CALIBRATION, SENSITIVITY AND RFI MITIGATION REQUIREMENTS FOR LOFAR

A.J. Boonstra[1], *S.J. Wijnholds*[1], *S. van der Tol*[2], *and B. Jeffs*[2,3]

[1]ASTRON, R & D Dept, Oude Hoogeveensedijk 4, P.O. Box 2 7990 AA Dwingeloo, The Netherlands
[2]Delft University of Technology, Dept of El. Eng., Mekelweg 4, 2628 CD Delft, The Netherlands
[3]Brigham Young University, Dept of El. & Comp. Eng., 459 CB Provo, Utah 84602

ABSTRACT

In radio astronomy, cosmic sources are observed which are many orders of magnitude weaker than the telescope system noise level. The necessary sensitivity is achieved by large telescope collecting areas, long integration times, and large bandwidths. In the coming two decades, telescopes are planned which are even one to two orders of magnitude more sensitive than the current generation. Examples are the Low Frequency Array (LOFAR), currently under construction in the Netherlands, and the Square Kilometer Array, for which the envisaged start of construction is in 2012. For this next generation of telescopes a dynamic range in the sky maps of over 10^6 is required. In order to reach these numbers, accurate calibration is needed. As these telescopes will observe with relatively large bandwidths, and because of the changing spectrum environment, interference mitigation techniques become increasingly important. In this paper, approaches for calibration and interference mitigation are presented, and results from the LOFAR initial phased array test station (ITS) are given.

1. INTRODUCTION

The expected sensitivity increase of the next generation of telescopes, the new telescope design concepts [1], and the changing spectrum environment impose new challenges with respect to calibration and interference mitigation. Consider for example traditional techniques for calibration and imaging, such as Selfcal [2] and CLEAN [3]. These techniques usually assume that the complex telescope gains are direction independent, whereas in wide field imaging with instruments such as LOFAR, the complex gains must be modeled as being direction dependent [4]. An aim in imaging is that instrument calibration errors and interference do not lead to a significant increase of the noise in the astronomical sky maps. In order to study this requirement, a mathematical framework was adopted, as described in [5, 6]. This paper aims at describing the calibration and interference mitigation problems in terms of this framework, and to present a few initial results obtained at the LOFAR Initial Test Station (ITS).

Notation: The transpose operator is denoted by ^{t}, the complex conjugate (Hermitian) transpose by $^{\mathrm{H}}$, an estimated value is denoted by $\hat{\cdot}$, and an expected value by $\mathcal{E}\{.\}$, $\odot$ is the element-wise matrix multiplication (Hadamard product), $\oslash$ is the element-wise matrix division, $\mathbf{1}$ is a vector containing ones, and diag(.) converts a vector into a diagonal matrix with the vector placed on the main diagonal.

THIS WORK WAS SUPPORTED IN PART BY THE STW UNDER DTC.5893 (VICI-SPCOM) AND DTC.5478, AND THE TU DELFT RESEARCH CENTER ICT.

2. DATA MODEL

Following [6], consider a telescope or antenna array with p elements. Let the output signal of the i^{th} antenna be denoted by $x_i(t)$, and define the array output vector $\mathbf{x}(t)$ by $\mathbf{x}(t) = [x_1(t), \cdots, x_p(t)]^t$. Assume that the narrow band condition holds, which implies that geometrical delay differences within the array can be represented by a phase shift of the signal. Consider further q astronomical source signals $s_k(t)$ with $k = 1 \cdots q$. Denote the spatial signature vector of each of the sources k by $\mathbf{a}_k$, and consider telescope noise signals $n_i(t)$ stacked in a $p \times 1$ vector $\mathbf{n}(t)$. Further let the telescope dependent gain g_i be defined by $\mathbf{g} = [g_1, \cdots, g_p]^t$, and in diagonal matrix form by $\mathbf{\Gamma} = \mathrm{diag}(\mathbf{g})$. Using these definitions, the array output vector can be expressed as

$$\mathbf{x}(t) = \mathbf{\Gamma}\left(\sum_{k=1}^{q} \mathbf{a}_k s_k(t)\right) + \mathbf{n}(t) \quad (1)$$

