

COMPARISON OF REDUNDANT WAVELET SCHEMES FOR MULTIPLE DESCRIPTION CODING OF VIDEO SEQUENCES

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ABSTRACT

Multiple description coding (MDC) recently appeared as a joint source-channel coding technique specifically designed for real-time multimedia applications over best effort switched packet networks such as Internet, in order to cope with packet losses due to transmission errors or network congestion. In this paper we compare several redundant wavelet decompositions in the framework of multiple description of scalable video coding. A special attention is paid to the optimal design of the central decoder. Simulation results are provided for motion-compensated filter banks so as to evaluate the efficiency of the central and side decoding strategies. Compared with other techniques, a key factor of the proposed analysed schemes is their reduced redundancy factor.

1. INTRODUCTION

In image and video coding, a number of wavelet bases have demonstrated good compression capabilities. In particular, in scalable video coding, there has recently been a growing interest in motion-compensated structures using the orthonormal Haar decomposition or biorthogonal 5-3 filter banks for the temporal decomposition [1], [2]. Scalable representations are useful for bitstream adaptation to bandwidth variations or receiver characteristics, but in case of video transmission over best effort switched packet networks such as Internet, additional difficulties in reconstruction are raised by packet losses. A joined source-channel design can increase the error resilience of the transmitted bitstreams, and multiple description coding [3] appeared as such a technique, introducing redundancy at the source in order to cope with channel losses. Basically, correlated descriptions of the input are created and transmitted over independent on-off channels. In case of channel failure due to network congestion, server failure etc, side decoders should be able to reconstruct with an acceptable quality the source, while the reception of all the descriptions by a central decoder leads to a high-quality reconstruction.

While several previous works addressed the multiple description of video by creating multiple loops in the temporal prediction loop of a hybrid codec [4], [5], we investigate in this work temporal multiple descriptions scheme based on a redundant $t + 2D$ wavelet decomposition. Our schemes combine thus the spatial/temporal/SNR scalability of a $t + 2D$ wavelet-based codec

with the error robustness conferred by coding correlated descriptions. Unlike other studies, in which the wavelet filter bank was applied independently in the spatial domain for each image [6], [7], in our approach the oversampled filter bank concerns the temporal decomposition. In a previous work [8], we have introduced temporal redundancy in a 3-band filter bank. Here, we first introduce an oversampled dyadic temporal filter bank and propose several ways of constructing correlated descriptions. The properties of such filter banks acting as channel codes over erasure channels have recently been investigated [9], [10], [11]. However, a special feature that we request from our schemes is a reduced redundancy factor, achieved by a further subsampling of the detail frames. In this case, the perfect reconstruction is not any more guaranteed for these schemes. We provide consequently a framework for studying the invertibility at the central decoder and reduce the effect of the quantization noise.

In the next section, we present the considered filter bank framework and introduce the proposed MDC schemes. In Section 3 the signal reconstruction is discussed. In Section 4, we focus on the video coding application and provide simulation results. The last section concludes this paper.

2. REDUNDANT FILTER BANK REPRESENTATIONS

Starting from a dyadic filter bank, we first investigate several redundant filter bank structures and describe possible configurations for creating multiple descriptions.

Let us consider a temporal input signal, $(x_n)_{n \in \mathbb{Z}}$ and denote by $(h_n)_{n \in \mathbb{Z}}$ (resp. $(g_n)_{n \in \mathbb{Z}}$) the low-pass (resp. high-pass) filter of a 2-band analysis filter bank with perfect reconstruction. The approximation coefficients are then given by

$$a_n^I = \sum_k h_{2n-k} x_k \quad (1)$$

and the detail coefficients are

$$d_n^I = \sum_k g_{2n-k} x_k. \quad (2)$$

By decimating at odd instants rather than even ones, the approxi-

mation/detail coefficient sequences become

$$a_n^{\text{II}} = \sum_k h_{2n-1-k} x_k \quad (3)$$

$$d_n^{\text{II}} = \sum_k g_{2n-1-k} x_k. \quad (4)$$

By considering all the above four set of coefficients, we obviously generate a redundant decomposition, the number of coefficients being multiplied by 2. It is however possible to build more “economical” representations while keeping the desired perfect reconstruction property. To do so, we will further decimate these sequences by a factor 2 and we will therefore find useful to introduce:

