ACTIVE SENSOR WAVEFORM DESIGN FOR MOTION INSENSITIVITY

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ABSTRACT

Waveforms for active sensors (e.g. radar or active sonar) are often designed to maximize sensitivity to target motion to enhance tracking performance. This paper addresses the complementary problem of waveform design to maximize *insensitivity* to motion. Insensitivity is desirable in cases where motion is a nuisance to the sensor system.

The motion insensitivity problem has been approached in the past using the trajectory diagram, related concepts of trajectory ambiguity, and generalized ambiguity functions. This paper discusses motion insensitivity in terms of both trajectory ambiguity and the generalized ambiguity function. The relationship between trajectory ambiguity and motion insensitivity via a generalized ambiguity function is described mathematically. The hyperbolic FM (HFM) signal is used as an example of a velocity insensitive signal.

1. INTRODUCTION

Waveforms for active sensors (e.g. radar or active sonar) are often designed to maximize sensitivity to target motion to enhance tracking performance. This paper addresses the complementary problem of waveform design to maximize *insensitivity* to motion. Insensitivity is desirable in cases where motion is a nuisance to the sensor system. For example consider a bat flying toward a tree [2]. The bat perhaps wants to know where the tree is located independent of how the bat is maneuvering. A velocity insensitive waveform would be advantageous in this case. Another example is an airborne radar experiencing motion drift due to inability to maintain constant-velocity level flight, leading to coherent gain loss.

The motion insensitivity problem has been approached in the past using the trajectory diagram, related concepts of trajectory ambiguity [1,3-6], and generalized ambiguity functions [2]. This paper discusses aspects of motion insensitivity in terms of both trajectory ambiguity and the generalized ambiguity function. The relationship between trajectory ambiguity and motion insensitivity via a generalized ambiguity function is described mathematically. The hyperbolic FM (HFM) signal is used as an example of a velocity insensitive signal.

This paper is organized as follows: Section II reviews the trajectory model for specular targets in a linear time-varying channel. Section III discusses trajectory ambiguity and motion insensitivity in phase. Finally, Section IV discusses the generalized ambiguity function approach to motion insensitive signal design and its relationship to trajectory ambiguity.

2. TRAJECTORIES IN A LINEAR CHANNEL

Trajectories can be described mathematically with a linear, time-varying channel model. For an active sensor this model is

$$y(t) = \int_0^\infty s(t-2\lambda)h(\lambda, t-\lambda)d\lambda \tag{1}$$

where s(t) is the transmitted signal, y(t) is the received signal, λ is range in units of time, and h(λ, t) is the time-varying impulse response of the channel. One spatial dimension is considered (pencil beam). The channel includes both target and multipath components, but for simplicity only "target" components are discussed. This model can be applied to radar, sonar, and other active sensors.

A dimension of information is lost in (1) and, regardless of the representation, the process is not generally reversible without ambiguity. This loss of information is the result of mapping the two-dimensional channel, h(λ,t), to a one-dimensional receive signal, y(t), via a linear operation with the transmitted signal: a projection of h onto a reduced dimension subspace defined by the signal.

Trajectories can be introduced into the model as specular (mirror-like) moving targets. A specular target can be modeled using a delta function. A delay that depends on the range to the target is applied to the delta function. Target motion is incorporated by allowing the range-dependent delay to vary with time. With a single trajectory this yields an impulse response of

$$h(\lambda, t) = \alpha\,\delta(\lambda - r(t)) \tag{2}$$

where α is the target amplitude and r(t) is the time-dependent range describing the motion path (trajectory) of the target. After adjusting the impulse response for propagation delay, (2) becomes

$$h(\lambda, t-\lambda) = \alpha\,\delta(\lambda - r(t-\lambda)) \,. \tag{3}$$

Given that the target's range-rate does not exceed the speed of propagation, $|\dot{r}(t)| < 1$, there is exactly one root of the argument of the delta function (one solution to $\lambda = r(t-\lambda)$). Dot notation is used for the derivative with respect to time, $\dot{r}(t) \equiv \partial r(t)/\partial t$. The impulse response in (3) is substituted into (1) to yield

$$y(t) = \alpha\, s(t - f(t)) \,, \tag{4}$$

where f(t) is the function that solves the root of the delta functional in (3).

As an example consider a constant velocity case with a specular target trajectory given by

$$r(t) = r_0 + vt \tag{5}$$

where r_0 is the target range at t equal to zero, and v is the target range rate. Substituting (5) into the argument of the delta function in (3) and solving the argument for zero gives

$$\lambda = r_0 + v(t-\lambda)$$

which simplifies to

$$\lambda = \frac{r_0 + vt}{1+v} \,. \tag{6}$$

Substituting the solution for λ in (6) into s(t - 2λ) and manipulating into the form of (4) yields the distortion function

$$f(t) = \frac{2r_0}{1+v} + \frac{2v}{1+v}t \,. \tag{7}$$

The distortion function of (7) indicates the signal received from a constant velocity target will experience a time delay (constant term) plus a time scaling (linear term).

