A NEW MAXIMUM A POSTERIORI CFAR BASED ON STABILITY IN SEA CLUTTER STATE-SPACE MODEL

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ABSTRACT

In this paper, a new constant false alarm rate (CFAR) for marine environment based on stability in sea clutter state-space model is proposed. In this CFAR, based on observation samples in the reference window, a maximum a posteriori (MAP) estimation for clutter power in the test cell is utilized for setting a threshold. The compound K model is the best model for high resolution sea clutter that includes a phenomenological insight into clutter formation, and it is most frequently cited in literature. The compound model for sea clutter is considered to be multiplication of two independent processes. This multiplicative model is linearized to form a state-space model for the sea clutter evolution. This state space model is established in the logarithmic domain. By introducing a stable distribution for innovations in the state-space model as a priori, we derive a new MAP estimator for the clutter power. The performance of the new CFAR named SLMAP-CFAR; stable linear MAP CFAR, is then compared to the existing LMAP.

1. INTRODUCTION

Radars often employ CFAR processors to adapt the detection threshold automatically to the local noise or clutter power in order to maintain an approximately constant false alarm. Under certain conditions the nature of radar back scatter from the sea surface is well known to depart significantly from the Rayleigh voltage form. Over the last few years the compound K-distribution model for sea clutter amplitude statistics has received much attention [1, 2], and it is now widely accepted that this provides a good phenomenological description of the sea clutter. The maximum improvements are obtained using ideal CFAR detection, where the local clutter power is assumed to be known exactly. Armstrong and Griffiths [3] undertook a study of the detection loss, relative to the ideal CFAR detection, of three different CFAR detectors where their study encompasses the cell averaging (CA), greatest of (GO) and order statistics (OS) CFAR, under a range of correlated, compound clutter conditions. They find that in the uncorrelated clutter, all three detectors suffer a large detection loss relative to the ideal CFAR, particularly in the case of spiky clutter. In short, the detectability loss suffered by the simple CFAR detectors described in [3] is close to the worst case loss, that is, the loss under conditions of the clutter modulation process being decorrelated from one range cell to the next. Bucciarelli et al. [4] study a number of different schemes, which attempt to take the spatial correlation structure of the clutter into account when estimating the local clutter power from the reference window. They have found that the estimator that linearly averages the radar returns in the logarithmic domain, where the weights are based on the covariance matrix of the modulating component, outperforms the CA CFAR scheme, hence, they introduce LMAP using MAP to estimate the local clutter power. Under certain conditions improving on LMAP, Noga [5] seeks for inherent nonlinearity in sea clutter evolution [1] and estimates the conditional heteroscedasticity; i.e., a time varying variance, model parameters for sea clutter and attempts to take the non-linearities in the structure of the clutter into account when estimating the local clutter power from the reference window. In [5] a new CFAR is introduced (referred to as CHLMAP CFAR) and the performance is also evaluated. Noga shows that little improvement can be accessed with CHLMAP compared to LMAP. In this paper, we introduce a correction in state-space model of [5]. Based on some evidence for stability in state-space model for sea clutter, a new CFAR scheme (referred to as SLMAPCFAR) is proposed and its performance is compared to LMAP CFAR. It is shown that an improvement in performance can be accessed. The clutter data needed for performance evaluation is generated according to T/Wv1 model, it is introduced in [6]. In this paper, we use the MAP principle to estimate the clutter power in the CFAR test cell and arrive at a nonlinear set of equations for a threshold that lead to higher CFAR performance obtained by LMAP CFAR. The paper is organized as follows: In section 2 we discuss the T/Wv1 model, and the state-space model for sea clutter enunciating the evidence for stability in the sea clutter; in this section, the new state-space model is also introduced. In section 3 the new CFAR scheme is introduced and the method to evaluate the performance of the proposed scheme is developed. The results of simulations are in section 4 and some concluding remarks are provided at the end.

2. SEA CLUTTER GENERATION MODEL

In this section, we provide the evidence for the sea clutter to follow a stable distribution; henceforth, a new CFAR scheme is proposed in the next section. The introduction of the compound form of the K-model was pioneered by Watts, Baker, and Ward, the model is made up of two components, the texture and speckle components. The range profile data needed for performance evaluation is generated by T/Wv1 model [6]; and for experimental compatibility, we generate 440 range profiles according to the properties reported in [5] from the real profiles. The histogram of the shape parameter and the correlation length of the range profiles are cited in [5].Bucciarelli et al. [4], Watts [7] and Armstrong and Griffiths [3] study the effects of the correlations in the modulating component on CFAR detection performance. A simple cell averaging CFAR detector is inherently incapable of approaching the ideal CFAR detection performance. The ideal CFAR performance, at least in an idealized speckle free environment, can be approached by adaptively setting the threshold based on a weighted average of the raw clutter samples in the neighboring range cells in the logarithmic domain, where the weighting coefficients are obtained from the correlation structure of the modulating component. A linearized, conditionally Gaussian state space model for high resolution sea clutter is developed by Noga [5]. It is argued that the (in)coherent sea clutter return in range cell n at time t (denoted by x_n^t) can be expressed as:

$$x_n^t = \nu_n^t s_n^t \tag{1}$$

where ν_n^t is the level of the underlying modulating process (sometimes referred to as the texture component) in range cell n at time t, and s_n^t refers to the corresponding speckle. Let's consider T consecutive clutter returns in range cell n, denoted by $\mathbf{x}_n = [x_n^t, \cdots, x_n^{t+T-1}]'$. Provided that T corresponds to a time period much shorter than the decorrelation time of the modulating component of sea clutter, the local power of the returns, $\sigma_n = 2\nu_n^2$, can be effectively considered to be constant. Noga [5] describes two alternative approaches that both are based on a Gaussian approximation to the posterior distributions. Noting that the likelihood for σ_n can be cast into a linear Gaussian form as:

$$\ln(\mathbf{x}_n' \Sigma^{-1} \mathbf{x}_n) - \psi^{(0)}(T) = \ln \sigma_n + \nu \sqrt{\psi^{(1)}(T)} \tag{2}$$

where T is the integration period and $\nu \sim \mathcal{N}(0,1)$, where $\mathcal{N}(\cdot)$ denotes a zero mean, variance of one Gaussian probability density function (PDF). For the modulating component linear and nonlinear models can be used that they form the state transition equation. The general form of state transition equation proposed in [5] is as follows:

$$y_n = \beta_0 + \sum_{\substack{j=-q \\ j\neq 0}}^{q} \beta_j y_{n+j} + \epsilon_n \sqrt{h_n}, \; \ln h_n = \alpha_0 + \sum_{\substack{i=-p \\ i\neq 0}}^{p} \alpha_i y_{n+i} \tag{3}$$

where $\epsilon_n \sim \mathcal{N}(0,1)$, in (3) we consider a form of heteroscedasticity in the clutter evolution. For this state-space model the logarithm of square of the speckle component is considered to be Gaussian. The state-space model as proposed in [5] is as follows:

$$\begin{aligned} \mathbf{w}_{-n} &= \mathbf{y}_{-n} + \sigma_{w|y} \nu_{-n}, \quad y_n = \mu_n + \epsilon_n \sqrt{h_n} \\ \mu_n &= \beta_0 + \sum_{\substack{j=-q \\ j\neq 0}}^{q} \beta_j y_{n+j}, \quad \ln h_n = \alpha_0 + \sum_{\substack{i=-p \\ i\neq 0}}^{p} \alpha_i y_{n+i} \end{aligned} \tag{4}$$

where $w_n = \ln(\mathbf{x}_n' \Sigma^{-1} \mathbf{x}_n) - \psi^{(0)}(T)$, $y_n = \ln \sigma_n$, and $\sigma_{w|y} = \sqrt{\psi^{(1)}(T)}$. The vectors with subscript n indicate the reference window samples. Two evidences for stability of $\mathbf{w}_{-n}$ as Nikias prescribes, are illustrated in [8] as tests for stability of a stochastic process, one is the test of infinite variance based on a property of stable processes [8]. For a non-Gaussian stable process there are no finite moments beyond a certain moment order. Practically, the divergence in the sample variance as the number of samples increases or jumps in the variance estimate can be known as an evidence for stability. The result of this test is shown in Figure 1 for range profile data generated by T/Wv1 model. For the stable PDF parameters based on the sample fractile method [8], symmetricity, the characteristic exponent, and the dispersion of the stable PDF are estimated. These parameters; symmetricity, the characteristic exponent α, the dispersion γ, and $c = \sqrt[\alpha]{\gamma}$ are estimated as 1, 1.75, 0.724, and 0.8310 for T/Wv1 model, respectively. The second test as an evidence for stability is that stable PDF can have a better fit for the innovation process of state-space model. In Figure 2 the empirical PDF is compared to the Gaussian and the stable PDFs, from this Figure the stable PDF has a better fit to the empirical PDF, also an analytical inspection; Kolmogorof-Smirnov test, verifies the visual inspection. Based on stability tests of the innovation process and assuming a linear model for the modulating component the state-space is now corrected as follows:

$$\mathbf{w}_{-n} = \mathbf{y}_{-n} + c_{w|y} \nu_{-n}, \quad y_n = \mu_n + \epsilon_n, \tag{5}$$

where $\nu \sim S\alpha S(0,1)$, $\epsilon_n \sim S\alpha S(0,1)$, and $S\alpha S(\cdot)$ denotes the standard alpha stable PDF, $c_{w|y} = \sqrt[\alpha]{\gamma}$ and μ_n are as (4). Based on this state-space model Noga [5] extends LMAP CFAR to CHLMAP CFAR which comprises the non-linear effects in the clutter evolution. Next, we introduce the SLMAP improving LMAP performance.

