ANTENNA ARRAY CONFIGURATION FOR HIGH-THOUGHPUT COMMUNICATIONS

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ABSTRACT

Strategically selecting the location of antennas in sparse arrays can dramatically improve a communication system's resilience to both multi-path fading and interference. By formulating the information theoretic capacity of a system employing an antenna array (both SIMO and MIMO) for a specified channel and interference model, we show how to optimally choose the locations of each antenna in both a 1- and 2-D space to maximize mutual information given prior constraints on the antenna positions.

1. INTRODUCTION

Communication systems are subject to multi-path interference and multi-path fading, as well as co-channel and hostile interference (jamming). Many modern receivers [1] employ smart antennas to combat interference by combining the weighted received signal from an N-element antenna array to construct an adaptive beam forming output expressed as

$$\hat{s} = \mathbf{w}^H \mathbf{y} \tag{1}$$

in which $\hat{s}$ is an estimate of the transmitted information-bearing signal s, $\mathbf{y}$ is the $N \times 1$ received signal vector, and $\mathbf{w}$ is the minimum variance distortionless response (MVDR) beam forming weight vector [2] which is expressed as

$$\mathbf{w} = \frac{\mathbf{R}_{i+n}^{-1}\mathbf{h}}{\mathbf{h}^H \mathbf{R}_{i+n}^{-1}\mathbf{h}} \tag{2}$$

In equation (2) $\mathbf{R}_{i+n}^{-1}$ is the $N \times N$ interference-plus-noise correlation matrix and $\mathbf{h}$ is the channel vector. Each element of the $N \times 1$ vector $\mathbf{h}$ corresponds to the complex channel coefficient binding the transmitter to each receiver. Extending (2) to address frequency selective fading is a straightforward extension of the formulation above [3].

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Equation (2) can be broadened to include multiple antennas on both transmit and receive by constructing the channel matrix $\mathbf{H} \in \mathbb{C}^{N \times M}$, where M is the number of transmitters. Noting that if both the interference and noise are Gaussian distributed, the minimum variance beam forming weight matrix is given by

$$\mathbf{W}^H = (\mathbf{H}^H \mathbf{R}_{i+n}^{-1}\mathbf{H})^{-1}\mathbf{H}^H \mathbf{R}_{i+n}^{-1} \tag{3}$$

and

$$\hat{\mathbf{s}} = \mathbf{W}^H \mathbf{y} \tag{4}$$

where $\hat{\mathbf{s}}$ is the estimate of the $M \times 1$ information-bearing signal vector.

Starting from equations (1) and (4), we will show that by optimizing the mutual information with respect to the channel coefficients we can determine the antenna locations that optimize communication capacity for a given power allocation; we will use the fact that the channel coefficients are a function of the receiver antenna positions in the optimization process.

Optimal antenna placement is a tradeoff between placing the antenna elements close together and moving the elements far apart. When antenna elements are far apart, the spatial diversity gain that helps to mitigate multi-path fading weakens interference mitigation by potentially introducing grating lobes and dispersion [4]. Conversely moving the antenna elements close to one another (e.g., $\leq \lambda/2$ spacing) may adversely effect the array's resilience to multi-path fading but potentially eliminates both grating lobes and dispersion. We will compare the results of optimizing the mutual information in the presence of multi-path fading and interference to determine the location of the antenna positions to those obtained using a minimum redundancy array formulation [5] for the 1-D case of a sparse linear array.

2. MUTUAL INFORMATION

We express the received signal vector as

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v}_{i+n} \tag{5}$$

where $\mathbf{v}_{i+n}$ is the interference plus noise vector and

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & & \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix} \tag{6}$$

is the channel matrix where for SIMO systems $M = 1$. It is easy to show that the distribution of $\hat{s}$ and $\hat{\mathbf{s}}$ from equations (1) and (4) are given as

$$\hat{s} \sim \mathcal{CN}(0, \| \mathbf{R}_{i+n}^{-1/2}\mathbf{h} \|_F^{-2} + \sigma_{ss}^2) \tag{7}$$

$$\hat{\mathbf{s}} \sim \mathcal{CN}(\mathbf{0}, (\mathbf{H}^H \mathbf{R}_{i+n}^{-1}\mathbf{H})^{-1} + \frac{\sigma_{ss}^2}{M}\mathbf{I}) \tag{8}$$

where the signal mean and variance are expressed as $E\{\hat{\mathbf{s}}\} = 0$, and $E\{\hat{\mathbf{s}}\hat{\mathbf{s}}^H\} = (\sigma_{ss}^2 / M)\mathbf{I}$, and the interference plus noise mean and variance are given by $E\{\mathbf{v}\} = 0$, and $E\{\mathbf{v}\mathbf{v}^H\} = \mathbf{R}_{i+n}$.

