Clutter Scattering Function Estimation and Ground Moving Target Detection from Multiple STAP Datacubes

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Abstract

Methods for estimating the clutter scattering function and detecting ground moving targets, using pulse-Doppler surveillance radar data, are described. The imaging problem is cast as one of structured covariance estimation with time-varying measurement models and illumination patterns. An Expectation-Maximization (EM) algorithm is derived and the computational issues arising from its use are discussed. The detection algorithm uses the estimated clutter statistics and a time-varying target model in a standard Generalized Likelihood Ratio Test (GLRT).

1. Introduction

In this paper we describe an estimation-theoretic technique for determining the statistics of land clutter from data normally associated with wide-area surveillance radar, then show how these statistics could be used for ground moving target detection. Our problem formulation is consistent with the model for space-time adaptive processing (STAP) presented in [1]. A pulse-Doppler radar platform with multiple transmit/receive elements emits several pulse train along an arbitrary flight path near the region of interest. Each pulse train is assumed to be perfectly coherent within one coherent processing interval (CPI), but different pulse trains are assumed noncoherent with respect to one another. The ground region is subdivided into pixels, or ground patches. The range and angle of each ground patch with respect to the platform for each transmitted pulse is assumed known, along with the illumination pattern. The received data for one pulse is modeled as the sum of the returns from all of the ground patches, each modulated by the transmit illumination. The data from all pulses or viewpoints is modeled in this way. Maximum-likelihood methodology is used to estimate the unknown scattering function.

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The clutter model assumed is the constant-γ model described by Barton [2] and others, which is appropriate for the medium grazing angles one might encounter in wide-area surveillance radar applications. In this model the Earth is treated as a rough, or Lambertian, reflector, in which the radar cross section is proportional to the sine of the grazing angle (or the projected area of the patch as seen from the radar platform) and the constant of proportionality is the terrain-dependent parameter γ describing the scattering effectiveness of the surface. We refer to his function $\gamma(n)$, expressed as a function of the Earth geographical coordinates, as the *clutter scattering function*.

The radar data are modeled as 0-mean complex Gaussian vectors whose covariances are linear transformations of the clutter scattering function. We apply the structured covariance approach to data modeling, and the Expectation-Maximization (EM) algorithm to the computation of the maximum-likelihood solution. This line of thinking extends a body of work by ourselves and others in structured covariance estimation and radar imaging to the case in which the desired parameters are modulated by a known spatial illumination pattern. Once the clutter model is established, it can be used for ground moving target detection.

2. Problem Formulation

We are interested in estimating the clutter scattering function $\gamma(n)$ of the Earth's surface in some region of interest. Suppose that this region is pixelized into N ground patches. The size of the patches is commensurate with the resolution of the radar along its highest-resolution dimension, typically range. The physical modeling of the patches could come from digital terrain elevation maps, which are becoming more prevalent in radar simulations and signal processing [3]. At each of K time instants, $k = 1 \cdots K$, a pulse waveform is transmitted from an airborne radar platform, with a known illumination pattern. The incident energy on patch n at time k is λ_{nk}. The received data across multiple sensors, and perhaps across multiple Doppler bins (depending on the radar), after pulse compression and quadrature demodulation, is a 0-mean complex Gaussian vector with M components denoted $\mathbf{z}_k$, given by the sum of the returns from all of the patches. That is,

$$\mathbf{z}_k \sim CN(\mathbf{0}, \mathbf{R}_k) \tag{2.1}$$

where the covariances $\mathbf{R}_k$ can be written

$$\mathbf{R}_k = \sum_{n=1}^{N} \mathbf{a}(n,k)\mathbf{a}^{\mathrm{H}}(n,k)\gamma_n\lambda_{nk} \quad . \tag{2.2}$$

$\mathbf{a}(n,k)$ is the response vector, or direction vector, for the n^{th} patch on the k^{th} pulse. $\mathbf{a}(n,k)$ represents the structural aspects of the received signal, whereas all the physical constants describing the transmitted and propagated energy, including the grazing angle to each patch, are incorporated into the model for λ_{nk}. We assume that the pulse compression is ideal, that is, the received data can be partitioned into range gates which decouple the data into disjoint sets. Put another way, each $\mathbf{R}_k$ is block-diagonal, with each block corresponding to one range gate. However, the grouping or association of ground patches into range gates varies from pulse to pulse because of the time-varying flight path or viewpoint of the radar platform.

