ASYMPTOTIC PERFORMANCE OF CODE-REFERENCE SPATIAL FILTERS FOR MULTICODE DS/CDMA

Francisco Rubio and Xavier Mestre

Centre Tecnològic de Telecomunicacions de Catalunya (CTTC)
Nexus I building, c/Gran Capità, 2-4, 2nd floor, Barcelona, 08034 Spain
URL: http://www.cttc.es/, e-Mail: {francisco.rubio, xavier.mestre}@cttc.es

ABSTRACT

We address the problem of code-reference spatial filtering for multicode DS/CDMA. The large-system analysis of the asymptotic performance of three spatial filters, respectively based on the matched filter, the decorrelator and a projector onto the span of the codes of the desired user, is presented. We derive analytical expressions for the asymptotic covariance and output signal-to-interference-plus-noise ratio (SINR) of these filters, assuming that both the spreading factor and the number of parallel code sequences increase without bound at the same rate. A superior performance of the projecting filter against the other two solutions is revealed: the performance of the spatial filters based on the matched filter and the decorrelator saturates both for increasing values of the input signal-to-noise ratio (SNR), whereas the projecting solution is able to sustain an increasingly high SINR.

1. INTRODUCTION

It is well known that the presence of multiuser interference constitutes the strongest limiting factor in the spectral efficiency of current and future wireless communication systems. In DS/CDMA systems, multiple antennas at base stations can be used to provide spatial diversity in order to mitigate multiple access interference (MAI) by reducing the amount of co-channel interference from other users within the same cell and neighboring cells, thereby improving the SINR and ultimately increasing the system capacity.

Joint space-time filtering has usually been proposed as a powerful signal processing solution. However, its higher complexity often precludes its application in standardized wireless systems. On the other hand, spatial filtering schemes yield architectures that are compatible with existing time-only processing methods, thus enabling a straightforward upgrade of current base stations. In [1], a semi-blind spatial filter for pilot-aided WCDMA is proposed that efficiently neutralizes the interference due to the training data transmitted by the user whose symbols are to be recovered. In this paper, we use a similar approach to analyze the performance of three different beamforming concepts for multicode DS/CDMA. The spatial filters considered here operate in a code-reference fashion, i.e. they are designed exploiting the spectral structure of the transmitted signal, so that no training information is needed and, hence, the associated loss of capacity is avoided. Moreover, unlike multiuser detection techniques (MUD), which rely upon the knowledge of the spreading sequences of all interfering users, the proposed filters make only use of the codes of the desired user.

This work was partially funded by the Spanish Ministry of Science and Technology (MCYT) under projects TIC2002-04594-C02-02, HU2002-0032, FIT-070000-2003-257 and the European Comission IST-2002-507525 and IST-2002-508009.

2. SIGNAL MODEL AND SPATIAL FILTERING

Separation of desired signals from unwanted disturbances impinging on an antenna array is a well-studied problem in the signal processing community. In this paper, we concentrate on methods that exploit the inherent redundance of the DS/CDMA signal to design the spatial filters without dedicated training signals or further knowledge of the array manifold. The main idea behind code-reference beamforming is to compare the spatial covariance matrix before and after a signal-enhancement filtering operation in the time domain. The main purpose of such a filter is to increase the signal power of the desired user while mantaining the noise-plus-interference component at a reasonable level. The spatial filter can be formulated as the generalized eigenvector associated with the maximum generalized eigenvalue of the matrix pencil $\left(\hat{\mathbf{R}}_2, \hat{\mathbf{R}}_1\right)$, where $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_2$ denote the sample correlation matrices before and after the signal-enhancement filter, respectively.

This idea has been widely exploited in the array processing literature. For example, in [2], the author derives an eigenstructure-based method applicable to gated signals, either in time or in frequency, that requires no knowledge of the array manifold but needs from a double eigendecomposition. This problem is solved in [3] for non-stationary signals by computing only one generalized eigendecomposition of two covariance matrices estimated in different time instants, assuming that noise and interference signals are stationary. A similar approach was applied in [4] to DS/CDMA systems, where the difference of two covariance matrices before and after despreading is used to obtain the array steering vector of the desired user. In conventional single-code DS/CDMA systems, traditional despreading (a filter matched to the user code signature) seems to be the natural choice for the signal-enhancement operation. However, since many current standards are based on multicode DS/CDMA [5], more sophisticated signal enhancement strategies can be considered for these systems. In the following, two alternative filtering schemes will be introduced and analyzed.

