HIDDEN MARKOV MODELS FOR RADAR PULSE TRAIN ANALYSIS IN ELECTRONIC WARFARE

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ABSTRACT

We present a new approach to radar pulse train analysis in electronic warfare. We consider an alternative to the classical Time-Of-Arrival (TOA) histogram technique commonly used for extraction of complex pulse patterns. We derive a Hidden Markov Model for the radar word templates, and develop a modi£ed version of the Viterbi algorithm to extract radar words from noisy and corrupted pulse sequences. We argue the advantages of this approach compared to the standard TOA histogram technique, and illustrate operation of the algorithm with computer simulation results.

1. INTRODUCTION AND PROBLEM STATEMENT

We present a new approach to radar pulse train analysis in radar Electronic Support (ES) systems of the defensive Electronic Warfare (EW) suite. The goal of ES is to collect and analyze potentially hostile radar signals. Responses to this information may range from simple warnings issued to the operator, to initiating soft- or hard-kill countermeasures. (See [1] for review of the £eld of Electronic Warfare). Speci£cally, ES must be able to decipher the tactical situation, including the type and threat-potential of radar emitters, using only the received signal.

One major dif£culty in this task is the overlapping RF channels used by military radar systems. The individual signals cannot be easily distinguished and must be de-interleaved into tracks that can be associated with unique emitters [2]. Moreover, the tracks must be classi£ed and identi£ed with speci£c emitter types using the data stored in a data base of Electronic Intelligence (ELINT). Such factors as the direction of arrival of electromagnetic waves, the carrier frequency, as well as pulse sequence structure are vital input variables to this decision process. After the source emitters have been identi£ed and pulses de-interleaved, the states of the radars and, consequently, the levels of threat associated with them, have to be estimated. This presents a broad range of opportunities for application of decision-theoretic, estimation, and discrete event system techniques. The approach presented here addresses some of the issues associated with detection and estimation of radar states encoded in radar signals.

An important goal of ES is to evaluate the level of threat posed by a potentially hostile radar [1]. Typically, the threat is closely associated with the generalized state of the radar *i.e.* Search, Acquisition, Tracking, or Missile Engagement. Track-While-Scan (TWS) radars and Multi-Function Radars (MFRs) can simultaneously perform multiple functions related to several targets. Therefore, evaluating the threat that such radars pose is one of the most challenging tasks of ES. The use of Hidden Markov Models (HMMs) to describe radar signals for this application was initially suggested in [3]. In [4], we presented a case study where we built a mathematical model of MFR and suggested an optimal state estimation algorithm to infer the radar state from observed data. We demonstrated the convenience of associating two levels of data organization with the MFR emitter signals – the *pulse level*, and the *word level*. Radar *words*, can be de£ned as static, or dynamically varying, groups of pulses that MFRs emit in different states. The ability to recognize these groups of pulses brings us closer to evaluating the radar state, and to estimating its threat level. Thus, one component of the pulse train analysis for ES is a problem of recognition of radar words. The radar words have to be extracted from the distorted pulse sequences recorded by the ES in a noisy environment. In Fig. 1, we show examples of word pulse structure for two different sample radar emitters. A detailed speci£cation for these words can be found in Appendix A.

Several dif£culties need to be addressed while solving the complex pattern recognition problem of extracting MFR words from the stream of observed pulses:

1. Radar pulses are observed through a stochastic non-stationary environment, characteristics of which may be unknown.
2. De-interleaving algorithms may fail to correctly separate distinct pulse sources. For example, pulses originating from one emitter "leak" into tracks that are predominantly associated with another emitter.
3. The observed pulse sequence is subject to quantization distortions due to absence of synchronization between the ES receiver and the radar.

The £rst two items can be modelled as various forms of electromagnetic pulse propagation channels. A simple binary, or binary erasure channel model is adequate for the £rst item. More sophisticated channel models such as Markov-Modulated channels can be used to emulate the effect of de-interleaver confusion. In this study, we only considered the binary channel in which pulses emitted by the radar could be lost with a probability p_{miss} (the probability that no pulse is detected at a particular quantization instant, given that one should have occurred in the corresponding signal), and spurious pulses could be introduced with a probability p_{spur} (the probability that a pulse is detected at some quantization instant, given that none was radiated).