In more compact notation, we can write the covariance $\mathbf{R}_k$ as

$$\mathbf{R}_k = \mathbf{A}_k(\Gamma\Lambda_k)\mathbf{A}_k^{\mathrm{H}} \tag{2.3}$$

where

$$\mathbf{A}_k = [\,\mathbf{a}(1,k) \cdots \mathbf{a}(N,k)\,] \quad , \tag{2.4}$$

$$\Gamma = diag(\gamma_1 \cdots \gamma_N) \quad , \tag{2.5}$$

and

$$\Lambda_k = diag(\lambda_{1k} \cdots \lambda_{Nk}) \quad . \tag{2.6}$$

The problem is to determine the maximum-likelihood estimate of Γ based on this model.

This problem can be viewed as one of structured covariance estimation, an area which has been well-studied in the past twenty years and which has been applied to radar imaging [4]. Our contribution can be viewed as an extension of that work in the following two ways: 1) the forward model is placed in an Earth-centered coordinate system through precise geolocation of the platform and the use of detailed geographic information, and 2) the time-varying measurement model is extended to the active case through the inclusion of the time-varying illumination term Λ_k. Although Λ_k could be incorporated directly in the definition of $\mathbf{A}_k$, reducing our estimation problem to that of [5], making a distinction between the role of the transmitter (Λ_k) and that of the receiver ($\mathbf{A}_k$) is valuable and facilitates the development of active-testing surveillance algorithms in future research efforts.

3. EM Algorithm

We apply the Expectation-Maximization (EM) algorithm [6] to the maximization of the log-likelihood for Γ, drawing on previous success with related structured covariance estimation algorithms. The EM algorithm for this problem is derived by first hypothesizing a set of *complete data*, which is related to the observed or *incomplete data* through a many-to-one mapping. The complete data is chosen so that the estimation algorithm would be trivial were it truly available. In our case, the complete data is a set of random variables u_{nk} which represent the observed returns from each of the individual ground patches, not seen through the array response transformations $\mathbf{A}_k$. That is,

$$\mathbf{u}_k \sim CN(\mathbf{0}, \Lambda_k\Gamma) \quad . \tag{3.1}$$

The incomplete-data $\mathbf{z}_k$ are related to the $\mathbf{u}_k$ through the many-to-one transformation

$$\mathbf{z}_k = \mathbf{A}_k\mathbf{u}_k \quad . \tag{3.2}$$

In each step of the EM algorithm, one begins with a current estimate or iterate of the diagonal parameter matrix Γ. Then the expected values of the sufficient statistics for the complete-data log-likelihood, conditioned on the observed data and assumed parameter values, are computed. In this case the sufficient statistics are the squared magnitudes $|u_{nk}|^2$. Then, these sufficient statistics are used to find the closed-form ML estimate for Γ, and the process is repeated.

Use the index p to denote the iteration number. The expected squared magnitude of u_{nk} is the squared magnitude of the conditional expectation, plus the variance. The conditional expectation is given by

$$\begin{aligned} \mathrm{E}\{\mathbf{u}_k \mid \mathbf{z}_k, \Gamma^{(p)}\} &= \mathbf{R}_{\mathbf{uz}}(p,k)\mathbf{R}_{\mathbf{zz}}^{-1}(p,k)\mathbf{z}_k \\ &= \Lambda_k\Gamma^{(p)}\mathbf{A}_k^{\mathrm{H}}\mathbf{R}_{\mathbf{zz}}^{-1}(p,k)\mathbf{z}_k \end{aligned} \tag{3.3}$$

where $\mathbf{R}_{\mathbf{zz}}(p,k)$ is the covariance of $\mathbf{z}_k$ at iteration p, and $\mathbf{R}_{\mathbf{uz}}(p,k)$ is the cross-covariance of $\mathbf{u}_k$ and $\mathbf{z}_k$, both predicated on the assumed value of $\Gamma^{(p)}$. These are given by

$$\mathbf{R}_{\mathbf{zz}}(p,k) \;=\; \mathbf{A}_k \Lambda_k \Gamma^{(p)} \mathbf{A}_k^{\mathrm{H}} \qquad (3.4)$$

and

$$\mathbf{R}_{\mathbf{uz}}(p,k) \;=\; \Lambda_k \Gamma^{(p)} \mathbf{A}_k^{\mathrm{H}} \quad . \qquad (3.5)$$