$$\hat{a}_n^{\text{I}} = a_{2n}^{\text{I}}, \quad (5)$$

$$\check{a}_n^{\text{I}} = a_{2n-1}^{\text{I}}, \quad (6)$$

similar notations being used for the other involved sequences. The vector $\bar{\mathbf{c}}_n$ which contains all the possible subsampled sequences is $\bar{\mathbf{c}}_n = (\hat{a}_n^{\text{I}} \check{a}_n^{\text{I}} \hat{a}_n^{\text{II}} \check{a}_n^{\text{II}} \hat{d}_n^{\text{I}} \check{d}_n^{\text{I}} \hat{d}_n^{\text{II}} \check{d}_n^{\text{II}})^{\text{T}}$. Let us now introduce the polyphase components of the analysis filters:

$$\forall i \in \{0, 1, 2, 3\}, \quad h_i(n) = h_{4n-i}, \quad g_i(n) = g_{4n-i} \quad (7)$$

and the corresponding z -transforms $H_i(z)$ and $G_i(z)$. Similarly, we define the four polyphase components of the input signal by:

$$\forall i \in \{0, 1, 2, 3\}, \quad x_n^{(i)} = x_{4n+i}. \quad (8)$$

The corresponding polyphase component vector is

$$\mathbf{x}_n = (x_n^{(0)} x_n^{(1)} x_n^{(2)} x_n^{(3)})^{\text{T}}. \quad (9)$$

It is then easily shown that Eqs. (1)-(4) are equivalent to the following polyphase representation:

$$\bar{\mathbf{C}}(z) = \bar{\mathbf{M}}(z) \mathbf{X}(z) \quad (10)$$

where $\bar{\mathbf{C}}(z)$ and $\mathbf{X}(z)$ are the z -transforms of the coefficient vector sequence and of the input signal and $\bar{\mathbf{M}}(z)$ is the global polyphase transfer matrix which is given by

$$\bar{\mathbf{M}}(z) = \begin{bmatrix} H_0(z) & H_1(z) & H_2(z) & H_3(z) \\ H_2(z) & H_3(z) & H_0(z)z^{-1} & H_1(z)z^{-1} \\ H_1(z) & H_2(z) & H_3(z) & H_0(z)z^{-1} \\ H_3(z) & H_0(z)z^{-1} & H_1(z)z^{-1} & H_2(z)z^{-1} \\ G_0(z) & G_1(z) & G_2(z) & G_3(z) \\ G_2(z) & G_3(z) & G_0(z)z^{-1} & G_1(z)z^{-1} \\ G_1(z) & G_2(z) & G_3(z) & G_0(z)z^{-1} \\ G_3(z) & G_0(z)z^{-1} & G_1(z)z^{-1} & G_2(z)z^{-1} \end{bmatrix}.$$

In order to generate schemes with lower redundancy, we propose to discard 2 of the components of $\bar{\mathbf{c}}_n$. The remaining 6-dimensional vector \mathbf{c}_n corresponds to a representation with a redundancy factor equal to 3/2. The resulting oversampled filter bank is such that $\mathbf{C}(z) = \mathbf{M}(z) \mathbf{X}(z)$, where $\mathbf{C}(z)$ is the z -transform of $(\mathbf{c}_n)_{n \in \mathbb{Z}}$ and $\mathbf{M}(z)$ is the polyphase transfer function of the considered scheme. Several possible decompositions may however be obtained depending on the choice of the submatrix $\mathbf{M}(z)$. More precisely, we have investigated the following four solutions, for which we specify the way of building two appropriate descriptions.

- **R-Scheme.** This scheme consists in splitting the detail coefficients of the classical critically subsampled analysis into two groups: even-index coefficients and odd-index coefficients, each group belonging to one of the descriptions. The approximation coefficients are simply duplicated. This corresponds to

$$\mathbf{c}_n = \underbrace{(\hat{a}_n^{\text{I}} \check{a}_n^{\text{I}} \hat{d}_n^{\text{I}})}_{\text{1st description}} \underbrace{(\hat{a}_n^{\text{II}} \check{a}_n^{\text{II}} \hat{d}_n^{\text{II}})}_{\text{2nd description}}^{\text{T}}. \quad (11)$$

In this case, $\mathbf{M}(z)$ is formed with the lines 1, 2, 5, 1, 2, 6 of $\bar{\mathbf{M}}(z)$ (lines 1, 2 are duplicated).

