LINEAR RECEIVERS FOR MULTIUSER MIMO CHANNELS

Ezio Biglieri *Alessandro Nordio* *Giorgio Taricco*

Politecnico di Torino, Italy • email: `<lastname>@polito.it`

ABSTRACT

We consider the use of multiple transmit and receive antennas (MIMO) in a wideband code-division multiple-access system with space–time coding. Using large-system asymptotic analyses, we examine the performance of linear receivers whereby multiuser detection and MIMO decoding are kept separate.

1. INTRODUCTION

The application of multiple antennas to multiple-access systems has been recently advocated (see, e.g., [3] and references therein). In this paper, we examine the performance of linear receivers in the uplink of a cellular system using wideband code-division multiple-access (CDMA) and multiple transmit and receive antennas with space–time coding. Specifically, we assume that a multiple-antenna subsystem is added to an already existing multiple-access system, so that the received signal is processed by two separate interfaces, one mitigating or suppressing the spatial interference and the other mitigating or suppressing the multiple-access interference. The performance of some among the possible two-stage receivers is compared to that of the single-stage receiver, designed to cope with both interferences at the same time, but requiring the whole receiver to be redesigned. We use large-system analyses, based on the assumptions that the spreading sequences are random and independent across transmit antennas, that the received signal is symbol-synchronous across users, that the receiver (the base station) has perfect channel-state information, and that the number of antennas as well as the number of users grow to infinity. Comparisons are based on the pairwise error probabilities (PEP) of the receivers, and, in particular, on the power loss caused by suboptimum processing of the received signal.

2. CHANNEL MODEL

In our channel model, K (mobile) users transmit to a single (base station) receiver. Each user transmits over t_k ($k = 1, \ldots, K$) antennas, and the receiver is provided with r antennas. The total number of transmit antennas is $t \triangleq \sum_{k=1}^{K} t_k$, and we define the average number of transmit antennas $\bar{t} \triangleq t/K$. To allow multiple access, direct-sequence CDMA is used, and all transmit antennas are assigned different spreading sequences, whose common length is S. The spreading sequence of user k and antenna $i = 1, \ldots, t_k$ is denoted by the row-vector $\mathbf{s}_{k,i} = (s_{k,i1}, \ldots, s_{k,iS}) \in \mathbb{C}^S$. Assume also that user k transmits the symbol $x_{k,i}$ on antenna $i = 1, \ldots, t_k$, so that the transmitted signal is $x_{k,i}\mathbf{s}_{k,i}$. Transmitted symbols have zero mean and variance $E_{s,k} = \mathbb{E}[|x_{k,i}|^2]$. The communication channel of user k is described by the gains from transmit antenna $j = 1, \ldots, t_k$ to receive antenna $i = 1, \ldots, r$, collected in the $r \times t_k$ channel matrices $\mathbf{H}_k$. The random channel matrices have i.i.d. entries which are zero-mean circularly symmetric complex Gaussian (ZMCSCG) random variables with variance $1/r$. Their distribution is denoted by $\mathcal{N}_c(0, 1/r)$. The receiver is affected by additive Gaussian noise represented by the $r \times S$ matrix $\mathbf{Z}$ with i.i.d. entries distributed as $\mathcal{N}_c(0, N_0)$. As a result of above assumptions, the channel model over one symbol interval is described by the following matrix equation:

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{X}_k \mathbf{S}_k + \mathbf{Z} = \mathbf{HXS} + \mathbf{Z} \,. \tag{1}$$

Here we assume that

- ▷ $\mathbf{H} = (\mathbf{H}_1, \ldots, \mathbf{H}_K) \triangleq (H_{ij})_{i,j=1}^{r,t} \in \mathbb{C}^{r \times t}$;
- ▷ $\mathbf{X}_k = \mathrm{diag}(x_{k,i})_{i=1}^{t_k} \in \mathbb{C}^{t_k \times t_k}$;
- ▷ $\mathbf{X} = \mathrm{diag}(\mathbf{X}_1, \ldots, \mathbf{X}_K) \triangleq \mathrm{diag}(x_1, \ldots, x_t) \in \mathbb{C}^{t \times t}$;
- ▷ $\mathbf{S}_k = (\mathbf{s}_{k,1}^T, \ldots, \mathbf{s}_{k,t_k}^T)^T \in \mathbb{C}^{t_k \times S}$;
- ▷ $\mathbf{S} = (\mathbf{S}_1^T, \ldots, \mathbf{S}_K^T)^T \triangleq (s_{i,j})_{i,j=1}^{t,S} \in \mathbb{C}^{t \times S}$.