The quantization distortions are due to the details of the speci£c hardware implementation of the ES. In general, the pulse se-

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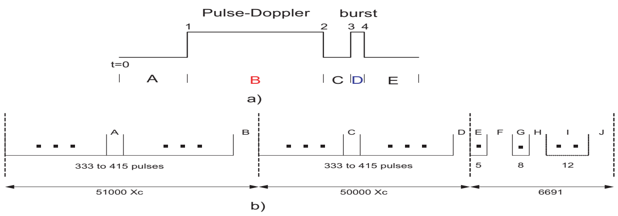

Fig. 1. Sample multi-function radar words.

quence quantization process is controlled by an observer clock described by a period T_{obs}. The theoretical synchronized pulse quantization model is de£ned by the following expression

$$n_i \equiv \left\lfloor \frac{t_i}{T_{obs}} \right\rfloor , \quad (1)$$

where t_i is the relative Time-Of-Arrival (TOA) of the received pulse, and n_i is an associated quantization index.

In practice, the pulse quantization model must include a uniformly distributed random phase $\varphi \in [0, T_{obs})$ to accommodate for the asynchronous nature of the radar-ES engagement,

$$n_i'(\varphi) = \left\lfloor \frac{t_i + \varphi}{T_{obs}} \right\rfloor = \begin{cases} n_i\,, & \text{with } 1 - p_i \\ n_i + 1\,, & \text{with } p_i \end{cases} . \quad (2)$$

The new quantization index $n_i'(\varphi)$ is a function of the random phase φ, and p_i is the *pulse splitting probability*

$$p_i \equiv \frac{t_i}{T_{obs}} - n_i\,. \quad (3)$$

Traditionally, the algorithms used for pulse train analysis in ES are based on Time-Difference-Of-Arrival histograms of either autocorrelations [2] or cross-correlations [5]. These algorithms are very simple and computationally ef£cient. However, their performance is not satisfactory for highly structured pulse sequences like those in Fig. 1 (see [6]). In the following sections, we present an algorithm rooted in the theory of HMMs that can be used to successfully solve the problem of word extraction from the deinterleaved MFR pulse sequences. We also show simulation results on some synthetic data generated for MFRs with the words shown in Fig. 1. In contrast with the traditional pulse train analysis algorithms, the performance of the proposed algorithms improves with the increasing complexity of the word pulse patterns.

2. HMMS AS RADAR WORD TEMPLATES

In this section, we derive a statistical model for radar word templates. The radar output can be represented in a quantized time domain de£ned by (2). Following [3], the appearance or absence of pulses at any given quantization instant can be represented, respectively, by 1 or 0 in a binary sequence. A single pulse, followed by a dead time can be viewed as a fundamental building block of any radar pulse sequence in the quantized time domain. This building block can be represented by a Markov chain shown in Fig. 2. The transition matrix for the Markov chain of the i^{th} pulse-interval within the k^{th} word is given by

$$\mathbf{A}_{k,i}^{'} = \begin{pmatrix} 0 & p_i & 1-p_i & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ 0 & 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} . \quad (4)$$

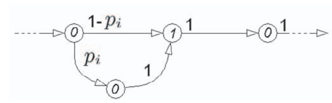

Fig. 2. Atomic Markov chain.

We refer to this structure as an *atomic Markov chain*. It is a *left-to-right* Markov chain [7], owing to the super-diagonal structure of the transition matrix. Furthermore, almost all non-zero transition probabilities are equal to 1; the only exception is the transition from the £rst state, which follows the PRI distribution of (3).

Any sequence of pulses emitted by a radar can be represented by the conjoining of an appropriate number of atomic Markov chains (4). By connecting these together, we can design models for words of arbitrary complexity, including those shown in Fig. 1. The transition probability matrix $\mathbf{A}_k$ of a composite Markov chain for the the k^{th} radar word template will have the following form

$$\mathbf{A}_k = \begin{pmatrix} \mathbf{A}_{k,1}^{'} & \mathbf{A}_{k,1}^{''} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{A}_{k,2}^{'} & \mathbf{A}_{k,2}^{''} & \mathbf{0} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{k,3}^{'} & \mathbf{A}_{k,3}^{''} & \mathbf{0} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} , \quad (5)$$

where

$$\mathbf{A}_{k,i}^{''} = \left(\begin{array}{c|c} \mathbf{0} & \mathbf{0} \\ \hline 1 & \mathbf{0} \end{array} \right) .$$