3. TRAJECTORY AMBIGUITY

Active sensor ambiguity can be represented in terms of ambiguous range versus time trajectories [4,5,7]. Using a trajectory ambiguity representation offers insight into a system's capability to resolve multiple targets exhibiting complex motion. Furthermore, the relationship between signal instantaneous frequency and trajectory ambiguity is useful for signal design.

For FM signals a phase ambiguity condition can be expressed as

$$\phi_1(t - f(t)) = \phi_2(t) + 2\pi n \tag{8}$$

where $\phi_1(t)$ and $\phi_2(t)$ are the phase functions of two signals, f(t) is the distortion function as described in (1)-(4), and n is an integer. Pairs of signals satisfying (5) are ambiguous in phase, and a receiver cannot discriminate between them. If (8) is solved using $\phi_1(t)$ equal to $\phi_2(t)$ an auto-ambiguity representation is obtained in the "trajectory domain." Solving for different phase functions yields a cross-ambiguity representation.

As an example of trajectory auto-ambiguity, consider a hyperbolic FM (HFM) signal (linear period modulation). Figure 1 shows a set of ambiguous target trajectories for an example HFM signal. Figure 2 shows the corresponding velocities for each of the trajectories in Figure 1.

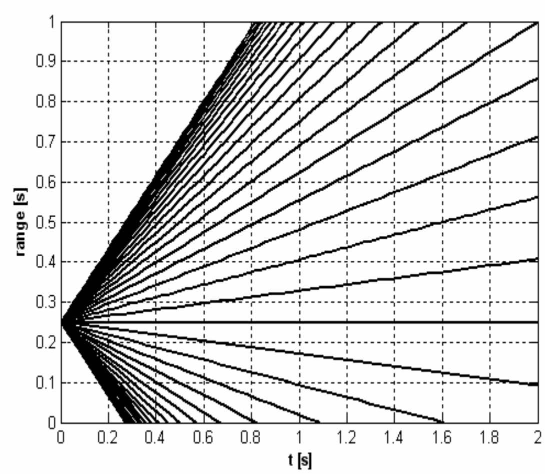

Figure 1. Example ambiguous trajectories for Hyperbolic FM (HFM) signal. Each trajectory is constant velocity.

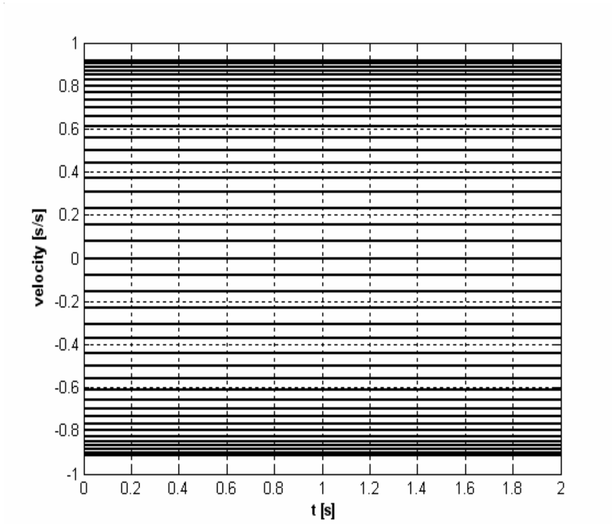

Figure 2. Velocity vs. time of ambiguous trajectories for HFM signal

The trajectories described by Figures 1 and 2 are constant velocity. It follows that the example HFM signal is insensitive in phase to the velocities shown in Figure 2. The edge regions with closely spaced trajectories in Figures 1 and 2 contain trajectories approaching the speed of sound (slope equal to one in Figure 1).

Equation (8) can be offset by π to solve for maximum sensitivity:

$$\phi_1(t - f(t)) = \phi_2(t) + \pi + 2\pi n \,. \tag{9}$$

For an HFM signal this will also yield constant velocity trajectories. For the example in Figures 1 and 2 the trajectories solving (9) lie between the ambiguous trajectories. The region of insensitivity can thus be characterized as a range of velocities with an extent that is small relative to the trajectory spacing and is periodic as indicated in Figure 2. As signal bandwidth is lowered the trajectory spacing will increase, increasing the effective range of insensitive velocities.

4. TRAJECTORY AMBIGUITY RELATIONSHIP TO AMBIGUITY FUNCTION INSENSITIVITY

Signal design for motion insensitivity has been treated in [2] using a generalized form of the ambiguity function. This section demonstrates how the ambiguity function approach is related to the trajectory ambiguity view of insensitivity.

Ambiguity function insensitivity to motion

A generalized ambiguity function for time delay and time distortion is given by

$$A_u(\tau,\varepsilon) = \int_{-\infty}^{\infty} \sqrt{1-\varepsilon \dot{f}(t-\tau)}\, s(t) s^*[(t-\tau) - \varepsilon f(t-\tau)]dt \tag{10}$$

where τ is time delay, f(t - τ) is a time distortion function equivalent to the distortion function described by (1) - (4), s(t) is the transmitted signal, and ε is a scale factor for the distortion [2]. Equation (10) generalizes the ambiguity function to handle delay and an additive time distortion.

