ENERGY-THROUGHPUT OPTIMIZATION FOR WIRELESS ARQ PROTOCOLS

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ABSTRACT

We consider energy-efficient resource allocation for wireless fading channels. We study the case where a sliding window ARQ protocol such as Go-Back-N is used to provide reliable communication. In particular, we consider power allocation policies that take into account the underlying window dynamics. An optimal dynamic programming approach and a sub-optimal approach based on renewal theory are given. Numerical results comparing these approaches are also presented.

1. INTRODUCTION

A key concern with mobile wireless devices is to judiciously utilize the available energy while providing acceptable Quality of Service (QOS). In this paper, we investigate energy efficient Automatic Repeat reQuest (ARQ) protocols that strike a balance between throughput and power consumption in a wireless data system. Traditionally ARQ protocols are thought of as operating at the data link layer and having little interaction with the underlying physical layer. However, in wireless environments, knowledge of channel conditions can greatly improve system performance. Indeed, in most recent wireless systems, ARQ is included along with various link adaptation techniques in a "radio link layer". Here, we consider the performance of ARQ protocols when physical layer parameters such as transmission power can be adapted based in part on the available Channel State Information (CSI). We examine the trade-offs between transmitted power and the efficiency of these adaptive protocols.

Several related approaches have been studied in the literature, e.g [1–5]. In most of this work, the window dynamics of the ARQ protocol are ignored. One of our goals is to study the impact of these effects. With this in mind, we consider allocating power for a Go-Back-N protocol with a window of size W. [1] In such a protocol, an error in the transmission of a packet results in the receiver discarding the next $W-1$ packets. Therefore, the optimal transmission power for a given packet will depend not only on the current channel state, but also on the delivery status of the previous $W-1$ packets. Moreover, when transmitting the current packet, the delivery status of all the previous packets will not be known at the transmitter because of feedback delays, and so can only be estimated based on the previous power levels and channel states. Assuming Markovian fading, we first formulate the optimal power allocation in a Markov decision framework [6]. However, as shown in Sect. 3, this type of formulation quickly becomes intractable. In particular, the state-space increases exponentially in the window size. In Sect. 4, we take a different approach and consider a simpler adaptation rule which is based on limited state information. This allows us to obtain a closed-form power solution whose performance is comparable to the optimal policy. Moreover, the complexity of this approach is independent of the window size and only polynomial in the number of channel states.

2. SYSTEM MODEL

We consider a slotted-time model, where each time-slot corresponds to the time to transmit one packet. The transmission rate is fixed, but the transmitter can adapt the power used to send each packet. Sending a packet at a higher power level results in a smaller probability of packet error. Specifically, during each time-slot, n, the probability of packet error is given by $\rho(P[n], \theta[n])$, where $P[n]$ is the transmission power and $\theta[n]$ represents the available CSI. For instance, in a narrow-band fading channel, $\theta[n]$ could represent either an estimate or the exact value of the fading level during the nth time-slot. Given the exact fading level, one example of $\rho(P, \theta)$ is

$$\rho(P, \theta) = \exp(-k\theta P), \tag{1}$$

where k is a constant depending on the modulation and coding used [7]. Other examples can be found in [4, 5]. The following applies to any $\rho(P, \theta)$ that is decreasing and convex in P for all θ. For simplicity, we assume that for each

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[1] In other protocols, such as Selective Repeat, window dynamics can also have an impact, e.g via "window stalls". Similar ideas can be applied there.

n, $\theta[n]$ takes values from a finite set $\Theta = \{\theta_1, \theta_2, \cdots, \theta_M\}$ and during each block, the value is independently chosen according to the probability distribution $p_\Theta(\cdot)$.

In the above setting, we consider power allocations for a Go-Back-N protocol that balance energy efficiency with throughput. To illustrate this, we first consider a power adaptation scheme, as in [4, 5], that is a function only of the current channel state θ_i, i.e., $P_i = f(\theta_i)$. We denote such a power allocation by $\mathbf{P} = \{P_1, P_2, \cdots, P_M\}$. For a given $\mathbf{P}$, the average success probability and average power expended are given, respectively, by

$$q(\mathbf{P}) = \sum_{i=1}^{M} (1 - \rho(P_i, \theta_i)) p_\theta(\theta_i),$$

and $\bar{P}(\mathbf{P}) = \sum_{i=1}^{M} P_i p_\theta(\theta_i)$. Consider a Go-Back-N protocol with window size W, which is matched to the round-trip delay, i.e., a packet that is successfully received will be acknowledged after $W - 1$ time-slots. Additionally, assume that the transmitter uses a time-out interval of $W - 1$ time-slots, so that if a packet is not acknowledged by this time it is retransmitted. These assumptions maximize the protocol's throughput in a channel without fading [8]. For simplicity, we assume that acknowledgments are never lost. For i.i.d fading, the needed average success probability for an average efficiency or throughput of η (packets/slot) is given by [8]

$$q(\mathbf{P}) = \frac{W}{W - 1 + \frac{1}{\eta}} \triangleq K_{gb-n}(\eta). \tag{2}$$