Combining the Markov chain (5) with the binary channel observation model for the ES discussed in Section 1, and de£ning the probability of missing a pulse as p_{miss}, and the probability of getting a spurious pulse as p_{spur}, we obtain an HMM for the radar word template as de£ned in [7] $\lambda_k = (\mathbf{A}_k, \mathbf{B}_k, \pi_k)$, where $\mathbf{B}_k$ is the observation probability matrix de£ned as

$$\mathbf{B}_k = \begin{pmatrix} 1 - p_{spur} & p_{spur} \\ 1 - p_{spur} & p_{spur} \\ p_{miss} & 1 - p_{miss} \\ 1 - p_{spur} & p_{spur} \\ \vdots & \vdots \end{pmatrix} , \quad (6)$$

and the initial state probability distribution is $\pi_k \equiv (1\,0\,0\,0\,\cdots)^T$.

3. VITERBI ALGORITHM FOR WORD EXTRACTION

Given the radar word template HMMs presented in Section 2, the problem of extracting radar words from noisy pulse sequence is equivalent to the problem of scoring the pulse sequence in the Viterbi sense. This procedure is described in detail in [7]. Thus, the word extraction algorithm involves the following steps:

1. Construction of *HMM templates* $\lambda_k = (\mathbf{A}_k, \mathbf{B}_k, \pi_k)$ for each radar emitter word k.
2. Calculation of the Viterbi log-score for each word template as described in [7].
3. The relative peaks in the Viterbi log-score correspond to candidate starting times for radar words in the quantized pulse sequence.

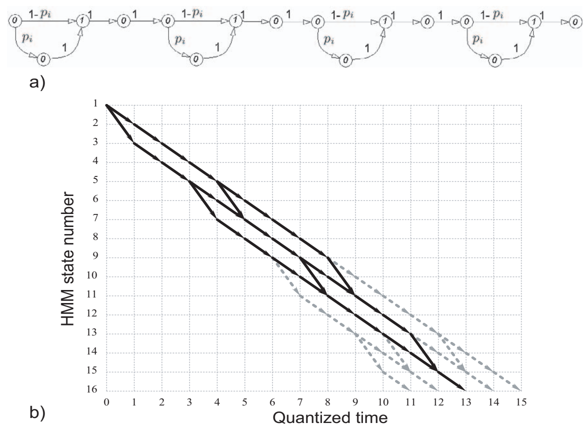

Fig. 3. Example of the trellis path of the Viterbi algorithm.

4. By executing the Viterbi backtracking procedure as described in [7], one can obtain the most likely distribution of individual pulse labels *i.e.* determine which pulses came from the radar, which should be declared spurious, and which ones were missing.

The generic Viterbi algorithm has a computational complexity $\mathcal{O}[LM_k^2]$ [7], where L is the length of the quantized pulse sequence, and M_k is the number of states in the HMM word template λ_k. For real-life radar systems, the number of states in HMM word templates can reach several hundreds of thousands. The higher the quantization precision of the ES (the smaller the observer clock period T_{obs}), the greater the number of states M_k. However, carefully exploiting the structure of the problem, we can show that a specialized Viterbi algorithm with the computational complexity having an upper bound of $\mathcal{O}[(N_p \times D)(M_k \times W_k)]$ can be developed. Here, $N_p \ll L$ is the number of pulses in the observed radar sequence, and D as well as W_k, are constants discussed below. Most importantly, the specialized Viterbi algorithm scales linearly with respect to M_k.

The £rst important observation that leads to reduction of the computational complexity of the scoring procedure is the following: the value of scores are signi£cant, and should be evaluated only at the positions in the quantized pulse sequence where pulses are present. The total length of the observation sequence is L, and it only contains N_p nonzero elements. Although both M_k and L increase with the decreasing T_{obs}, the number of observed pulses N_p is independent of T_{obs}. However, by using this technique, if the £rst several pulses of the word are missing in the observed sequence, the whole word may not be found. One possible solution to this problem is to introduce depth-D backtracking in the scoring algorithm. In that case, D scores are calculated for each observed pulse in a sequence. For the binary channel, the probability of missing several pulses in a row decreases exponentially. Therefore, D can be chosen so that this probability is driven below a certain threshold of tolerance, and the complexity bound becomes $\mathcal{O}[(N_p \times D)M_k^2]$.