Eq. (1) can be expanded in the form

$$Y_{ij} = \sum_{\ell=1}^{t} H_{i\ell} s_{\ell j} \;\, x_\ell + Z_{ij} \,.$$

which shows that (1) can be rewritten so as to have a single channel matrix $\widetilde{\mathbf{H}} \in \mathbb{C}^{rS \times t}$ encompassing the effects of MIMO channel and spreading sequences. The resulting channel equation is

$$\mathbf{y} = \widetilde{\mathbf{H}}\mathbf{x} + \mathbf{z} \,, \tag{2}$$

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where, if $\mathrm{vec}(\mathbf{A})$ denotes the vector obtained by stacking the columns of $\mathbf{A}$ on top of each other, we have defined $\mathbf{x} \triangleq \mathrm{vec}(\mathbf{X})$, $\mathbf{y} \triangleq \mathrm{vec}(\mathbf{Y})$, $\mathbf{z} \triangleq \mathrm{vec}(\mathbf{Z})$, and $\widetilde{H}_{i+(j-1)r,\ell} = H_{i\ell} s_{\ell j}$ for $i = 1, \ldots, r$, $j = 1, \ldots, S$, and $\ell = 1, \ldots, t$.

In the following we assume that the spreading sequences are randomly generated with i.i.d. complex entries having constant magnitude and random phase uniformly distributed over $(0, 2\pi)$. Thus, we have $\mathbb{E}[\mathbf{S}\mathbf{S}^\dagger] = \mathbf{I}_t$, where $(\cdot)^\dagger$ denotes Hermitian conjugation.

The structure of channel equations (1) and (2) suggests that two different types of linear receivers can be designed:

1. A *separate* (two-stage) receiver based on (1), and aimed at mitigating separately the effect of $\mathbf{H}$ and $\mathbf{S}$ on the transmitted signal (spatial and multiple-access interference, respectively).

2. A *joint* receiver based on (2), and aimed at mitigating the effect of $\widetilde{\mathbf{H}}$ on the transmitted signal, i.e., the joint effect of spatial and multiple-access interference.

3. SEPARATE VS. JOINT RECEIVERS

3.1. Separate receivers

The single-user matched filter (SUMF) is based on the assumption that both spatial and multiple-access interference can be modeled as independent Gaussian noise. Thus, by rewriting the channel equation (1) as

$$\mathbf{Y} = \mathbf{H}_1\mathbf{X}_1\mathbf{S}_1 + \sum_{k=2}^{K} \mathbf{H}_k\mathbf{X}_k\mathbf{S}_k + \mathbf{Z}$$

and setting user 1 as the reference user, the SUMF receiver implements the transformation

$$\mathbf{Y} \mapsto \widehat{\mathbf{Y}} \triangleq \mathbf{H}_1^\dagger \mathbf{Y} \mathbf{S}_1^\dagger \approx \mathbf{X}_1 \, .$$

A general linear separate multiuser receiver can also be defined. It operates in two stages. First, it mitigates the spatial interference by processing linearly $\mathbf{Y}$ to obtain $\widetilde{\mathbf{Y}} \approx \mathbf{X}\mathbf{S}$. Next, it mitigates the multiple-access interference by processing linearly $\widetilde{\mathbf{Y}}$ to obtain $\widehat{\mathbf{Y}} \approx \mathbf{X}$. The latter matrix is fed to the decoder.

Specifically, we may have the following interfaces:

1. Decorrelator (or zero-forcing, ZF): [1]

$$\mathbf{Y} \mapsto \widetilde{\mathbf{Y}} = \mathbf{H}^+\mathbf{Y} = (\mathbf{H}^\dagger\mathbf{H})^{-1}\mathbf{H}^\dagger\mathbf{Y}.$$

2. Linear minimum mean-square error (LMMSE) filter:

$$\mathbf{Y} \mapsto \widetilde{\mathbf{Y}} = \mathbf{F}\mathbf{Y}$$

where $\mathbf{F}$ is chosen in order to minimize the mean-square error (MSE) of the received signal after whitened matched filtering, namely,

$$\mathbb{E}[\|(\widetilde{\mathbf{Y}} - \mathbf{X}\mathbf{S})\mathbf{S}^\dagger(\mathbf{S}\mathbf{S}^\dagger)^{-1/2}\|^2 \mid \mathbf{H}] \, . \qquad (3)$$