Here, $K_{gb-n}(\eta)$ is a "protocol constant" which depends on the ARQ protocol and the required efficiency. Given η, the power allocation that minimizes the long-term average power is the solution to:

$$\begin{aligned} \min_{\{\mathbf{P}: P_i \geq 0 \ \forall \ i\}} \quad & \bar{P}(\mathbf{P}), \\ \text{subject to:} \quad & q(\mathbf{P}) = K_{gb-n}(\eta). \end{aligned} \tag{3}$$

This problem was considered in [4, 5]. Under our assumption that ρ is convex, this problem can be solved using convex programming. This can be extended to other ARQ protocols by changing the protocol constant $K_{gb-n}(\eta)$.

When the throughput requirements are high, the optimal solution to (3) results in packet transmissions in every channel state (i.e. $P_i > 0$ for all i) [4, 5]. As the throughput requirement is decreased, the optimal power allocation might involve setting suspending transmission (i.e., setting $P_i = 0$) in one or more channel states. In the above formulation, this results in the packet being lost with probability one and all subsequent transmissions in the current window will then be discarded by the receiver. Clearly, a better approach would be to (*i*) take into account the probability that a packet will be accepted when deciding on the power to use and (*ii*) instead of dropping a packet when $P[n] = 0$, simply defer the transmission of the packet until the next time-slot. Next we consider the design of energy efficient protocols that take these considerations into account.

3. POWER ALLOCATIONS: DYNAMIC PROGRAMMING FORMULATION

As stated above, we consider a Go-back-N protocol where, when $P[n] = 0$, the corresponding packet is held and transmitted in the next time-slot (unless the transmitter times out, in which case it goes back and re-sends a previous packet). This requires more interaction between the physical and data link layers. In particular, the timer for a packet should not begin until the packet is actually transmitted by the physical layer.

For such a protocol, we formulate the optimal power allocation as a Markov decision problem (MDP), where the control action is the power level used for each packet and, as in (3), the objective is to minimize the average power for a given throughput requirement. We define the *transmitter window* at time n to be the set of packets which could potentially be received, but have not yet been acknowledged. The system's dynamics depend on whether any of the packets in the transmitter window are in error[2], as well as the current channel state and control action. This can be summarized by defining a system state $(\theta[n], A[n])$ where $A[n] = i$, when the first error in the transmitter window is at time $n - i$; in this case, a transmission at time n will not be accepted at the receiver. If there are no errors in the transmitter window, we set $A[n] = W$, unless $P[n - 1] = 0$, in which case we set $A[n] = W + 1$, indicating the previous transmission was suspended. When $P[n] = 0$, the state transition probabilities are:

$$\Pr(S_i | S_j, P = 0) = \begin{cases} 1, & i = j + 1, \, j < W - 1, \text{ or} \\ & \quad i = W + 1, \, j \geq W - 1 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

When $P[n] > 0$, no transition takes places to $A[n+1] = S$, and the state transition probabilities are:

$$\Pr(S_i | S_j, P > 0) = \begin{cases} 1, & i = j + 1, j < W - 1 \\ \rho(P, \theta), & i = 1, \, j \geq W - 1 \\ 1 - \rho(P, \theta), & i = W, \, j \geq W - 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The per stage cost $C(P[n], A[n])$ is given by $P[n] + \lambda$ if $A[n] = W$, and is $P[n]$, otherwise, where λ is a Lagrange multiplier for the throughput constraint. The objective is to specify a power allocation to minimize the average cost. A difficulty is that at time n, the transmitter does not observe

[2] Here we define a packet as being in error if it is received correctly, but can not be accepted because of an earlier error.