Consider an observation in which the signal is sampled with sample period T, and define the data sample matrix $\mathbf{X} = [\mathbf{x}(T), \mathbf{x}(2T), \cdots, \mathbf{x}(NT)]$. The (short term) covariance estimate $\widehat{\mathbf{R}}$ takes the form $\widehat{\mathbf{R}} = N^{-1}\mathbf{X}\mathbf{X}^H$. Define the source power by $\sigma_{s_k}^2 = \mathcal{E}\{|s_k(t)|^2\}$, and stack the source signals $s_k(t)$ in a $q \times 1$ vector $\mathbf{s}(t)$. Assuming the sources are mutually independent, the source signal covariance $\mathbf{\Sigma}_s = \mathcal{E}\{\mathbf{s}(t)\mathbf{s}(t)^H\}$ is diagonal: $\mathbf{\Sigma}_s = \mathrm{diag}(\boldsymbol{\sigma}_s)$, where $\boldsymbol{\sigma}_s = [\sigma_{s_1}^2, \cdots, \sigma_{s_q}^2]^t$. Assuming the noise $\mathbf{n}(t)$ is Gaussian and independent also results in a diagonal covariance matrix: $\boldsymbol{\Sigma}_n = \mathrm{diag}(\boldsymbol{\sigma}_n)$, where $\boldsymbol{\sigma}_n = [\sigma_{n_1}^2, \cdots, \sigma_{n_p}^2]^t$, and $\sigma_{n_i}^2 = \mathcal{E}\{|n_i(t)|^2\}$. Assuming the spatial signature vectors are deterministic, and are stacked in a $p \times q$ matrix $\mathbf{A}$, the expected value $\mathbf{R} = \mathcal{E}\{\widehat{\mathbf{R}}\}$ has model

$$\mathbf{R} = \boldsymbol{\Gamma}\mathbf{A}\boldsymbol{\Sigma}_s\mathbf{A}^H\boldsymbol{\Gamma}^H + \boldsymbol{\Sigma}_n \tag{2}$$

3. CALIBRATION APPROACHES

One of the possible large scale telescopes configurations, such as used in LOFAR, consists of many antenna elements grouped in stations. These antenna elements operate as a phased array, and (multiple) station beams can be formed. The beams from the stations are correlated, integrated and stored. From these covariance matrices, the astronomical sky images can be deduced. In the following two sections, the calibration problem is stated first from a station perspective and then from a full array calibration perspective.

The antenna gains of the individual antenna elements of a station are direction dependent but assumed known. In the station calibration problem, this direction dependency can be absorbed in the known sky source fluxes matrix $\boldsymbol{\Sigma}_s$, and the data model described above can be used. For the full telescope calibration problem, the data model above can be used as well, with x_i being the beamformed station output. In this case, the data model will be extended with a direction dependent gain matrix $\mathbf{G}$.

3.1. Station calibration

If the geometrical phase factors $\mathbf{A}$ of the sources and their powers $\sigma_{s_k}^2$ are known from the telescope geometry and astronomical catalogues, the calibration problem can be formulated in terms of a least squares cost function:

$$\{\hat{\mathbf{g}}, \hat{\boldsymbol{\sigma}}_n\} = \arg\min_{\mathbf{g},\boldsymbol{\sigma}_n} \| \boldsymbol{\Gamma}\mathbf{A}\boldsymbol{\Sigma}_s\mathbf{A}^H\boldsymbol{\Gamma}^H + \boldsymbol{\Sigma}_n - \widehat{\mathbf{R}} \|^2 . \tag{3}$$

This cost function is similar to the cost function to be minimised in Selfcal and CLEAN.

A first approach is assuming $\boldsymbol{\Sigma}_n = \mathbf{0}$, and a solution of the calibration problem would be

$$\widehat{\mathbf{g}\mathbf{g}^H} = \widehat{\mathbf{R}} \oslash \left(\mathbf{A}\boldsymbol{\Sigma}_s\mathbf{A}^H\right) \tag{4}$$

from which $\widehat{\mathbf{g}}$ can be extracted by eigenvalue decomposition of $\widehat{\mathbf{g}\mathbf{g}^H}$. Note that this is not the Least Squares solution, and that in the (rare) case an entry of $\mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^H$ is zero, the estimation accuracy is reduced. Calibrations using celestial sources have been successfully completed on ITS [7] due to the fact that it is a sky noise limited system so the approximation $\boldsymbol{\Sigma}_n = \mathbf{0}$ holds.

In [8] it was shown that it is possible to derive a matrix $\mathbf{M}$ with entries $m_{ij} = g_i/g_j$ based on the off-diagonal elements of $\widehat{\mathbf{R}}$ and $\mathbf{A}\boldsymbol{\Sigma}_s\mathbf{A}^{\mathrm{H}}$ which are not affected by the diagonal matrix $\boldsymbol{\Sigma}_n$. Once the gains are estimated, they can be inserted in (3) which can then be used to solve for the system noises of the individual receivers.