We have two possible types of linear separate multiuser receivers: ZF and LMMSE. Their description can be summarized as follows.

$$\begin{array}{rcl} \mathbf{Y} & \mapsto & \widetilde{\mathbf{Y}} = \mathbf{F}_h\mathbf{Y} \approx \mathbf{X}\mathbf{S} \\ \widetilde{\mathbf{Y}} & \mapsto & \widehat{\mathbf{Y}} = \widetilde{\mathbf{Y}}\mathbf{S}^+ \approx \mathbf{X} \, , \end{array} \qquad (4)$$

where

$$\mathbf{F}_h = \begin{cases} (\mathbf{H}^\dagger\mathbf{H})^{-1}\mathbf{H}^\dagger & \text{(ZF)} \\ (\mathbf{H}^\dagger\mathbf{H} + \delta_s\mathbf{I}_t)^{-1}\mathbf{H}^\dagger & \text{(LMMSE)} \end{cases} \qquad (5)$$

and $\delta_s \triangleq N_0/E_s$.

3.2. Joint receivers

The joint receiver is based on (2). Focusing on linear receivers, we can represent its operation by the map

$$\mathbf{Y} \mapsto \widetilde{\mathbf{Y}} = \mathbf{F}_j\mathbf{Y} \, .$$

Here we have two receiver structures defined by matrix $\mathbf{F}_j$:

$$\mathbf{F}_j = \begin{cases} (\widetilde{\mathbf{H}}^\dagger\widetilde{\mathbf{H}})^{-1}\widetilde{\mathbf{H}}^\dagger & \text{(ZF)} \\ (\widetilde{\mathbf{H}}^\dagger\widetilde{\mathbf{H}} + \delta_s\mathbf{I}_t)^{-1}\widetilde{\mathbf{H}}^\dagger & \text{(LMMSE)} \end{cases} \, . \qquad (6)$$

4. RECEIVER PERFORMANCE

We study the performance of the linear separate receivers by calculating the corresponding pairwise error probabilities (PEPs). We take as a reference the performance of the maximum-likelihood (ML) receiver with metric $\|\mathbf{Y} - \widetilde{\mathbf{H}}\mathbf{X}\|^2$, whose PEP is given by

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}}) = Q\left(\frac{\|\boldsymbol{\Delta}\|}{\sqrt{2\,N_0}}\right)$$

where $\boldsymbol{\Delta} \triangleq \mathbf{X} - \widehat{\mathbf{X}}$ [2].

4.1. Separate receivers: SUMF

In this case, the receiver metric for the reference user is

$$\mu(\mathbf{X}_1) \triangleq \|\mathbf{H}_1^\dagger\mathbf{Y}\mathbf{S}_1^\dagger - \mathbf{X}_1\|^2 \, . \qquad (7)$$

For notational convenience, we set

$$\begin{array}{rcl} \mathbf{J}_1 & \triangleq & \displaystyle\sum_{k=2}^{K} \mathbf{H}_k\mathbf{X}_k\mathbf{S}_k \, , \\ \mathbf{Z}_1 & \triangleq & \mathbf{J}_1 + \mathbf{Z} \, . \end{array}$$

[1] $(\cdot)^+$ denotes the Moore-Penrose left or right pseudo-inverse.

Hence, conditionally on $\mathbf{H}_1$ and $\mathbf{S}_1$, after some algebra the PEP can be given the form[2]

$$\mathbb{P}(\mathbf{X}_1 \to \widehat{\mathbf{X}}_1 \mid \mathbf{H}_1, \mathbf{S}_1) = Q\left(\frac{\begin{array}{c}\|\boldsymbol{\Delta}_1\|^2 + 2\,(\mathbf{H}_1^\dagger \mathbf{H}_1 \mathbf{X}_1 \mathbf{S}_1 \mathbf{S}_1^\dagger - \mathbf{X}_1, \boldsymbol{\Delta}_1) \\ +2\,(\mathbf{H}_1 \boldsymbol{\Delta}_1 \mathbf{S}_1, \mathbf{J}_1)\end{array}}{\sqrt{2\,N_0\,\|\mathbf{H}_1 \boldsymbol{\Delta}_1 \mathbf{S}_1\|^2}} \right) \quad (8)$$

where $\boldsymbol{\Delta}_1 \triangleq \mathbf{X}_1 - \widehat{\mathbf{X}}_1$.