The mutual information is expressed as [6]

$$I(\hat{\mathbf{s}};\mathbf{s} \mid \mathbf{H}) = H(\hat{\mathbf{s}} \mid \mathbf{H}) - H(\hat{\mathbf{s}} \mid \mathbf{s}, \mathbf{H}) \tag{9}$$

where $H(\hat{\mathbf{s}} \mid \mathbf{H})$ and $H(\hat{\mathbf{s}} \mid \mathbf{s}, \mathbf{H})$ are the differential entropy. Using equations (5) and (8) we get

$$\begin{aligned} H(\hat{\mathbf{s}} \mid \mathbf{s}, \mathbf{H}) &= K + \frac{1}{2}\log_2 |(\mathbf{H}^H \mathbf{R}_{i+n}^{-1}\mathbf{H})^{-1}| \\ H(\hat{\mathbf{s}} \mid \mathbf{H}) &= K + \frac{1}{2}\log_2 |(\mathbf{H}^H \mathbf{R}_{i+n}^{-1}\mathbf{H})^{-1} + \frac{\sigma_{ss}^2}{M}\mathbf{I}| \end{aligned} \tag{10}$$

where $K = \frac{M}{2}\log_2(2\pi e)$ so that

$$I(\hat{\mathbf{s}};\mathbf{s} \mid \mathbf{H}) = \log_2 |\mathbf{I} + \frac{\sigma_{ss}^2}{M}\mathbf{H}^H \mathbf{R}_{i+n}^{-1}\mathbf{H}| \tag{11}$$

Equation (11) represents the mutual information at the beam forming output of the MIMO receiver as defined by equation (4). The results generalize to the SIMO case as

$$I(\hat{s}; s \mid \mathbf{h}) = \log_2(1 + \sigma_{ss}^2 \| \mathbf{R}_{i+n}^{-1/2}\mathbf{h} \|_F^2) \tag{12}$$

3. THE CHANNEL AND INTERFERENCE MODEL

The physical properties of the channel model are based on the far-field approximation that the transmitted signal may be approximated as a plane wave at the receiver.

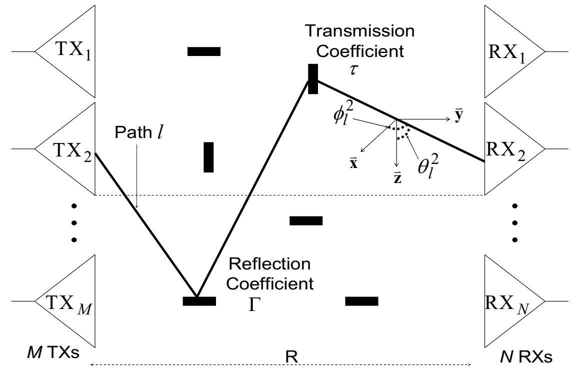


Fig. 1. Scattering environment with plane wave propagation. Multi-path amplitude and phase is a function of path length and the reflection and transmission coefficients of the scatterers.

In the channel model each scatterer is assigned a coefficient of reflection and transmission to account for both reflection and refraction, and the model can be generalized to include each antenna's beam shape and power-aperture as well as diffraction [7]. As illustrated in Fig. 1, the physical channel model is represented as

$$\mathbf{H}(\mathbf{P}) = \sum_{l=1}^{N_p} \mathbf{A}(\mathbf{P}, l)\mathbf{B}(l) \tag{13}$$

where $\mathbf{A}(\mathbf{P}, l) \in \mathbb{C}^{N \times M}$, $\mathbf{B}(l) \in \mathbb{C}^{M \times M}$, N_p is the total number of propagation paths between the transmit and receive array and

$$\mathbf{A}(\mathbf{P}, l) = \begin{bmatrix} e^{-\frac{j2\pi \mathbf{p}_1^T \mathbf{a}_1^l}{c}f} & \cdots & e^{-\frac{j2\pi \mathbf{p}_1^T \mathbf{a}_M^l}{c}f} \\ \vdots & \ddots & \vdots \\ e^{-\frac{j2\pi \mathbf{p}_N^T \mathbf{a}_1^l}{c}f} & \cdots & e^{-\frac{j2\pi \mathbf{p}_N^T \mathbf{a}_M^l}{c}f} \end{bmatrix} \tag{14}$$

