to the theoretical performance limit of WZC than any other previously reported. They indicate that we are approaching the theoretical limits of Wyner-Ziv coding!

Wyner-Ziv video coding is a promising technique for many-to-one "uplink" video communication systems. Puri and Ramchandran [26] first presented a PRISM coder that swaps the encoder/decoder complexity in standard coders by achieving compression without exploiting the inter-frame correlation via motion estimation at the encoder; instead, this correlation is exploited at the decoder. Girod *et al.* [27] proposed a distributed video compression scheme based on turbo codes and addressed its error resilience property. Sehgal *et al.* [28] discussed how coset-based Wyner-Ziv video coding can alleviate the problem of prediction mismatch. Xu and Xiong [29] used nested quantization and LDPC code based SWC to construct a layered Wyner-Ziv video coder that approaches the rate-distortion performance of the conventional H.26L fine granular scalability (FGS) coder. Because the Wyner-Ziv enhancement layer in [29] is generated "blindly" at the encoder without knowing the base layer (or side information at the decoder), the problem of error propagation associated with standard FGS coding due to channel errors in the base layer is alleviated.

For the Gaussian CEO problem [23], Pradhan and Ramchandran [6] provided a code design with fixed-rate scalar quantizers and trellis codes. Although capable of trading off

[1] All rate points within the inner bound are achievable, while those outside the outer bound are not.

transmission rates between the two encoders, the design in [6] performs far away from the theoretical limits, especially at low rates. Yang *et. al* have addressed the MT source code design problem in [30] for both direct and indirect quadratic Gaussian cases with two encoders/sensors. Two approaches are taken in [30]: the first relies on WZC and source splitting and the second employs symmetric SWC [18] for arbitrary rate allocation between the two sources after each of them is quantized. Assuming ideal quantization (source coding) and SWC, the code design proposed in [30] is shown to be capable of achieving any point on the inner bound for both MT coding problems. Practical designs based on TCQ for source coding and turbo/LDPC codes for SWC perform only 0.29 b/s away from the inner bound.

4. DISTRIBUTED JOINT SOURCE-CHANNEL CODING (DJSCC)

4.1. Theory

The case of DJSCC presents different scenarios. The first one, the *asymmetric case*, considers that one of the sources, Y, is available at the decoder and used as a side information for the decoding of the other source X, which is transmitted through a noisy channel. Under these circumstances, the separation theorem proved in [31] asserts that the entropy of the source $H(X)$ in the standard separation theorem should be replaced by $H(X|Y)$. An extension to lossy source-channel coding with side information is given in [32].

In the second scenario, the *independent channels case*, each source is transmitted through an independent noisy channel to the common receiver. Under these conditions, it has been recently shown [33] that the separation principle between source and channel coding still holds. In the final scenario, the *MAC case*, the correlated sources are transmitted through a multiple access channel (MAC). In this case, as discussed in [34], the separation principle does not apply, and the theoretical limits are unknown in general.

4.2. Practice

Practical designs for the *asymmetric case* have been proposed in [13, 35, 36], and the practical schemes designed for joint source-channel coding of single sources can also be applied in a straightforward manner. For example, the design of [36] exploits systematic IRA codes. The main idea is to view the system as transmitting the source over two channels: the first one is the actual noisy channel; the second is the "virtual" correlation channel between the source and side information. Systematic IRA codes are well suited for DJSCC because parity bits instead of syndrome bits can be generated to achieve the same compression ratio as in SWC while allowing both channels to be considered in the design of a bigger size code for improved error robustness. This parity-based approach for DJSCC is thus more general than Wyner's syndrome-based approach [4], which is optimal for SWC (with only one correlation channel), and the two approaches are equivalent under the SWC setup. Combining the advantage of IRA codes for DJSCC and that of LT codes [37] for erasure protection in an overall Raptor code [38] design is done in [39] for video streaming over packet erasure channels.

Designs for the *independent channels case* and the *MAC case* have been proposed in [8, 40, 41, 42, 43, 44]. In both cases, the proposed designs utilize channel codes that encode the sources at the desired information rates, producing pseudo-random codewords. Obviously, the codes used in these joint source-channel coding approaches have to be less powerful than in a separated scheme. However, the 'weakness" of the codes in the joint source-channel approach is compensated by exploiting the correlation between sources at the decoder. Compared with a separated scheme, the proposed joint source-channel coding approach allows a channel code of a single rate to be used in combination with sources having arbitrary joint entropy rates, with the modifications to maintain efficient coding involving only processing in the decoder. It is important to remark that in the *MAC case* the proposed designs are able to outperform the Shannon separation limit.

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