Consider the uplink of a K-user multicode DS/CDMA system using $P > 1$ antenna elements at the base station. The symbols of each user are transmitted using Q different parallel spreading codes with spreading factor equal to L. The bandpass signal received by the P sensors is simultaneously sampled and downconverted, and a collection of ML baseband samples is gathered into a common matrix with complex entries $\mathbf{X} \in \mathbb{C}^{P \times ML}$. Hence, the received sample matrix is defined as

$$\mathbf{X} = \mathbf{a}_1 (\mathbf{C}_1 \mathbf{s}_1)^H + \sum_{i=2}^{K} \mathbf{a}_i (\mathbf{C}_i \mathbf{s}_i)^H + \mathbf{N}, \tag{1}$$

where the columns of $\mathbf{C}_i$ contain the Q parallel code sequences of the ith user (linearly independent), to which the symbol stream $\mathbf{s}_i \in \mathbb{C}^Q$ is to be mapped, and $\mathbf{a}_i$ is its steering vector, both assumed to be unknown. $\mathbf{N} \in \mathbb{C}^{P \times ML}$ models the spatial noise samples at each antenna. Without loss of generality, user 1 is considered to be the desired user. Throughout this paper, let $(\cdot)^H$ denote complex conjugate and transpose.

Now, we define the sample covariance matrix at the input of the antenna array as

$$\hat{\mathbf{R}}_1 = \frac{1}{ML} \mathbf{X}\mathbf{X}^H. \tag{2}$$

To recover the symbols of the desired user, the following signal-enhancement filters are considered: the matched filter (MF), the decorrelator (DEC) and a third solution based on the orthogonal projection onto the subspace spanned by the spreading sequences (PC). The received signal at the array after signal enhancement is

$$\mathbf{Y}_{MF} = \mathbf{X}\mathbf{C}_1 \tag{3}$$

$$\mathbf{Y}_{DEC} = \mathbf{X}\mathbf{C}_1 \left(\mathbf{C}_1^H \mathbf{C}_1\right)^{-1} \tag{4}$$

$$\mathbf{Y}_{PC} = \mathbf{X}\mathbf{C}_1 \left(\mathbf{C}_1^H \mathbf{C}_1\right)^{-1/2} \tag{5}$$

for each scheme, respectively. Note that the third matrix filter consists of an orthogonal basis for the span of the codes. Thus, the sample covariance matrix after the signal-enhancement filter can be similarly expressed as

$$\hat{\mathbf{R}}_2 = \frac{1}{ML} \mathbf{Y}\mathbf{Y}^H = \frac{1}{ML} \mathbf{X}\mathbf{C}_1 \left(\mathbf{C}_1^H \mathbf{C}_1\right)^{-\xi} \mathbf{C}_1^H \mathbf{X}^H, \tag{6}$$

where ξ takes the values 0, 1 and 2 for the matched filter, projector and decorrelator, respectively. Due to the signal enhancement, the SINR at the output of the spatial filter will be approximately proportional to the ratio $\mathbf{w}^H \hat{\mathbf{R}}_2 \mathbf{w} \,/\, \mathbf{w}^H \hat{\mathbf{R}}_1 \mathbf{w}$. Consequently, it seems reasonable to design $\hat{\mathbf{w}}$ by maximizing this ratio, i.e. as the generalized eigenvector associated with the maximum generalized eigenvalue of the matrix pencil $\left(\hat{\mathbf{R}}_2, \hat{\mathbf{R}}_1\right)$.

3. ASYMPTOTIC PERFORMANCE PREDICTION

If an infinite number of samples were available ($M \to \infty$), the three architectures proposed above would be equivalent to the optimum beamformer (i.e. the one maximizing the output SINR). However, since the spatial filters are in practice designed with sample covariance matrices, the performance of the three solutions will be different, and it is a priori difficult to identify the best one. In order to reveal the behavior in a finite-sample-size situation, our asymptotic analysis is derived under the assumption that both the observation window length ML and the number of parallel spreading sequences Q grow without bound, whereas the ratio between them ($\alpha = Q/ML$) remains constant. With this strategy, we obtain results that are more representative of real non-asymptotic situations because, as in practical scenario, both quantities are assumed to have the same order of magnitude. Results from random matrix theory [6] and free probability [7] are used to obtain the asymptotic matrix pencil for each spatial filter as a function of the moments of the limiting empirical eigenvalue distribution of the random matrix $\left(\mathbf{C}_1^H \mathbf{C}_1\right)$ and the parameter ξ associated with each particular spatial filtering solution.