The largest reduction in complexity of the Viterbi algorithm is achieved by taking advantage of the highly sparse nature of the transition probability matrix $\mathbf{A}_k$. Fig. 3 (a) and (b) illustrate how this feature can be exploited to reduce the number of paths used by the Viterbi algorithm. Suppose we have the HMM word template shown in Fig. 3 (a). Fig. 3 (b) shows the trellis for all possible transitions of this HMM. The rows on this diagram represent the sequential state numbers in the HMM (1-16), and the columns are the quantized time steps of the Viterbi algorithm (0-15). The shortest path through the trellis (state 16 at time 11) corresponds to the case where no pulse-splitting has occurred in the observation sequence. The longest path (state 16 at time 15), corresponds to the case where pulse-splitting has occurred for every single pulse. We de£ne $W_k = 15 - 11 = 4$ as the maximum width of the diagonal belt of the Viterbi trellis. In general, $W_k = W(\lambda_k)$ is an explicit function of the template structure, with its upper bound being the total number of pulses in the word. Clearly, the computational complexity of such an algorithm is no greater than $\mathcal{O}[(N_p \times D)(M_k \times W_k)] \ll \mathcal{O}[LM_k^2]$.

In practice, neither the shortest, nor the longest paths through the trellis are possible. In fact, the true radar word template has a certain duration (see Fig. 1 (b) for example), and this duration in the quantized domain can be evaluated using (1). Therefore, the valid path is located between the shortest and the longest one, and is unique for each word template. Suppose that in Fig. 3 the quantized word duration is 13. Thus, all the paths shown by grey dashed lines in Fig. 3 (b) are invalid, and should be eliminated. This amounts to roughly half of all possible paths. For HMM word templates with several hundreds of thousands of states, this is a substantial gain in performance.

4. SIMULATION RESULTS

In this section, we present some simulation results for the radar emitters with word structures shown in Fig. 1 and explained in Appendix A. We have performed a number of experiments where we generated synthetic word sequences for these emitters, and corrupted them with various levels of noise. Then, we performed scoring and word extraction on this data using the Viterbi algorithm as presented in Section 3.

Fig. 4 (a) shows results for approximately $35ms$ of data from the radar emitter having the word structure shown in Fig. 1 (a). Here, $T_{obs} = 1.1\mu s$, and about 20% of radar pulses were missed. The spur rate $\rho = \frac{p_{spur}}{T_{obs}}$ was set to $6000 pulses/s$. The top graph of Fig. 4 (a) shows the Viterbi scores, and the bottom graph plots the sequence of words extracted from the scores of the top graph. The emitter words and their respective scores are color-coded so that the words W1, W3, W5, W6, and W7 are represented, respectively, by red, blue, pink, black, and green. In this experiment, only these emitter words were detected. The scores for each word peak sharply in the location of the start of the word in the sequence. The extracted word sequence exactly matched the actual sequence emitted by the radar. This particular emitter is capable of searching for and tracking multiple targets in a time-multiplexed fashion. The transmission is arranged into blocks of four sequential words, each corresponding to either a tracking sequence or a searching pattern. In Fig. 4 (a), the emitter was tracking one target using a block consisting of four repetitions of the seventh word, W7-W7-W7-W7. The subsequent block was allocated to tracking a second target using a W1-W6-W6-W6 sequence. The next three blocks (or 12 words) follow a different pattern used to search for additional targets. The radar then resumes tracking the £rst target using the W7-W7-W7-W7 block and the entire £ve-block pattern repeats, with minor variations. This £gure shows how much information we can obtain about the state of a potentially hostile radar emitter simply by being able to extract and identify the words encoded into radar pulse sequences.