with

$$\mathbf{B}(l) = diag(b_1^l \quad b_2^l \quad \cdots \quad b_M^l) \tag{15}$$

where f and c are the signal frequency and speed of light, respectively, and b_i^l is the fading coefficient of the l^{th} path between transmit antenna i and each antenna on receive. In equation (14), $\mathbf{p}_j^T = [x_j \quad y_j \quad z_j]$ is the position of the j^{th} antenna in Cartesian coordinates where the columns of the matrix $\mathbf{P} = [\mathbf{p}_1 \quad \mathbf{p}_2 \quad \cdots \quad \mathbf{p}_N]$

represents the positions of each of the N receive antennas, and $\mathbf{a}_l^k = [\sin\theta_l^k \cos\phi_l^k \quad \sin\theta_l^k \sin\phi_l^k \quad \cos\phi_l^k]$ is the unit vector corresponding to the direction-of-arrival at the receiver of the signal from the l^{th} path originating from transmit antenna k, where θ and ϕ are the angle-of-arrival of the signal in elevation and azimuth, respectively.

The channel matrix associated with P interfering sources, $\mathbf{H}_{\text{int}} \in \mathbb{C}^{N\times P}$, has the same form as (13). With the interference denoted by $\mathbf{i}$, the correlation matrix $\mathbf{R}_{i+n}$ is then expressed as

$$\mathbf{R}_{i+n} = \frac{1}{B}\int_{f_c-\frac{B}{2}}^{f_c+\frac{B}{2}} E\{\mathbf{H}_{\text{int}}^H \mathbf{i}\mathbf{i}^H \mathbf{H}_{\text{int}}\} + \sigma_n^2 \mathbf{I}\, df \qquad (16)$$

where the jk^{th} element of the matrix is given by

$$[\mathbf{R}_{i+n}]_{jk} = \sigma_i^2 \sum_{l=1}^{N_{p_i}} \sum_{m=1}^{N_{p_i}} \alpha_{jk}(l,m) b_j^l b_k^{m*} + \sigma_n^2 \delta(j-k) \qquad (17)$$

with

$$\alpha_{jk}(l,m) = \sum_{r=1}^{N} e^{-\frac{j2\pi \mathbf{p}_r^T(\mathbf{a}_j^l - \mathbf{a}_k^m)}{c} f_c} \operatorname{sinc}\left(\frac{\mathbf{p}_r^T(\mathbf{a}_j^l - \mathbf{a}_k^m)}{c} B\right) \qquad (18)$$

where $E(\mathbf{i}\mathbf{i}^H) = \sigma_i^2 \mathbf{I}_p$, $\delta(\cdot)$ is the Dirac delta function, and Npi is the total number of paths between the interference source and receive array. Due to the potential for the dispersion of a signal received by a sparse array whose elements are spaced at distance that is greater than $\lambda/2$ (half wavelength), the expression in equation (16) is integrated over the signal bandwidth B at center frequency f_c [8]. Note that the spectral response of $\mathbf{R}_{i+n}$ is assumed flat in the formulation of the correlation matrix in equations (16) through (18) above.

4. ANTENNA POSITION OPTIMIZATION

In equation (11) the mutual information is formulated with respect to a fixed channel model. Given a probability distribution for both the angle-of-arrival and fading coefficients of the desired signal and interference that are derived from measurements and/or generated from a model [7], we can express the probability that the channel matrix $\mathbf{H}$ is equal to $\mathbf{H}_k$ and the interference plus noise covariance $\mathbf{R}_{i+n}$ is equal to $\mathbf{R}_{i+n}^{k'}$ as

$$p(\mathbf{H} = \mathbf{H}_k, \mathbf{R}_{i+n} = \mathbf{R}_{i+n}^{k'}) = \gamma_{k,k'} \qquad (19)$$

where (19) corresponds to a discrete version of the probability density characterizing the channel and interference that is derived from a statistical model or measurements. Using (19) the weighted optimization of the mutual information with respect to each of the antennas in the receive array can then be formulated as

$$C_P = \max_{\forall \mathbf{p}_j \in \mathbf{P}} \log_2 \left| \prod_{k=1}^{Q} \prod_{k'=1}^{Q'} \left(\mathbf{I} + \frac{\sigma_{ss}^2}{M} \mathbf{H}_k^H \mathbf{R}_{i+n}^{-1k'} \mathbf{H}_k\right)^{\gamma_{k,k'}} \right| \qquad (20)$$
$$\text{s.t.} \quad f(\mathbf{P}) \succeq 0$$