In our analysis, the asymptotic limits will be derived under the following statistical assumptions:

(**As1**) The elements of the noise matrix $\mathbf{N}$ are i.i.d. circularly symmetric Gaussian random variables with zero mean and variance σ^2.

(**As2**) The transmitted symbols are modeled as i.i.d. circularly symmetric random variables with zero mean, unit variance and bounded higher order moments. They are also independent of the received noise.

(**As3**) The code sequences are also modeled as i.i.d. circularly symmetric r. v. with zero mean and variance $1/Q$. They are also independent of the received noise and transmitted symbols.

Proposition 1 (Asymptotic Spatial Filters) *Under (As1) - (As3) and as $Q, ML \to \infty$ at the same rate, the three spatial filters proposed in Section 2 converge in probability to the same limit (up to a scalar factor), which is given by the generalized eigenvector associated with the maximum generalized eigenvalue of the matrix pencil $(\mathbf{R}_2, \mathbf{R}_1)$, where*

$$\mathbf{R}_1 = \mathbf{R}_S + \mathbf{R}_N \ , \ \mathbf{R}_2 = m^{-\xi+2} \mathbf{R}_S + m^{-\xi+1} \mathbf{R}_N \tag{7}$$

and

$$\mathbf{R}_S = \mathbf{a}_1 \mathbf{a}_1^H , \mathbf{R}_N = \mathbf{A}_1 \mathbf{A}_1^H + \sigma^2 \mathbf{I}_P. \tag{8}$$

Here, $\mathbf{A}_1$ is the matrix with the steering vectors of the interferers and m^i is defined as the limiting normalized trace $\frac{1}{ML} \operatorname{tr}\left[\left(\mathbf{C}_1^H \mathbf{C}_1\right)^i\right]$.

Proof. See [8] for a proof. ■

We see from this result that under the asymptotic conditions considered here, the three proposed beamvectors tend to the same limit (up to a scalar factor) as $Q, ML \to \infty$ at the same rate. The three solutions are asymptotically proportional to the eigenvector associated with the maximum eigenvalue of the generalized eigenproblem $(\mathbf{R}_S, \mathbf{R}_N)$, which is denoted by $\mathbf{w}_o$. This is, at the same time, the solution for the beamformer maximizing the SINR at the output of the antenna array, i.e.

$$\mathbf{w}_o = \arg \max_{\mathbf{w}} \frac{\mathbf{w}^H \mathbf{R}_S \mathbf{w}}{\mathbf{w}^H \mathbf{R}_N \mathbf{w}}. \tag{9}$$

The three proposed designs for the despreading operation yield then spatial filters that are asymptotically optimal in the sense of maximizing the SINR. Therefore, from the point of view of the asymptotic solution, there is no difference in performance between these three implementations. This does not mean, however, that they are equivalent in a real (non-asymptotic) scenario. The difference in performance will be given by the difference in the asymptotic covariance of the beamvector weights around the optimum value. A different asymptotic covariance matrix will translate into a different SINR at the output of the spatial filter. In this paper, we define the (average) output SINR of a particular beamformer $\hat{\mathbf{w}}$ as

$$SINR(\hat{\mathbf{w}}) = \frac{\mathbb{E}\left[\hat{\mathbf{w}}^H \mathbf{R}_S \hat{\mathbf{w}}\right]}{\mathbb{E}\left[\hat{\mathbf{w}}^H \mathbf{R}_N \hat{\mathbf{w}}\right]} = \frac{\mathbf{w}^H \mathbf{R}_S \mathbf{w} + \operatorname{tr}\left[\mathbf{R}_S \mathbf{C}_{\hat{\mathbf{w}}}\right]}{\mathbf{w}^H \mathbf{R}_N \mathbf{w} + \operatorname{tr}\left[\mathbf{R}_N \mathbf{C}_{\hat{\mathbf{w}}}\right]} \tag{10}$$

where the expectation in the middle term is taken with respect to the statistics of the estimate $\hat{\mathbf{w}}$, $\mathbf{w} = \mathbb{E}[\hat{\mathbf{w}}]$ is its mean beamvector and $\mathbf{C}_{\hat{\mathbf{w}}} = \mathbb{E}\left[(\hat{\mathbf{w}}-\mathbf{w})(\hat{\mathbf{w}}-\mathbf{w})^H\right]$ its asymptotic covariance matrix [9]. We prefer to use this performance measure rather than the expectation of the instantaneous SINR because (10) is much simpler to compute (only second-order statistics are needed), whereas both performance measures can be demonstrated to be very close in practical situations (see ([10]) for further details).