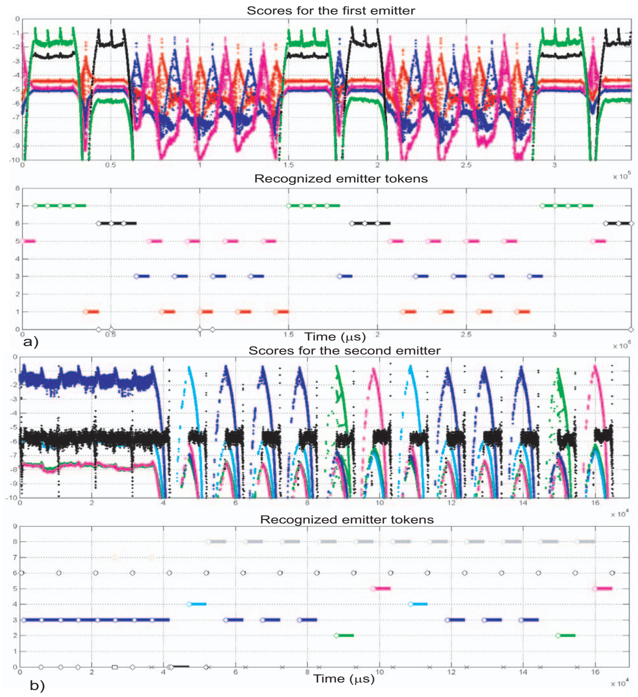

Fig. 4. Simulation results showing Viterbi scores. (a) shows the extracted word sequence for the radar words in Fig. 1 (a), and (b), for the radar words in Fig. 1 (b).

Fig. 4 (b) shows results of a similar experiment performed with $17ms$ of data generated by the emitter having the word structure shown in Fig. 1 (b). Although this particular radar emitter has only 5 words, we have considered the termination character and the dead time between segments in Fig. 1 (b), as well as an occasional period of emitter silence (absence of any pulse radiation for a known period of time), as separate words. Therefore, the total number of processor words is equal to 8. In this example, the radar was performing target acquisition. The extracted word sequence matched the expectations almost exactly. The algorithm failed to detect word W8 around $4.2ms$ due to some local bursts of noise in the observation sequence, preventing the word sequencer from making a reliable decision.

5. CONCLUSION

In this paper, we have presented a novel approach to the pulse train analysis in Electronic Support. We have solved the dif£cult Electronic Warfare problem of analyzing highly structured pulse sequences produced by Multi-Function Radar emitters with the aim of extracting radar words, that are related to the radar state. Currently, to the best of our knowledge, no complete solution to this problem is available in the unclassi£ed radar signal processing literature. The key element of this approach is the novel HMM statistical model of radar word templates. We have demonstrated promising simulation results. Due to the dynamic nature of HMMs, the presented approach can accommodate for an even larger class of MFRs, where the words may be ¤exible entities, whose pulse representations may be dynamically varying and dependent on the context in which they are found. The proposed approach could be used as the pulse train analysis stage of the radar state estimation algorithm that was introduced in [4], providing a complete solution to the radar state estimation problem.

A. APPENDIX: RADAR WORD SPECIFICATION

In Fig. 1, we show examples of word pulse structures for two different radar emitters, chosen for demonstration purposes. The £rst emitter has a vocabulary of 9 words (W1-W9), all of which have the same pulse envelope, shown in Fig. 1 (a). The envelope structure can be separated into 5 distinct sections (A through E). Sections A, C, and E are dead times of known duration, where no pulses are radiated by the emitter. Section B is the £xed Pulse Repetition Interval (PRI) pulse-Doppler component. The nine distinct words of the radar are distinguished by different PRIs of this section in the pulse envelope. Finally, section D contains 12 pulses at a nominally £xed PRI. All pulse-Doppler PRIs in section B are $< 100\mu s$, and pulses in section D are $\cong 4\mu s$.

The second emitter has a smaller vocabulary, including four active words (W1-W4), and one blank word (W5) in which no pulses are radiated for a speci£ed period of time. The words of this radar are emitted in pairs followed by a short termination character. Fig. 1 (b) illustrates the three distinct, £xed-length components of the pulse sequence. The £rst component has a length of 51000 crystal clock counts (Xc $\approx 0.095\mu s$), the second component is 50000Xc, and the termination character is 6691Xc. Depending on the speci£ed pair of words, sections A and C may each contain 333-415 pulses, and are followed, respectively, by dead time zones B and D of variable length. The termination character is the same for all states of the radar; it consists of three groups of pulses – E, G, and I, with respectively, 5, 8, and 12 £xed-PRI pulses, separated by dead time zones of £xed durations – F, H, and J.

B. REFERENCES

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