- **D1-Scheme** We distribute the detail coefficients according to the same scheme as above, but in the second description, instead of repeating the approximation coefficients given by Eq. (1), we use those given by Eq. (3) This corresponds to

$$\mathbf{c}_n = \underbrace{(\hat{a}_n^{\text{I}} \check{a}_n^{\text{I}} \hat{d}_n^{\text{I}})}_{\text{1st description}} \underbrace{(\hat{a}_n^{\text{II}} \check{a}_n^{\text{II}} \hat{d}_n^{\text{II}})}_{\text{2nd description}}^{\text{T}}. \quad (12)$$

and $\mathbf{M}(z)$ is formed with the lines 1, 2, 5, 3, 4, 6 of $\bar{\mathbf{M}}(z)$.

- **D2-Scheme** We add more “diversity” in the choice of the detail coefficients by taking

$$\mathbf{c}_n = \underbrace{(\hat{a}_n^{\text{I}} \check{a}_n^{\text{I}} \hat{d}_n^{\text{I}})}_{\text{1st description}} \underbrace{(\hat{a}_n^{\text{II}} \check{a}_n^{\text{II}} \hat{d}_n^{\text{II}})}_{\text{2nd description}}^{\text{T}}. \quad (13)$$

We deduce that $\mathbf{M}(z)$ is formed with the lines 1, 2, 5, 3, 4, 7 of $\bar{\mathbf{M}}(z)$.

- **D3-Scheme** By selecting the odd-subsampled detail coefficients in Eq. (3) rather than the even-subsampled one, we get

$$\mathbf{c}_n = \underbrace{(\hat{a}_n^{\text{I}} \check{a}_n^{\text{I}} \hat{d}_n^{\text{I}})}_{\text{1st description}} \underbrace{(\hat{a}_n^{\text{II}} \check{a}_n^{\text{II}} \check{d}_n^{\text{II}})}_{\text{2nd description}}^{\text{T}}. \quad (14)$$

and $\mathbf{M}(z)$ is formed with the lines 1, 2, 5, 3, 4, 8 of $\bar{\mathbf{M}}(z)$.

In the absence of quantization, the first two schemes are obviously invertible since they include all the coefficients resulting from the classical critically subsampled analysis. For the latter two schemes the invertibility is not guaranteed a priori. By considerations which are detailed in [12], we have shown that these schemes *can be inverted by FIR synthesis filters* for usual choices of the analysis filters.

3. SYSTEM RECONSTRUCTION

3.1. Solution of the System Inversion

In this section, we investigate how to reconstruct the signal from the proposed redundant representations. In terms of MDC, this amounts to design the synthesis scheme used at the central decoder. With the notations used in the previous section, we can formulate the problem as follows: we want to find an $N \times K$ transfer function $\mathbf{W}(z)$ such that

$$\mathbf{W}(z)\mathbf{M}(z) = \mathbf{I}_{N \times N} \quad (15)$$

where $\mathbf{W}(z) = [W_{i,j}(z)]_{1 \leq i \leq N, 1 \leq j \leq K}$ and $\mathbf{M}(z) = [M_{i,j}(z)]_{1 \leq i \leq K, 1 \leq j \leq N}$. We remind that $K = 6$ is the number of

coefficient sequences and $N = 4 < K$ is the number of polyphase components of the input signal. The maximum length of the scalar filters with transfer function $W_{i,j}(z)$ (resp. $M_{i,j}(z)$) is assumed to be equal to $P \in \mathbb{N}^*$ (resp. $Q \in \mathbb{N}^*$). For (15) to be satisfied, we thus have to solve N^2 scalar polynomial equations.