the entire state, i.e this is a partially observed MDP [6]. In this case, the optimal policy will depend on the information state $I[n]$, which is a sufficient statistic for the true state [6]. Here, $\{A[n-W]$,$P[n-W]$, $\theta[n-W]$,$P[n-W-1]$,$\theta[n-W-1]$, $\cdots$, $P[n-1]$,$\theta[n-1]\}$ can be shown to be an information state. Let $\mathcal{I}$ represent the set of all information states. A power allocation is then given by a policy $\pi : \mathcal{I} \to \mathcal{P}$, where $\mathcal{P} = \{P_1, P_2, \cdots, P_L\}$ is a set of feasible transmission powers.[3] For each $I \in \mathcal{I}$, the average cost associated with policy π is,

$$V(I) = \lim_{N \longrightarrow \infty} \frac{1}{N} \mathbb{E}_\pi \left[\sum_{j=0}^{N-1} C(P[j], A[j]) | I_0 = I \right]. \quad (6)$$

The optimal policy that minimizes $V(I)$ in (6) can be obtained by solving the Bellman equation [6]:

$$V^*(I) + \mu^* = \min_{\mathcal{P}} \left\{ \mathbb{E}[C(P, A)|I] + \sum_{I' \in \mathcal{I}} p(I'|I) V^*(I') \right\}. \quad (7)$$

We solve this using relative value iteration. The complexity of this is $\mathcal{O}((ML)^{W-1})$, where L is the number of power levels used in $\mathcal{P}$.

4. POWER ALLOCATIONS: RENEWAL THEORY

The complexity involved in finding the optimal policy in Sect. 3 motivates us to consider simpler power allocations. Assuming there are N "active" channel states (defined to be the states with non-zero powers), we consider policies of the form

$$P[n] = \begin{cases} P_i, & \text{if } \theta[n] = \theta_i, \ i \le N, \\ 0, & \text{if } i > N. \end{cases} \quad (8)$$

If $P[n] = 0$, i.e., a packet is suspended in the current slot, it is transmitted in the next slot, with power $P[n+1]$ determined as in (8). In other words, as in Sect. 2, the power allocation only depends on the current channel state; however, here when transmission is suspended, the corresponding packet is held until the next slot. We employ a renewal theory argument to determine the average throughput and power for this type of policy. Packet transmission sequences are assumed to be divided into renewal cycles as in [9]. Each cycles is terminated by the occurrence of a transmission error in an "active" state. Figure 1 shows an example of this for a two-state channel when transmission is suspended in state θ_2. In this case, each cycle is terminated by a transmission error in state θ_1. Note that the suspended channel states are interspersed within streaks of successful transmissions. In each cycle, let X be the number of packets transmitted successfully, and Y be the number of suspended slots not including the first W slots. For instance, during the first three cycles in Fig. 1, X is 4, 0, and 2 while Y is 1, 3,and 0, respectively. The start of each cycle will be a renewal event. From renewal reward theory, the efficiency is given by,

$$\begin{array}{lcccccccccccccc}
\text{Channel:} & | & \overbrace{\theta_1 \cdots}^{W} & \theta_1 & \theta_2 & \theta_1 & \theta_1 & \theta_1 & | & \overbrace{\theta_1 \cdots}^{W} & \theta_2 & \theta_2 & \theta_2 & | & \overbrace{\theta_1 \cdots}^{W} \ \theta_1 \ \theta_1 \\
\text{Delivery:} & & \text{F} & \text{S} & & \text{S} & \text{S} & \text{S} & & \text{F} & & & & & \text{F} \quad \ \text{S} \ \ \text{S} \\
\text{Cycle:} & & \textbf{I} & & & & & & & \textbf{II} & & & & & \textbf{III}
\end{array}$$

Fig. 1. An example packet transmission sequence.

$$\eta = \frac{\mathbb{E}[X]}{W + \mathbb{E}[X+Y]}. \quad (9)$$

Assuming transmissions occur in only N active states ($P_i = 0, i > N$), the above expectations can be calculated by modeling each cycle as a Markov chain with a trapping state corresponding to the occurrence of an error and by assigning an appropriate reward r_i to each state θ_i. The reward aggregated before an error occurs, when the initial state is θ_i, is given by

$$\bar{r_i} = (1 - \rho(P, \theta_i))[r_i + \sum_{j=1}^{M} p_\theta(\theta_i) \bar{r_j}] \ \forall \ i. \quad (10)$$

Here, $\mathbb{E}[X]$ can be calculated by assigning the rewards of 1 and 0 for staying in active and inactive states, respectively. Since, $\mathbb{E}[X+Y]$, is the average number of slots before an error occurs, it can be found by setting $r_i = 1$ for all i. Given the efficiency of the protocol η, the needed average success probability can be derived from (9) as

$$\sum_{i=1}^{N} p_\theta(\theta_i)(1 - \rho(P_i, \theta_i)) = \frac{\eta((W-1)\sum_{i=1}^{N} p_\theta(\theta_i) + 1)}{(W-1)\eta + 1}, \quad (11)$$

Assuming N active channel states, the average power expended per slot is then given by

$$\bar{P}(N) = \sum_{i=1}^{N} P_i p_\theta(\theta_i). \quad (12)$$

Thus, given N active states, the optimal power allocation is obtained by minimizing (12) subject to the constraint in (11). The overall optimal power allocation can then be found by iterating over the number of active states. However, the following proposition simplifies this iterative procedure. Here, N_{min} is the minimum number of channel states needed so that the throughput constraint in (11) remains feasible.