Along the τ axis (the time autocorrelation function of the signal) for small τ, the square magnitude of A_u is a function of the RMS bandwidth of the signal, σ_B

$$\left|A_u(\tau,0)\right|^2 \approx 1 - \sigma_B^2 \tau^2 \,.$$

Larger σ_B^2 leads to a narrower time autocorrelation function and smaller range resolution. A similar relationship exists along the epsilon axis where the distortion autocorrelation function is given by

$$\left|A_u(0,\varepsilon)\right|^2 \approx 1 - \eta^2 \varepsilon^2 \tag{11}$$

where η^2, a function of the signal and the distortion function, is analogous to σ_B^2, a squared "bandwidth" with respect to the distortion, f(t). Minimizing η^2 minimizes the sensitivity to the motion described by f(t), flattening the ambiguity function along the motion distortion axis.

Motion Insensitivity in Phase

The generalized ambiguity function of (10) can be used to derive a signal that maximizes phase insensitivity to motion. The signal is assumed to have the form

$$u(t) = a(t) e^{j\phi(t)} \tag{12}$$

where a(t) is the amplitude of the signal and ϕ(t) is the phase. The motion bandwidth η^2 from (11) is a function of both the amplitude and the phase of the signal. It is shown in [2] that, for small epsilon, η^2 can be minimized with respect to the phase term by using a signal that satisfies

$$\dot{\phi}(t) = \frac{1}{Kf(t)} \,. \tag{13}$$

Thus motion sensitivity is minimized by matching the instantaneous frequency of the signal, $\dot{\phi}(t)$, to the reciprocal of the distortion function. Equivalently, (13) can be interpreted as matching the instantaneous period of the signal, $1/\dot{\phi}(t)$, to the distortion.

Connection to phase insensitivity results

The motion insensitivity results derived using the ambiguity function are consistent with the trajectory ambiguity results. To analyze the relationship, the phase function in (12) can be expanded about ε equal to zero for small ε to yield

$$\phi(t-\varepsilon f(t)) \approx \phi(t) - \varepsilon f(t)\dot{\phi}(t) .$$

Choosing a signal such that

$$f(t)\dot{\phi}(t) = K$$

yields the approximation

$$\phi(t-\varepsilon f(t)) \approx \phi(t) - \varepsilon K$$

which satisfies the trajectory ambiguity phase insensitivity condition of (8) to within a constant. For larger ε higher order terms can be included in the expansion to yield

$$\phi(t-\varepsilon f(t)) = \phi(t) + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \varepsilon^n f^n(t)\phi^{(n)}(t) . \quad (14)$$

where $\phi^{(n)}(t)$ is the nth derivative of the phase function with respect to time. Equation (14) specifies that for complete phase insensitivity the nth derivative of signal phase multiplied by the nth power of f(t) must be a constant. There are only two cases where a complete match is known to exist:

Linear distortion (constant velocity) and the hyperbolic FM waveform

The linear distortion function in this case is given by $f(t) = st$, and the corresponding signal instantaneous frequency is $\dot{\phi}(t) = K/t$ where K is a constant. Equation (14) is satisfied considering

$$f^n(t) = s^n t^n , \; \phi^{(n)}(t) = \frac{(-1)^{n-1} K}{(n-1)!} t^{-n}$$

and thus

$$f^n(t)\phi^{(n)}(t) = C_n .$$

Constant distortion and constant frequency (CW)

The constant distortion function is given by $f(t) = \tau$, and the corresponding signal instantaneous frequency is $\dot{\phi}(t) = K$ where K is a constant. Since $\phi^{(n)}(t) = 0, n > 1$, equation (14) is satisfied for the CW case.

For other scenarios the insensitivity condition of (13) is approximate, but by generalizing (10) the higher-order terms of (14) can be matched as well. For example, the additive distortion function in (10) cannot model arbitrary velocity and acceleration simultaneously. By adding an additional distortion parameter to the ambiguity function this could be addressed.

5. CONCLUSION

Waveform design to maximize *insensitivity* to motion is an important problem in cases where motion is a nuisance to the active sensor system. In the past, design for motion insensitivity has been approached using both trajectory ambiguity and a generalized ambiguity function. This paper discussed motion insensitivity as represented by both approaches.

The generalized ambiguity function result obtains phase insensitivity by matching the instantaneous period of the signal to the distortion, effectively flattening the ambiguity function along the motion axis. It was shown how this result can also be derived using trajectory phase ambiguity.

The flatness along a motion axis of the ambiguity function (motion insensitivity) is analogous to sets of ambiguous trajectories in the trajectory domain (trajectory ambiguity). Both representations describe the same underlying ambiguity, and both are useful tools for understanding signal design for motion insensitivity.

6. REFERENCES

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