Let $\mathbf{W}(z)$ and $\mathbf{M}(z)$ in (15) be explicitly written as Laurent polynomial matrices of the form:

$$\mathbf{W}(z) = \sum_{p=-P+1}^0 \mathbf{W}_p z^{-p} \quad (16)$$

$$\mathbf{M}(z) = \sum_{q=0}^{Q-1} \mathbf{M}_q z^{-q} \quad (17)$$

where, for all p (resp. q) \mathbf{W}_p (resp. \mathbf{M}_q) is an $N \times K$ (resp. $K \times N$) matrix. The global $N \times N$ transfer function in the left-hand side of (15) reads

$$\mathbf{G}(z) = \mathbf{W}(z)\mathbf{M}(z) = \sum_{s=-P+1}^{Q-1} \mathbf{G}_s z^{-s} \quad (18)$$

where, for all s ,

$$\mathbf{G}_s = \sum_{p=\max(-P+1, s-Q+1)}^{\min(0, s)} \mathbf{W}_p \mathbf{M}_{s-p}. \quad (19)$$

This shows that the solution $\mathbf{W}(z)$ of (15) is obtained by solving a system of $N^2(Q + P - 1)$ linear equations. On the other hand, the number of unknown variables in $\mathbf{W}(z)$ is NKP .

Our goal is to find an inverse $\mathbf{W}(z)$ which satisfies (15) and is optimal in a sense that will be made more precise in Section 3.2. Using (19), Relation (15) may be rewritten in the following matrix form:

$$\mathcal{M}\mathbf{W} = \mathcal{U} \quad (20)$$

where \mathbf{W} and \mathcal{U} are real matrices of sizes $KP \times N$ and $N(Q + P - 1) \times N$ respectively, which are given by

$$\mathbf{W}^T = [\mathbf{W}_{-P+1} \ \dots \ \mathbf{W}_0] \quad (21)$$

$$\mathcal{U}^T = \underbrace{[\mathbf{0}_{N \times N} \ \dots \ \mathbf{0}_{N \times N}]}_{P-1 \text{ times}} \mathbf{I}_{N \times N} \underbrace{[\mathbf{0}_{N \times N} \ \dots \ \mathbf{0}_{N \times N}]}_{Q-1 \text{ times}} \quad (22)$$

whereas \mathcal{M}^T is the $KP \times N(Q + P - 1)$ generalized Sylvester matrix:

$$\mathcal{M}^T = \begin{bmatrix} \mathbf{M}_0 & \mathbf{M}_1 & \dots & \mathbf{M}_{Q-1} & \mathbf{0}_{K \times N} & \dots & \mathbf{0}_{K \times N} \\ \mathbf{0}_{K \times N} & \mathbf{M}_0 & \mathbf{M}_1 & \dots & \mathbf{M}_{Q-1} & \mathbf{0}_{K \times N} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}_{K \times N} & \dots & \dots & \mathbf{0}_{K \times N} & \mathbf{M}_0 & \mathbf{M}_1 & \dots \end{bmatrix}$$

Eq. (20) has a solution when $\text{rank}([\mathcal{M} \ \mathcal{U}]) = \text{rank}(\mathcal{M})$. If $\mathbf{M}(z)$ can be inverted by a Laurent polynomial matrix, we can claim that there exists a minimal value of P for which this equality is reached.

3.2. Optimal Reconstruction

In terms of coding, it is of main importance to study the influence of the choice of \mathbf{W} on the effect of the quantization noise. Modelling the quantization as the addition of a noise on all sequences at the output of $\mathbf{M}(z)$, we aim at reducing as much as possible the influence of this noise on the reconstruction $(\mathbf{x}'_n)_{n \in \mathbb{Z}}$ of $(\mathbf{x}_n)_{n \in \mathbb{Z}}$. We assume in the sequel that the noise vector sequence $(\mathbf{b}_n)_{n \in \mathbb{Z}}$ is zero-mean, independent and identically distributed with non singular covariance matrix $\mathbf{\Lambda}$. We obviously have

$$\mathbf{x}'_n = \mathbf{x}_n + \mathbf{v}_n \quad (23)$$

where $(\mathbf{v}_n)_{n \in \mathbb{Z}}$ is the multivariate moving average process defined by $\mathbf{v}_n = \sum_p \mathbf{W}_p \cdot \mathbf{b}_{n-p}$. The autocovariance matrix for \mathbf{v}_n is

$$\mathbb{E}\{\mathbf{v}_n \mathbf{v}_n^T\} = \sum_p \mathbf{W}_p \mathbf{\Lambda} \mathbf{W}_p^T. \quad (24)$$