4.2. Multiuser separate receivers

In this case the receiver metric is

$$\mu(\mathbf{X}) \triangleq \|\mathbf{F}_h \mathbf{Y} \mathbf{S}^+ - \mathbf{X}\|^2 \quad (9)$$

where $\mathbf{F}_h$ and $\mathbf{S}^+$ are functions of $\mathbf{H}$ and $\mathbf{S}$ only. Hence, conditionally on the matrices $\mathbf{H}$ and $\mathbf{S}$, the PEP is given by

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}} \mid \mathbf{H}, \mathbf{S}) = Q\left(\frac{\|\boldsymbol{\Delta}\|^2 + 2\,(\mathbf{F}_h \mathbf{H} \mathbf{X} - \mathbf{X}, \boldsymbol{\Delta})}{\sqrt{2\,N_0\,\|\mathbf{F}_h^\dagger \boldsymbol{\Delta} (\mathbf{S}^+)^\dagger\|^2}} \right) \quad (10)$$

where, again, $\boldsymbol{\Delta} \triangleq \mathbf{X} - \widehat{\mathbf{X}}$. For the ZF receiver:

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}} \mid \mathbf{H}, \mathbf{S}) = Q\left(\frac{\|\boldsymbol{\Delta}\|^2}{\sqrt{2\,N_0\,\mathsf{Tr}[(\mathbf{H}^\dagger\mathbf{H})^{-1} \boldsymbol{\Delta} (\mathbf{S}\mathbf{S}^\dagger)^{-1} \boldsymbol{\Delta}^\dagger]}} \right) \quad (11)$$

4.3. Joint receivers

We calculate the PEP by assuming the following receiver metric:

$$\mu(\mathbf{x}) \triangleq \|\mathbf{F}_j \mathbf{y} - \mathbf{x}\|^2 \quad (12)$$

where $\mathbf{F}_j$ is a function of $\widetilde{\mathbf{H}}$. Conditionally on $\widetilde{\mathbf{H}}$, we have

$$\mathbb{P}(\mathbf{x} \to \widehat{\mathbf{x}} \mid \widetilde{\mathbf{H}}) = Q\left(\frac{\|\boldsymbol{\delta}\|^2 + 2\,((\mathbf{F}_j \widetilde{\mathbf{H}} - \mathbf{I}_t)\mathbf{x}, \boldsymbol{\delta})}{\sqrt{2\,N_0\,\|\mathbf{F}_j^\dagger \boldsymbol{\delta}\|^2}} \right) \quad (13)$$

where $\boldsymbol{\delta} \triangleq \mathbf{x} - \widehat{\mathbf{x}}$.

5. ASYMPTOTICS

5.1. $t < \infty$ and $r, S \to \infty$

We first assume that $t < \infty$ and $r, S \to \infty$, and we consider the cases of the separate and joint receiver.

[2] We denote by $(\mathbf{A}, \mathbf{B}) \triangleq \mathrm{Re}\mathsf{Tr}(\mathbf{A}\mathbf{B}^\dagger)$ the inner matrix product. Notice that $(\mathbf{A}, \mathbf{A}) = \|\mathbf{A}\|^2$.

5.1.1. Single-user matched filter

In this case, we examine the asymptotic behavior of the PEP described in eq. (8). Since $t_1 \le t < \infty$, we have

$$\mathbf{H}_1^\dagger \mathbf{H}_k \to \delta_{1k} \mathbf{I}_{t_1} \quad \text{and} \quad \mathbf{S}_1 \mathbf{S}_1^\dagger \to \mathbf{I}_{t_1} ,$$

where $\delta_{1k} = 1$ for $k = 1$ and 0 otherwise. Therefore,

$$\mathbb{P}(\mathbf{X}_1 \to \widehat{\mathbf{X}}_1 \mid \mathbf{H}_1, \mathbf{S}_1) \to Q\left(\frac{\|\boldsymbol{\Delta}_1\|}{\sqrt{2\,N_0}} \right) \quad (14)$$

which shows that the PEP depends only on the code of user 1.