3. NEW CFAR SCHEME AND ITS PERFORMANCE EVALUATION

In the previous section a new state-space model for sea clutter evolution was proposed. Based on the new model we propose a new CFAR scheme in which the clutter power estimate in the test cell is a MAP estimate conditioned on the observations in the reference window. To obtain the MAP estimate with the stability assumption which is referred to as SLMAP we assume that y is a priori jointly Gaussian with mean $\mathbf{m}_y$ and the covariance matrix Σ_y, estimates of which can be obtained from the time history of clutter returns. This assumption greatly simplifies the analysis, while retaining the most important ingredient in the analysis, the covariance structure of the modulating component. From the observation equation, the joint PDF conditioned on observations is as follows:

$$p(\mathbf{w}_{-n}|\mathbf{y}) \quad = \quad \prod_{i=1}^{L} \zeta\left(\frac{(w_i - y_i)}{c_{w|y}}; \alpha, \beta\right), \qquad (6)$$

where $\zeta(x; \alpha, \beta)$ is the standard stable PDF [8]. To obtain the MAP estimate, $p(\mathbf{y}|\mathbf{w}_{-n})$, the posteriori distribution should be maximized. From the priori about $\mathbf{y}$:

$$p(\mathbf{y}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{y}-\mathbf{m}_y)'\Sigma_y^{-1}(\mathbf{y}-\mathbf{m}_y)\right)}{(2\pi)^{-(2L+1)/2}|\Sigma_y|} \qquad (7)$$

and the relation between posteriori and priori distributions, $p(\mathbf{y}|\mathbf{w}_{-n}) = p(\mathbf{w}_{-n}|\mathbf{y})p(\mathbf{y})$, the posteriori distribution is obtained. To obtain the MAP estimate of the clutter power in the test cell, with the aid of maximizing the log likelihood function, a set of nonlinear equations should be solved resulting from the maximization of (7):

$$(\mathbf{y}-\mathbf{m}_y)'\Sigma_{yn}^{-1} + \frac{1}{c_{w|y}}\frac{\zeta'((w_i - y_i)/c_{w|y}; \beta)}{\zeta((w_i - y_i)/c_{w|y}; \alpha, \beta)} = 0 \qquad (8)$$

$$(\mathbf{y}-\mathbf{m}_y)'\Sigma_{yL+1}^{-1} = 0$$

where $i = 1, \cdots, L$ and Σ_{yn}^{-1} is the nth column of Σ_y^{-1}. From the above set of equations $\mathbf{y}$ is estimated and $\hat{y}_n = \mathbf{y}^{\text{MAP}}(L+1)$ is the estimate of clutter power in the test cell. Based on this estimate the SLMAP CFAR threshold can be considered as $T = G\exp(\hat{y}_n)$. The performance prediction of the new CFAR necessitates determining the threshold distribution, however, analytical performance evaluation proves to be nontrivial. With the aid of the stable random processe properties the approximate performance can be determined, considering the normalized threshold $\tau_n = \ln((V_T(n)/G)^2 = \mu_{n|w}$, $\mu_{n|w} = \beta_0 + \sum_{j=-q, j\neq 0}^{q} \beta_j y_{n+j}$, conditioned on the reference window samples, where the distribution of the normalized threshold is stable. The mean and c parameter of this distribution are as follow:

$$a = \beta_0 + \sum_{\substack{j=-q \\ j\neq 0}}^{q} \beta_j y_{n+j}, \quad c_\tau = c\sqrt[\alpha]{\sum_{j\neq 0} \beta_j^\alpha}. \qquad (9)$$

Knowing the distribution of normalized threshold, analytically determining the distribution of $V_T(n) = \exp(\tau_n/2)$, the main threshold, seems not to be a simple task. So for the main threshold distribution, a normal maximum likelihood ML estimate was utilized and its parameters μ_V and σ_V are ML estimates. Knowing the threshold distribution parameters, probability of detection and false alarm, conditioned on the modulating component, can be determined the same way as Watts [7] proposed for CA CFAR.

4. SIMULATION RESULTS

The single pulse target detection results are obtained using a SLMAP CFAR and LMAP CFAR detectors for the previously discussed database of 440 range profiles using T/Wv1 model. The average probability of false alarm for each profile was set to $P_{fa}=10^{-4}$, with the average probability of detection for Swerling I and II targets set to $P_d=0.5$. The reference window size was fixed at 10 range cells (with no guard cells and 4 range cells for the prediction error variance estimate of the CHLMAP CFAR detector). The performance of SLMAP CFAR relative to LMAP is shown in Figure 4 and of CHLMAP CFAR relative to LMAP is shown in Figure 3, a better performance in Figure 4 is attributed to the closer match of the true PDF by the stable PDF contrary to the Gaussian assumption. The performance results indicate that SLMAP CFAR detector ostensibly provides improvement relative to the LMAP CFAR detector.