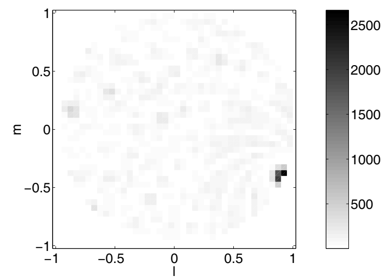

Fig. 1. Full sky image, based on observed correlation matrices for a 9.77 kHz wide frequency channel at 18.916 MHz, from a 6.7 s duration LOFAR-ITS observation. The image is completely dominated by the transmitter source on the south-eastern horizon. The image coordinates are direction cosines (l,m).

A third approach to station calibration is based on using a strong transmitter source. In [7] it is demonstrated that the system noise of the elements of LOFAR-ITS is just a fraction of the sky noise. Since the sky noise is already negligible compared to the power of the transmitter source, the receiver noise can be neglected as well. Assuming the transmitter and interfering sources can be modeled in the same way as astronomical sources, the model becomes

$$\mathbf{R} = \boldsymbol{\Gamma}\mathbf{a_r}\sigma_r^2\mathbf{a_r}^H\boldsymbol{\Gamma}^H, \tag{5}$$

where $\mathbf{a}_r$ and σ_r^2 denote the phase factors and the power of the transmitter. Since the system noise of the elements is negligible small, it does not need to be estimated. Therefore the calibration problem reduces to

$$\{\hat{\mathbf{g}}\} = \arg\min_{\mathbf{g}} \| \boldsymbol{\Gamma}\mathbf{a_r}\sigma_r^2\mathbf{a_r}^{\mathrm{H}}\boldsymbol{\Gamma}^{\mathrm{H}} - \widehat{\mathbf{R}} \|^2 \tag{6}$$

As in (4), a solution is $\widehat{\mathbf{g}\mathbf{g}^H} = \widehat{\mathbf{R}} \oslash \left(\mathbf{a}_r\sigma_r^2\mathbf{a}_r^H\right)$. The gain vector estimate $\widehat{\mathbf{g}}$ is now the eigenvector corresponding to the largest eigenvalue of $\widehat{\mathbf{g}\mathbf{g}^H}$.

This method was used to calibrate the data imaged in figure 1 by estimating the position of the source from the data to set $\mathbf{a}$ and taking a source power of 1, since the intrinsic power of the source is not known. After applying the corrections found in the calibration, a second calibration was done to estimate the gains after calibration. The phase of the complex gains before and after calibration are shown in figure 2. The standard deviation of the phases was $26.5°$ before calibration and $0.0011°$ after calibration. The errors after calibration are the combined result from the finite sample effect and ignoring the sky and system noise

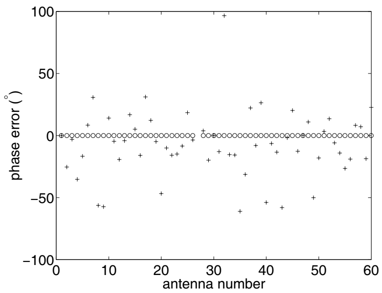

Fig. 2. Phase errors of the individual signal paths before (+) and after (o) calibration on the transmitter source.

contributions. The fact that $\mathbf{a}_r$ is estimated based on the uncalibrated data may introduce a systematic pointing offset in (l, m) coordinates, but the relative positions of the celestial sources remain the same.

3.2. LOFAR Full Array Calibration

The new generation of large array instruments such as SKA and LOFAR present calibration problems significantly more challenging than previously encountered. Existing algorithms exploit an assumption that the unknown array response terms can be modeled with a single complex gain for each array element, plus possibly an unknown phase gradient across the field of view. Using known array element locations and exact power levels and directions to bright point source objects (from a star catalog) provides the required external information to make calibration unambiguous.