5.1.2. Multiuser separate receivers

In this case, we examine the asymptotic behavior of the PEP described in eq. (10). Since $t < \infty$, we have

$$\mathbf{H}^\dagger \mathbf{H} \to \mathbf{I}_t \quad \text{and} \quad \mathbf{S}\mathbf{S}^\dagger \to \mathbf{I}_t .$$

Then, from eq. (11), we have for the ZF receiver:

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}}) \to Q\left(\frac{\|\boldsymbol{\Delta}\|}{\sqrt{2\,N_0}} \right) .$$

For the LMMSE receiver, since $\mathbf{F}_h = (\mathbf{H}^\dagger \mathbf{H} + \delta_s \mathbf{I}_t)^{-1} \mathbf{H}^\dagger$, we have $\mathbf{F}_h \mathbf{H} \to (1 + \delta_s)^{-1} \mathbf{I}_t$ and $\mathbf{F}_h \mathbf{F}_h^\dagger \to (1 + \delta_s)^{-2} \mathbf{I}_t$. Thus, we obtain from (10), under the assumption of constant-energy code words (so that $\|\widehat{\mathbf{X}}\| = \|\mathbf{X}\|$):

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}}) \to Q\left(\frac{\|\boldsymbol{\Delta}\|}{\sqrt{2\,N_0}} \right)$$

for the LMMSE separate multiuser receiver. Its performance depends on all users' codes.

5.1.3. Joint receiver

In this case we can see that $\widetilde{\mathbf{H}}^\dagger \widetilde{\mathbf{H}} \to \mathbf{I}_t$ and $\mathbf{S}\mathbf{S}^\dagger \to \mathbf{I}_t$. Thus,

$$\mathbf{F}_j \to \begin{cases} \widetilde{\mathbf{H}}^\dagger & \text{(ZF)} \\ (1 + \delta_s)^{-1} \widetilde{\mathbf{H}}^\dagger & \text{(LMMSE)} \end{cases}$$

As a result, we have, assuming again constant-energy code words:

$$\mathbb{P}(\mathbf{x} \to \widehat{\mathbf{x}}) \to Q\left(\frac{\|\boldsymbol{\Delta}\|}{\sqrt{2\,N_0}} \right) \qquad (\mathrm{ZF}, \mathrm{MMSE}) \quad (15)$$

We see that, under these asymptotic assumptions, no loss is incurred by two-stage receivers.

5.2. $t, r, S \to \infty$

In this section we consider the case when the system parameters t_i $(i = 1, \dots, K)$, r, and S grow to infinity, while the ratios t_i/r and t_i/S approach finite constants. More precisely, we assume that t_i $(i = 1, \dots, K), r, S \to \infty$ while $t_i/r \to \alpha_i < 1$ and $t_i/S \to \beta_i < 1$. For convenience, we denote $\alpha \triangleq \sum_i \alpha_i$ and $\beta \triangleq \sum_i \beta_i$. Our analysis is based on Free Probability Theory (see, e.g., [2]).

5.2.1. *Single-user matched filter*

The numerator and denominator of (10) converge to deterministic limits that can be derived from the following results:[3] $t_1^{-1}(\mathbf{H}_k\mathbf{X}_k\mathbf{S}_k, \mathbf{H}_1\mathbf{\Delta}_1\mathbf{S}_1) \to \delta_{1k}t_1^{-1}(\mathbf{X}_k, \mathbf{\Delta}_1)$ and $t_1^{-1}\|\mathbf{H}_1\mathbf{\Delta}_1\mathbf{S}_1\|^2 \to t_1^{-1}\|\mathbf{\Delta}_1\|^2$. Thus, the asymptotic PEP is

$$\mathbb{P}(\mathbf{X}_1 \to \widehat{\mathbf{X}}_1) \to Q\left(\frac{\|\mathbf{\Delta}_1\|}{\sqrt{2\,N_0}}\right)$$

where $\mathbf{\Delta}_1 \triangleq \mathbf{X}_1 - \widehat{\mathbf{X}}_1$. This shows that the performance depends only on the code of user 1.

5.2.2. *Multiuser separate receiver*

Consider first the LMMSE receiver. We have

$$t^{-1}\|\mathbf{F}_h^\dagger\mathbf{\Delta}(\mathbf{S}^+)^\dagger\|^2 \to \frac{\alpha_s - \sqrt{\alpha_s^2 - 4\,\alpha}}{2\,\alpha\sqrt{\alpha_s^2 - 4\,\alpha}}(1-\beta)^{-1}t^{-1}\|\mathbf{\Delta}\|^2$$

(where $\alpha_s \triangleq 1 + \alpha + \delta_s$) and

$$t^{-1}(\mathbf{F}_h\mathbf{H}\mathbf{X}, \mathbf{\Delta}) \to \frac{\alpha_s - \sqrt{\alpha_s^2 - 4\,\alpha}}{2\,\alpha}t^{-1}(\mathbf{X}, \mathbf{\Delta})\,.$$