The next proposition gives the asymptotic expression for the covariance[1] of the maximum generalized eigenvalue eigenvector associated with the matrix pencil $\left(\hat{\mathbf{R}}_2, \hat{\mathbf{R}}_1\right)$.

Proposition 2 (Asymptotic Covariance) *Under (As1) - (As3) and as $Q, ML \to \infty$ at the same rate, for almost every realization of signature sequences, the covariance matrix of the properly normalized spatial filters around the optimum value $\mathbf{w}_o$ behaves as,*

$$ML\mathbf{C}_{\hat{\mathbf{w}}} \to \vartheta \mathbf{W}_1 \mathbf{W}_1^H \tag{11}$$

where $\mathbf{W}_1 = \left[\begin{array}{ccc} \mathbf{w}_2 & \ldots & \mathbf{w}_P \end{array}\right]$ and the parameter ϑ can be generally expressed for the three filters as a function of ξ and γ_o, defined as the optimum SINR (i.e. the SINR obtained with the optimum beamvector $\mathbf{w}_o$ in (9)),

$$\vartheta = \frac{1}{\left(m^{-\xi+2} - m^{-\xi+1}\right)^2 \gamma_o^2}\Bigg[m^{-2\xi+2}(1+\gamma_o) + \\ + m^{-2\xi+3}\gamma_o(1+\gamma_o) - 2m^{-\xi+2}\gamma_o\left(m^{-\xi+2}\gamma_o + m^{-\xi+1}\right) - \\ -2m^{-\xi+1}\left(m^{-\xi+2}\gamma_o + m^{-\xi+1}\right) + \left(m^{-\xi+2}\gamma_o + m^{-\xi+1}\right)^2\Bigg]$$

Proof. See [8] for a proof. ■

The consistency of the sample weights is obvious from (11), since its covariance goes to zero as $O\left(\frac{1}{ML}\right)$. After noting that $\mathbf{W}_1\mathbf{W}_1^H$ is identically defined for all three spatial filters, their covariance is seen to be proportional to the parameter ϑ. Particularizing the previous expression to the matched filter and the decorrelating solutions, we obtain

$$\vartheta_{MF} = \frac{1}{\gamma_o}\left[1 + 2\alpha + \frac{\alpha}{\gamma_o}\right] + \alpha \tag{12}$$

$$\vartheta_{DEC} = O\left(\frac{1}{\gamma_o}\right) + \frac{\alpha}{1-\alpha}. \tag{13}$$

whereas for the projecting approach we have

$$\vartheta_{PC} = \frac{1}{\gamma_o(1-\alpha)}\left[1 + \frac{\alpha}{\gamma_o}\right] \tag{14}$$

Furthermore, by inserting the covariance matrices into (10), the average output SINR for each proposed spatial filter in the asymptotic regime are well approximated by [8]

$$SINR(\hat{\mathbf{w}}_o) = \frac{\gamma_o}{1 + \frac{P-1}{ML}\vartheta(1+\gamma_o)}. \tag{15}$$

Note that only for the solution projecting onto the span of the known spreading codes does the covariance factor vanish as the optimum SINR gets higher. In this limiting situation, the normalized covariance of the other two solutions approaches a floor value that depends on the parameter α. This fact exemplifies the different behavior of the projection solution compared to the other two methods in terms of output SINR. The following behavior of the different covariance matrices can be inferred from (12) - (14): as the ratio between the number of parallel codes and the length of the observation window goes to zero, all expressions become the same. This is due to the fact that, for a fixed Q, as the length of the linearly independent code sequences increases, they become more and more orthogonal to each other. Analogously, for a fixed L, as the number of parallel codes gets smaller, the contribution to the interference or degree of correlation between sequences decreases.

[1] Since the spatial weight vector is defined up to a constant factor, care must be taken in imposing some amplitude and phase constraints to avoid ambiguities in the definitions of the weight vector covariance [8].