Theorem 1 *For $\rho(P_i, \theta_i) = exp(-k\theta_i P_i)$, $\bar{P}(N)$ is unimodal in $N \in [N_{min}, M]$, if $\eta < \frac{1.72}{W-1}$.*

We omit the proof due to space considerations.

The computation of the optimal policy in this case is $\mathcal{O}(M^2)$. This compares favorably against the complexity of the dynamic programming approach in Sect. 3 .

[3] The set $\mathcal{P}$ is assumed to be finite, so that $\mathcal{I}$ remains finite.

5. NUMERICAL RESULTS

We study the performance of the two approaches discussed, through a numerical example and compare them with the policy obtained in (3). We shall refer to the policy in (3) as "weak interaction", to differentiate it from the other schemes that require a stronger interaction between the data link and physical layers. Consider an i.i.d. fading channel with four possible channel states representing fade levels of 1dB, 3dB, 5dB and 10dB, and steady-state probabilities of 0.33, 0.15, 0.21 and 0.31, respectively. The error function is as in (1) with $k = 1.5$. Figure 2 shows the efficiency as a function of the average power for the three schemes with $W = 2$. There is little difference between the dynamic programming and renewal-based approaches. However, both these approaches can reduce the average power by approximately 80 % over the weak interaction case. Figure 3 examines the performance of the renewal-based approach as the window size increases (for such window sizes the complexity of the dynamic programming approach becomes prohibitive.) The power savings obtained through this approach (over the weak interaction) increases with the window size. This is because at low throughputs, the transmitter is able to selectively transmit in the good channel states, thereby reducing the chance of transmission errors.

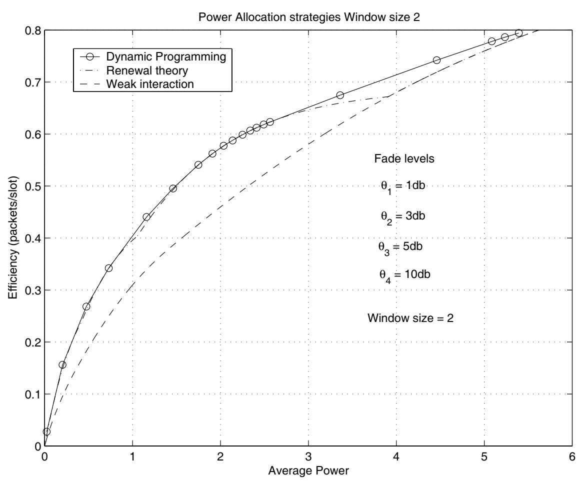

Fig. 2. Power allocations for Go-Back-N with W = 2.

6. CONCLUSIONS

We presented several energy-efficient variants of a Go-Back-N protocol for wireless environments, which require some amount of coupling between the data link and physical layers. These protocols exploit different amount of knowledge of the channel conditions and the previous decisions to achieve different balances between complexity and performance.

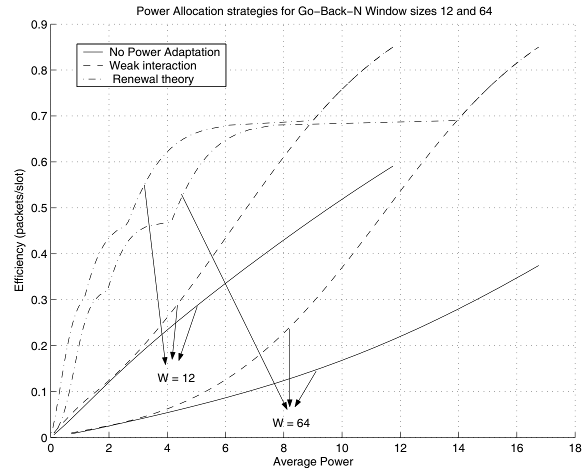

Fig. 3. Power allocations for Go-Back-N with large window sizes.

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