However, at the lower frequencies observed by LOFAR, signals interact strongly with the ionosphere so calibration parameters are source direction dependent and a calibration solution must be found for every observed object. The single-complex-gain-term-per-element assumption no longer holds. Once a LOFAR station is calibrated for each antenna (as discussed above) we consider the steered station beam as a single element in the larger full array and must estimate a complex gain for each object in each station beam. Figure 3 illustrates how space objects are seen through a random refractive layer that varies in thickness on the same scale as distances between sources of interest in the image field. Over the large (200 km) aperture, most stations see completely independent ionospheric patches. The expected value of $\widehat{\mathbf{R}}$ given in (2) now becomes

$$\mathbf{R} = (\mathbf{G} \odot \mathbf{A})\mathbf{\Sigma}_s(\mathbf{G} \odot \mathbf{A})^{\mathrm{H}} + \mathbf{\Sigma}_n, \qquad (7)$$

where $\mathbf{G}$ is a $p \times q$ matrix containing a complex gain factor per antenna and per source. $\mathbf{A}$ depends on the known positions for the q calibrator sources in the field of view, $\mathbf{\Sigma}_s$ depends on the known source brightness, and $\mathbf{G}$ and $\mathbf{\Sigma}_n$ must be estimated.

If the number of sources is greater than one there are multiple solutions to (7) so additional physical models must be imposed to enforce a unique solution. Using these constraints will require developing new and innovative calibration algorithm strategies. We have identified four ap-

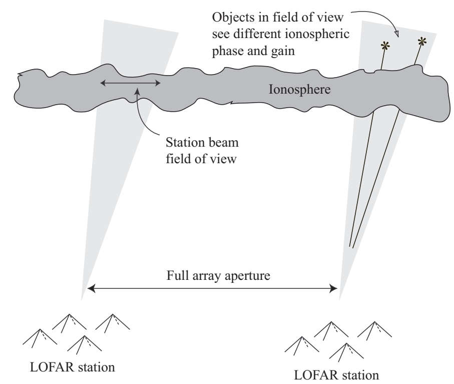

Fig. 3. The problem of LOFAR calibration through ionospheric refraction. Unknown complex gains through the ionosphere are different for each source at each station. (after C. Lonsdale)

proaches to resolving this calibration ambiguity.

First, the geometry of LOFAR is designed such that the central core consists of tightly packed stations which see a common ionosphere as shown in figure 4. It can be shown that if the number of stations in this core subarray exceeds the number of bright calibrator sources then the full array, including distant stations as illustrated in figure 3, can be uniquely calibrated.

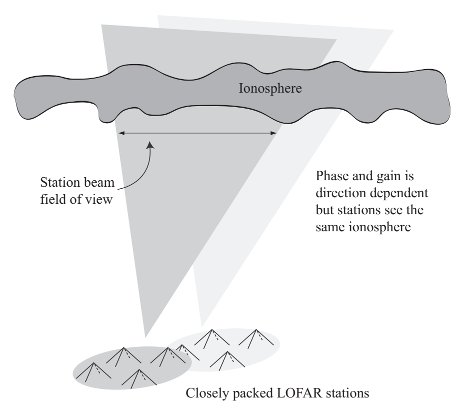

Fig. 4. Calibration scenario for closely spaced LOFAR central core stations. Due to beam overlap at ionospheric altitude, each station sees the same direction dependence which is cancelled out in the cross correlation computation. (after C. Lonsdale)

Second, in most observing conditions the predominant ionospheric effect is a phase rotation with little attenuation.

The unknown gain term is mostly due to antenna (direction dependent) and electronics (direction independent) which are relatively stable. Assuming the amplitude gain matrix, $\mathbf{\Gamma}$, has been estimated separately, the remaining problem is to find the phases. This problem can be formulated as follows.

$$\mathbf{R} = (\mathbf{\Phi} \odot \mathbf{\Gamma} \odot \mathbf{A})\mathbf{\Sigma}_s(\mathbf{\Phi} \odot \mathbf{\Gamma} \odot \mathbf{A})^{\mathrm{H}} + \mathbf{\Sigma}_n, \quad (8)$$

where $\mathbf{G} = \mathbf{\Phi} \odot \mathbf{\Gamma}$ and the entries of $\mathbf{\Phi}$ are all unit-magnitude with phases to be estimated. It can been shown that in some cases multiple solutions exist, but that this is of little practical concern since these cases are rare.