4. NUMERICAL VALIDATION

In this section we present a numerical validation via simulations of the asymptotic study presented in the last section. To demonstrate the convergence of the covariance matrix to the asymptotic expression obtained theoretically, we considered a scenario with five users transmitting from the azimuth angles (in degrees) [10 (desired user), 40, 25, -35, -20 (interfering signals)] and impinging on a uniform linear array of eight elements situated half a wavelength apart. All the interfering users were received with a mean power 20 dB above the noise floor. In order to show the rate of convergence toward the asymptotic expressions derived in the previous section, we steadily increase the value of Q and ML at the same rate. We fixed the two signal parameters ML and Q to $ML = 8k$ and $Q = k$ and let k vary from 1 to 64. Figure 1 shows this convergence in terms of the quantity $L\frac{\mathrm{tr}[\mathbf{C}_{\hat{\mathbf{w}}}]}{\|\mathbf{w}_o\|_2^2}$ versus k for the beamformers under consideration and for two different values of received desired SNR, which is defined as $\mathrm{SNR}_{input} = p_{user1}/\sigma^2$, namely 0 dB and -10 dB. The covariance results are averaged over 100 realizations of the symbol and code sequences, which were all randomly drawn from a QPSK alphabet. We observe that the rate of convergence is quite reasonable and that the derived asymptotic expressions are very accurate in practical situations, even for unfavorable values of input SNR.

In Figure 2, asymptotic and simulated output SINR versus input SNR in a realistic scenario are depicted. In this case, the same scenario as in the previous example is considered, but an antenna array of only four elements is used to show the filtering properties of the proposed schemes in a situation with more users than antenna elements. The simulated points are seen to match very well the underlying theoretical curves. Furthermore, note that the simulation results confirm the previously predicted asymptotic behavior and consequently expose the differences in the output SINR between the proposed spatial filters, as identified in Section 3: the performance of the matched filter and the decorrelator saturates as $\mathrm{SINR}_{opt} \to \infty$, whereas the projector is able to benefit from an increasing input SNR. This indicates that the actual performance of the spatial filters is not only interference-limited, but also estimation-error limited: the residual uncertainty in the estimation of the beamvector does not vanish for increasing values of the input SNR. The same phenomenon was also observed in [1] for a pilot-aided spatial filter for multicode WCDMA, and in [11] in the context of asymptotic efficiency of blind and group-blind linear MMSE multiuser receivers. Note that due to this finite sample-size effect, the output SINR at the output of the projector does not reach the optimum value, although the loss in output SINR is less significant.

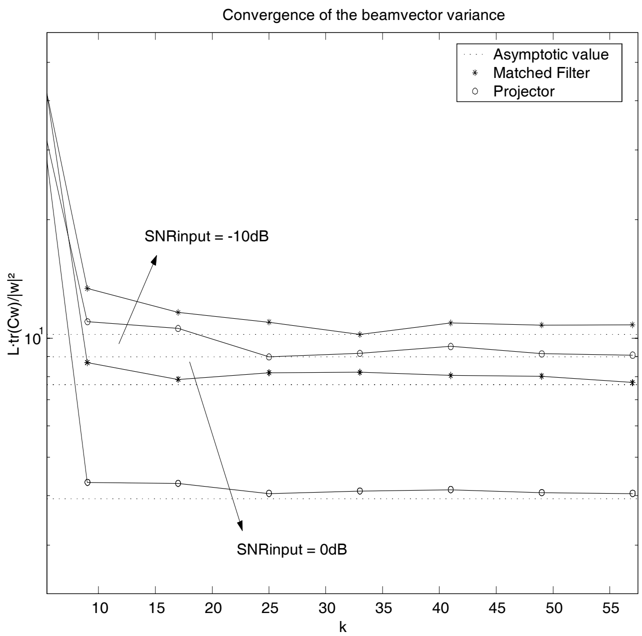

Fig. 1. Convergence of the normalized beamvector variance. We fixed $ML = 8k$, $Q = k$, and k is varied from 1 to 64.