5. CONCLUSION

In this paper, we proposed a new scheme for CFAR detector in compound clutter environment that results in improvement relative to the LMAP CFAR detector. The new scheme utilizes a more powerful linearization tool for the state-space model of sea clutter. By replacing the usual Gaussian assumption for the sea clutter by the stable model a naturally realistic formulation is determined, we obtain a better performance relative to LMAP and CHLMAP CFARs. The simulation results show the higher performance of the introduced method in terms of less SNR needed for a specified P_d and for a constant P_{fa}.

6. REFERENCES

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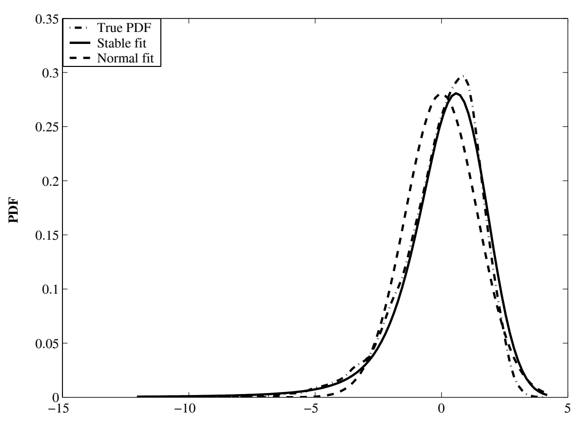

Figure 1: Stable PDF fit compared to Gaussian PDF.

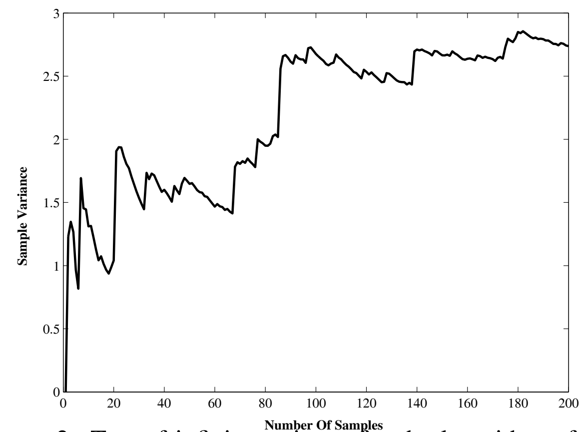

Figure 2: Test of infinite variance for the logarithm of the squared Rayleigh.

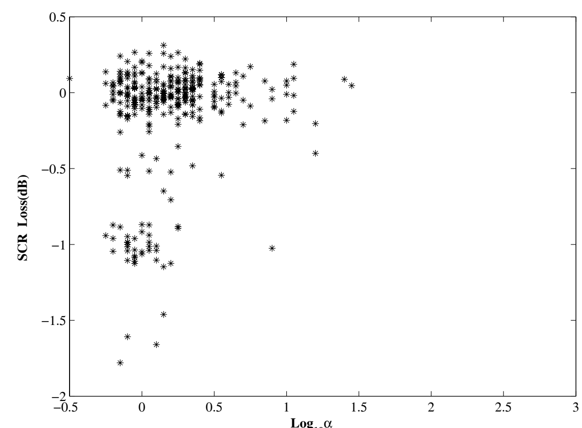

Figure 3: Detection loss for the CHLMAP CFAR detector relative to the LMAP CFAR detector with P$_d$=0.5 and P$_{fa}$=10^{-4}, with 10 cell window.

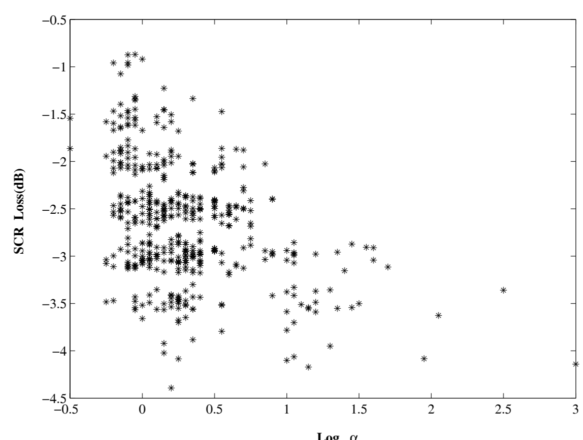

Figure 4: Detection loss for the SLMAP CFAR detector relative to the LMAP CFAR detector with P$_d$=0.5 and P$_{fa}$=10^{-4}, with 10 cell window.