Then, we can write

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}}) \to Q\left(\frac{A\|\mathbf{\Delta}\|^2 + B(\mathbf{X}, \mathbf{\Delta})}{\sqrt{2\,N_0\|\mathbf{\Delta}\|^2}}\right), \qquad (16)$$

where

$$A = \left[\frac{2\,\alpha\sqrt{\alpha_s^2 - 4\,\alpha}}{\alpha_s - \sqrt{\alpha_s^2 - 4\,\alpha}}(1-\beta)\right]^{1/2},$$

and

$$B = \frac{\alpha_s - 2\,\alpha - \sqrt{\alpha_s^2 - 4\,\alpha}}{\alpha}A.$$

In this case we have interaction between the two user stages. The asymptotic PEP of the ZF separate receiver is obtained by letting $\delta_s \to 0$. The corresponding coefficients are $A = (1-\alpha)^{1/2}(1-\beta)^{1/2}$ and $B = 0$. Thus, the two receiver stages do not interact.

5.2.3. *Joint receiver*

In this case we assume that $t, r, S \to \infty$ with $t/(rS) \to \widehat{\alpha}$. Notice that this asymptotic analysis is not compatible with the one performed for the two-stage receiver, where $t, r, S \to \infty$ with $t/r \to \alpha$ and $t/S \to \beta$ unless the product $\alpha\beta \to \infty$ with the same order as t.

Consider the LMMSE receiver first. We have

$$\mathbb{P}(\mathbf{X} \to \widehat{\mathbf{X}}) \to Q\left(\frac{A\|\boldsymbol{\delta}\|^2 + B(\mathbf{x}, \boldsymbol{\delta})}{\sqrt{2\,N_0\|\boldsymbol{\delta}\|^2}}\right), \qquad (17)$$

[3] $\tau(\mathbf{A}) \triangleq n^{-1}\mathsf{Tr}(\mathbf{A})$ for every $n \times n$ square matrix $\mathbf{A}$.

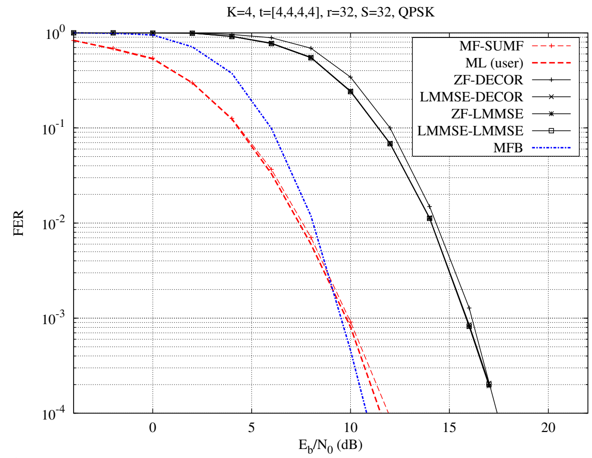

Fig. 1. *Performance of the proposed receivers for* $K = 4, t_k = 4, r = 32, S = 32$.

where

$$A = \left(\frac{2\,\widehat{\alpha}\sqrt{\widehat{\alpha}_s^2 - 4\,\widehat{\alpha}}}{\widehat{\alpha}_s - \sqrt{\widehat{\alpha}_s^2 - 4\,\widehat{\alpha}}}\right)^{1/2}$$

and

$$B = \frac{\widehat{\alpha}_s - 2\,\widehat{\alpha} - \sqrt{\widehat{\alpha}_s^2 - 4\,\widehat{\alpha}}}{\widehat{\alpha}}\,A\,.$$

The asymptotic PEP of the ZF separate receiver is obtained by letting $\delta_s \to 0$. The corresponding coefficients are given by $A = (1-\widehat{\alpha})^{1/2}$ and $B = 0$. The above suggests that an approximation to the gain of the joint ZF receiver versus the separate ZF receiver is given by $\gamma = \frac{1-t/(rS)}{(1-t/r)(1-t/S)}$. It can be shown that $\gamma > 1$ if $t < r$ and $t < S$, as we assumed above.

6. CONCLUSIONS

We have examined the performance of linear receivers in the uplink of a cellular system using CDMA and multiple transmit and receive antennas. The performance of a two-stage receiver was compared to that of the single-stage receiver. Large-system analyses were used, based on the assumption that the guidelines they provide are still useful when the system parameters take on finite values., as demonstrated in [1]. An example is shown in Fig. 1.

7. REFERENCES

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