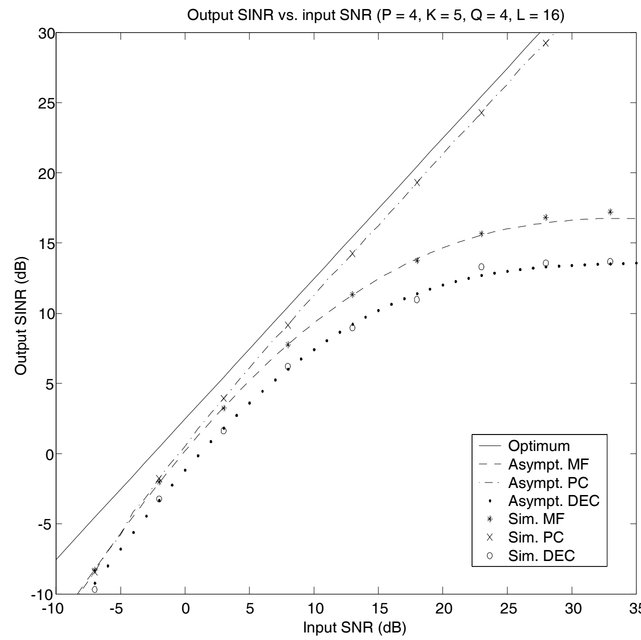

Fig. 2. Simulated and asymptotic output SINR as a function of the input SNR.

It can also be observed from Figure 2 that, for increasing values of output SINR, the matched filter performs even better than the decorrelating solution. This can be understood by noting that the code separation capabilities of the different schemes turn out to be unimportant for the beamforming operation at hand.

5. CONCLUSIONS

The analysis of the asymptotic performance of three blind spatial filters for multicode DS/CDMA is presented. The three methods provide the solution for the beamvector in a single eigenvalue computation, avoiding the use of iterative techniques. We have derived analytical expressions for the asymptotic covariance of the beamvectors and its output SINR as both the spreading factor and the number of parallel code sequences increase without bound at the same rate. The fast convergence properties of the techniques used for our large-system analysis allow for a good match between the analytical results and the system performance in realistic non-asymptotic situations. These results are quite general because they do not depend on realizations of the signature sequences and little is assumed regarding other spatio-temporal characteristics of the received signals. The usual despreading operation based on correlating the received signal with the code of the desired user is shown to yield a poor output SINR if the finite sample size effect is taken into consideration. Two other alternative and more sophisticated signal-enhancement filters have been evaluated: the performance of the spatial filters built upon the matched filter and the decorrelator schemes is seen to saturate for increasing values of the input SNR, whereas a significant performance gain can be achieved with a filter projecting onto the span of the codes as this solution is able to sustain an increasingly high output SINR.

6. REFERENCES

[1] X. Mestre and J.R. Fonollosa, "Spatial filtering for pilot-aided WCDMA systems: A semi-blind subspace approach," *IEEE Trans. on ASSP*, vol. 51, pp. 2665–2678, October 2003.

[2] M. Viberg, "Sensor array processing using gated signals," *IEEE Trans. on ASSP*, vol. 37, pp. 447–450, March 1989.

[3] A. Souloumiac, "Blind source detection and separation using second order non-stationarity," in *Proc. ICASSP*, May 1995.

[4] B. Suard, A. F. Naguib, G. Xu, and A. Paulraj, "Performance of CDMA mobile communication systems using antenna arrays," in *Proc. of ICASSP*, April 1993, vol. 4, pp. 153–156.

[5] 3GPP Techn.Spec. 25.211, "Physical channels and mapping of transport channels onto physical channels (FDD)," Online available: www.3gpp.or, December 1999.

[6] Z. D. Bai, "Methodologies in spectral analysis of large dimensional random matrices, a review," *Statistica Sinica*, vol. 9, pp. 611–677, 1999.

[7] D. V. Voiculescu, K. Dykema, and A. Nica, *Free random variables*, vol. 1, CRM Mon. Series. Am. Math. Soc., 1992.

[8] F. Rubio and X. Mestre, "Large-system performance analysis of code-reference spatial filters for multicode DS/CDMA," Technical Report CTTC/RC/2004-02, Available at http://www.cttc.es/drafts/cttc-rc-2004-002.pdf.

[9] A. Host-Madsen, "Performance of blind and group-blind multiuser detectors," *IEEE Trans. on Inf. Theory*, July 2002.

[10] X. Mestre, *Space Processing and Channel Estimation: Performance Analysis and Asymptotic Results*, Ph.D. thesis, Univ. Politècnica de Catalunya, Barcelona, Spain, 2002.

[11] J. Zhang and X. Wang, "Large-system performance analysis of blind and group-blind multiuser receivers," *IEEE Trans. on Inf. Theory*, vol. 48, pp. 2507–2523, September 2002.