The third approach dubbed "peeling" exploits the fact that due to Earth rotation the relative positions of the array and the sources change with time [4]. Over a limited time and frequency span the ionospheric gains can be treated as constant while the variation in $\mathbf{A}$ adds enough diversity to find a unique solution. A straightforward way to exploit this diversity is to take the average over a time-frequency domain, while compensating for the phase changes of the k^{th} single source (the brightest). The resulting matrix is:

$$\begin{aligned} \mathcal{R}_k &= \frac{1}{N_f N_t} \sum_{l=1}^{N_f} \sum_{m=1}^{N_t} \mathrm{diag}(\mathbf{a}_k)^{-1}\mathbf{R}(f_l, t_m)\mathrm{diag}(\mathbf{a}_k^{\mathrm{H}})^{-1} \\ &\approx \mathbf{\Gamma}_k \mathbf{1} \sigma_k^2 \mathbf{1}^t \mathbf{\Gamma}_k^H + \mathbf{\Sigma}_n \end{aligned} \quad (9)$$

This compares to the unambiguous single source calibration problem. Gains can be computed for this source, then its contribution to the $\mathbf{R}(f_l, t_m)$ is subtracted out, and the process is repeated for the next brightest source.

A fourth approach exploits the fact that the complex ionospheric gain terms depend in a known deterministic way with frequency (phase perturbation is proportional to wavelength). It can be shown that jointly estimating calibration gains for more frequency bins than the number of calibrator sources leads to an unambiguous solution.

4. INTERFERENCE MITIGATION

In the station calibration section it was shown that the phase errors after calibration on a transmitter source are very small. This demonstrates that the transmitter is calibrated very accurately to the estimated position. The interfering source therefore can be subtracted from covariance matrix data yielding the filtered matrix $\tilde{\mathbf{R}}$

$$\tilde{\mathbf{R}} = \widehat{\mathbf{R}} - \hat{\sigma_r}^2 \hat{\mathbf{a}}_r \hat{\mathbf{a}}_r{}^H \quad (10)$$

A sky map based on the filtered matrix $\tilde{\mathbf{R}}$ is shown in figure 5 [7]. Several other spatial filtering techniques were applied as well, such as described in [6]. For example, a projection matrix $\mathbf{P}$ can be defined: $\mathbf{P} = \mathbf{I} - \mathbf{a}_r(\mathbf{a}_r^H \mathbf{a})^{-1}\mathbf{a}_r^H$. It has the property $\mathbf{P}\mathbf{a}_r = 0$, yielding the filtered matrix $\tilde{\mathbf{R}} = \mathbf{P}\widehat{\mathbf{R}}\mathbf{P}$. Applying this technique yielded similar results. Interference attenuation numbers up to 30 dB were achieved, and are limited by finite estimation accuracies.

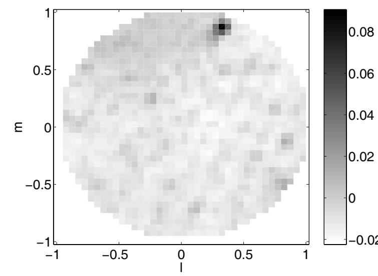

Fig. 5. Observed sky map after removing the horizon transmitter by source subtraction. The upper right source is the astronomical source Cas.A.

5. REFERENCES

[1] J. D. Bregman, "Concept design for a low-frequency array," in *Proc. SPIE Vol. 4015, p. 19-32, Radio Telescopes, Harvey R. Butcher; Ed.*, July 2000, pp. 19–32.

[2] T. J. Pearson and A. C. S. Readhead, "Image Formation by Self-Calibration in Radio Astronomy," *Annual Rev. Astronomy and Astrophysics*, vol. 22, pp. 97–130, 1984.

[3] J.A. Hogbom, "Aperture synthesis with a non-regular distribution of interferometer baselines," *Astron. Astrophys. Suppl.*, vol. 15, pp. 417–426, 1974.

[4] J. E. Noordam, "LOFAR calibration challenges," in *Proceedings of the SPIE, Vol. 5489, pp. 817-825 (2004).*, Oct. 2004, pp. 817–825.

[5] J.P Hamaker, "Understanding radio polarimetry. IV. the full-coherency analogue of scalar self-calibration: Self-alignment, dynamic range and polarimetric fidelity," *Astron. and Astrophys. Suppl. Ser.*, vol. 143, pp. 515–534, May 2000.

[6] A. Leshem, A.J. van der Veen, and A.J. Boonstra, "Multichannel interference mitigation techniques in radio astronomy," *The Astrophysical Journal Supplement Series*, vol. 131, no. 1, pp. 355–373, November 2000.

[7] S.J. Wijnholds, J.D. Bregman, and A.J. Boonstra, "Sky noise limited snapshot imaging in the presence of RFI with LOFAR's Initial Test Station," *Exp. Astr.*, accepted for publication.

[8] A.J. Boonstra and A.J. van der Veen, "Gain calibration methods for radio telescope arrays," *IEEE trans. sig. proc.*, vol. 51, no. 1, pp. 25–38, Jan 2003.