The global noise power on the components of \mathbf{x}'_n is $\mathbb{E}\{\|\mathbf{v}_n\|_2^2\}$. We deduce that the inverse system minimizing the effect of the quantization error is

$$\mathbf{W} = (\mathbf{\Lambda}')^{-1/2} (\mathcal{M}(\mathbf{\Lambda}')^{-1/2})^\# \mathcal{U} \quad (25)$$

where $\mathbf{\Lambda}'$ is the block-diagonal matrix of size $(PK) \times (PK)$ given by $\mathbf{\Lambda}' = \text{Diag}(\mathbf{\Lambda}, \dots, \mathbf{\Lambda})$ and $\mathbf{A}^\#$ designates the pseudo-inverse of a matrix \mathbf{A} . A particular case of interest is when the coefficients sequences are quantized with the same precision, which can be modelled by $\mathbf{\Lambda} = \sigma^2 \mathbf{I}_{K \times K}$. Then, the optimal choice reduces to

$$\mathbf{W} = \mathcal{M}^\# \mathcal{U}. \quad (26)$$

4. SIMULATIONS RESULTS

When comparing the four proposed MDC schemes in terms of global noise power on the reconstructed sequences, the R and D2 schemes appeared to provide lower performance than D1 and D3. Therefore, we only present comparisons concerning the two latter schemes.

We have implemented $J = 3$ levels of motion-compensated temporal lifting Haar decomposition [13], the last level consisting of one of the two analysed schemes D1 or D3. Note that in this way the overall redundancy of the structure is decreased to $1 + 2^{-J}$. The detail frames obtained at resolution levels $j < J$ have been alternately distributed between the two descriptions in an identical manner for the two schemes.

Concerning the side decoders, a pseudo-inverse approach was also applied to deduce the optimal reconstruction for each scheme.

The proposed schemes have been tested on several CIF sequences at 30fps. On the first two temporal decomposition levels a full pel motion compensation is involved in the lifting transform, while at the last level no motion estimation is performed. Temporal subband frames have been decomposed with 9/7 biorthogonal wavelets. The spatio-temporal wavelet coefficients and motion vectors have been coded as for the non robust codec, by using the MC-EZBC algorithm [14].

In Tab. 1 we compare the rate-distortion performance of the central and side decoders for the D1 and D3 schemes. Note that the central decoder of the D1 scheme outperforms the central decoder

of D3, as can be predicted from the theoretical framework. One of the side decoders (denoted by “A” in Tab. 1) is identical for the two schemes. However, due to an asymmetrical construction of the two descriptions in the D1 scheme, one of its side decoders (denoted by “B” in Tab. 1) exhibits a poorer performance.

“FOREMAN” D1 scheme						
bitrate	250	500	750	1000	1500	3000
central	29.48	32.19	33.85	34.98	36.85	40.53
side A	26.05	27.20	27.78	28.13	28.66	29.51
side B	24.32	24.84	25.06	25.16	25.29	25.43
“FOREMAN” D3 scheme						
bitrate	250	500	750	1000	1500	3000
central	29.27	32.01	33.68	34.79	36.68	40.39
side A	26.05	27.20	27.78	28.13	28.66	29.51
side B	25.26	26.16	26.62	26.88	27.28	27.96
“MOBILE” D1 scheme						
bitrate	250	500	750	1000	1500	3000
central	19.89	22.18	23.54	24.88	26.55	30.61
side A	18.96	20.15	20.78	21.24	21.91	23.07
side B	18.36	19.24	19.70	19.93	20.37	20.90
“MOBILE” D3 scheme						
bitrate	250	500	750	1000	1500	3000
central	19.70	21.95	23.33	24.61	26.33	30.43
side A	18.96	20.15	20.78	21.24	21.91	23.07
side B	18.81	19.83	20.35	20.72	21.28	22.23

Table 1. Rate-distortion comparison of the two schemes for the central and side decoders: YSNR (dB) at different bitrates (Kbs), for “FOREMAN” and “MOBILE” sequences (CIF at 30fps) on three levels of wavelet decomposition.

5. CONCLUSIONS AND FUTURE WORK

In this paper we have compared several temporal MDC schemes based on redundant wavelet decompositions with a reduced redundancy factor. We have proposed a general framework for analysing the perfect reconstruction of the schemes and optimal design of the central decoder. The studied schemes have been applied to robust scalable video coding and their merits have been compared. Further work is under investigation to improve the behaviour of the side decoders, by better taking into account the motion compensations involved in the temporal decompositions.

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