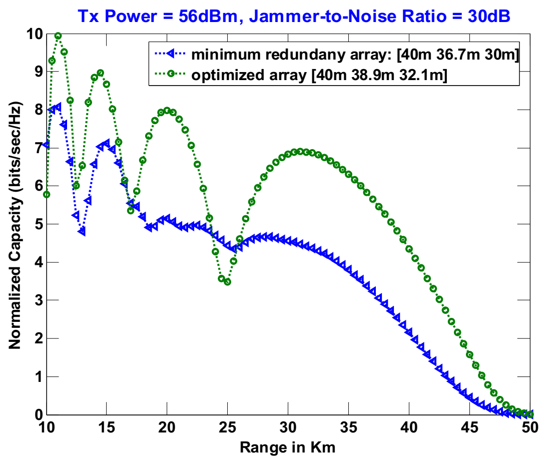
with

$$f(\mathbf{P}) = \begin{bmatrix} \mathbf{P} - \mathbf{K}_l \\ \mathbf{K}_u - \mathbf{P} \end{bmatrix} \qquad (21)$$

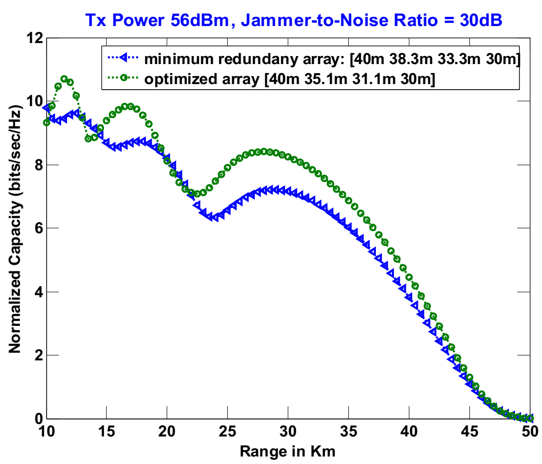
where C_P represents the average normalized information-theoretic capacity (bits/second/Hz), $Q\times Q$' are the number of quantized points in the probability mass function of (19) and $\mathbf{K}_l$ and $\mathbf{K}_u$ are the upper and lower bounds on the position of the receiver's antennas. Equation (20) represents a determinant maximization problem with matrix inequality constraints that is solvable using interior point methods [9].

5. SIMULATION RESULTS

Our simulation environment consists of ship-borne assets operating in a littoral environment employing a single C-band transmit antenna and either three or four antennas on receive (SIMO). The receivers were subject to hostile main-beam jamming (jammer-to-noise ratio = 30dB) from two stand-off airborne assets with the first asset fixed at 0.35 degrees off the horizon, and the other asset swept from .35 to .7 degrees off the horizon. Each ship-borne asset engaged in time division pair-wise communications in a 12 KHz instantaneous bandwidth and employed synchronized frequency hopping to avoid co-channel interference. Communication distances between two ship-borne assets ranged from 10Km to 50Km with a constant transmit power of 56dBm. Using equation (20) with Q = 49000, Q' = 2000, $\gamma_{i,i'} = \gamma_{j,j'}$ for all [i,i'], [j,j'] (corresponding to a uniform distribution) and the constraint that each receive antenna could be positioned between 30m and 40m in elevation, we determined the optimum placement for each receive antenna given a transmit antenna positioned at an elevation of 40m. The comparison of the performance of the ship-borne communication system with antenna placement derived using (20) as described above to that of a communication system operating in the same environment but whose antenna positions where chosen using a minimum redundancy array formulation are illustrated in Fig. 2 below.



(a)



(b)

Fig. 2. Information-theoretic capacity of two units 10Km to 50Km apart communicating over water in the presence of one fixed and one mobile main beam jammer. (**a**) Three receive antennas. (**b**) Four receive antennas.

The illustrations in Fig. 2 express the averaged information-theoretic capacity with respect to each of the distances over which two ships will engage in communications. The fall-off in communication capacity as the distance between two ships approaches 50Km (near the horizon in our round earth model) is due to the near perfect cancellation of the direct path signal by the specular. We have found that the improvement in throughput of a communication system using antenna elements positioned using the constrained determinant maximization in (20) is due to the balancing of the beneficial effects of separating the antennas to spatially decorrelate the multi-path and minimize the number of the of element pairs with the same spatial correlation lag while mitigating the deleterious effects of grating lobes. For both the 3- and 4-element array case as illustrated in Fig. 2, the optimized array outperforms the minimum redundancy array at nearly every communication range of interest.

6. CONCLUDING REMARKS

Intelligently selecting the position of antennas in a sparse array can dramatically improve the performance of a communication system that is subject to both fading and interference. We showed that by maximizing the weighted sum of the mutual information over all channel and interference models of interest we could determine the optimum spatial positions of the elements of an antenna array in 1- or 2-dimensions. We then demonstrated in a representative environment the improvement in performance of a communication system whose antenna positions were selected using the formulation in equation (20) as compared to a communication system whose antenna positions were chosen using a minimum redundancy array formulation.

7. REFERENCES

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