The conditional covariance of $\mathbf{u}_k$ is given by

$$cov\{\mathbf{u}_k \mid \mathbf{z}_k, \Gamma^{(p)}\} \qquad (3.6)$$

$$= \mathbf{R}_{\mathbf{uu}}(p,k) \;-\; \mathbf{R}_{\mathbf{uz}}(p,k)\mathbf{R}_{\mathbf{zz}}^{-1}(p,k)\mathbf{R}_{\mathbf{zu}}$$

$$= \Lambda_k \Gamma \;-\; \Lambda_k \Gamma^{(p)} \mathbf{A}_k^{\mathrm{H}} \mathbf{R}_{\mathbf{zz}}^{-1}(p,k) \mathbf{A}_k \Gamma^{(p)} \Lambda_k \quad .$$

The sufficient statistics we seek are the squared magnitudes of (3.3) plus the diagonal elements of (3.6).

If one were given the $|u_{n,k}|^2$ directly, the maximum-likelihood estimates of the γ_n would be given by

$$\hat{\gamma}_n \;=\; \frac{1}{K} \sum_{k=1}^{K} \frac{|u_{n,k}|^2}{\lambda_{nk}} \qquad (3.7)$$

or in more compact notation

$$\hat{\Gamma} \;=\; \frac{1}{K} \sum_{k=1}^{K} \Lambda_k^{-1} diag(\mathbf{u}_k \mathbf{u}_k^{\mathrm{H}}) \quad . \qquad (3.8)$$

Substituting the conditional expectations in place of the actual squared magnitudes in (3.7) we have finally the EM iteration given by

$$\Gamma^{(p+1)} \;=\; \Gamma^{(p)} \qquad (3.9)$$

$$+ \; \frac{1}{K} \sum_{k=1}^{K} \Lambda_K diag\Big[\Gamma^{(p)} \mathbf{A}_k^{\mathrm{H}} \mathbf{R}_{\mathbf{zz}}^{-1}(p,k) \mathbf{z}_k \mathbf{z}_k^{\mathrm{H}} \mathbf{R}_{\mathbf{zz}}^{-1}(p,k) \mathbf{A}_k \Gamma^{(p)}\Big]$$

$$- \; \frac{1}{K} \sum_{k=1}^{K} \Lambda_K diag\Big[\Gamma^{(p)} \mathbf{A}_k^{\mathrm{H}} \mathbf{R}_{\mathbf{zz}}^{-1}(p,k) \mathbf{A}_k \Gamma^{(p)}\Big] \quad .$$

4. Computational Considerations

The EM algorithm as described above has very high computational requirements and thus every effort must be made to streamline the computations where possible. We briefly describe some of these implementation issues here; for more information see [8].

Each data cube can be viewed as a vector of length $LM_d M_s$, where L is the number of range gates, M_d the number of pulses, or Doppler bins, and M_s the number of sensors. The partitioning of data into range gates means that the matrices to be inverted in (3.9) are size $M \times M$, where $M = M_d M_s$; nevertheless there are L such systems to be inverted for each data cube.

Special attention must be paid to the organization of the calculations to accomodate the fact that the data from different range bins are independent, but that the grouping of pixels into range bins varies from measurement to measurement. Given some "master" ordering of the parameters $\gamma_1 \cdots \gamma_N$, there exists some permutation $\mathbf{P}_k$ that organizes the parameters into range bins, for the purpose of computing the sufficient statistics in the EM algorithm. Once computed, these sufficient statistics must be permuted back into the master ordering of the parameter vector before the sum over k can be taken in (3.9).

The central calculation of the EM algorithm is that of finding the conditional mean and covariance of a certain multivariate Gaussian distribution, given the data and a prior mean and covariance. Numerically stable and efficient data-domain algorithms, based on the solution of the least-squares problem

$$\mathbf{y} \;\approx\; \mathbf{A}_k \Gamma^{\frac{1}{2}} \mathbf{x} \qquad (4.1)$$

have been derived and implemented. These methods avoid the full calculation of the matrix $\mathbf{R}_k$ at each iteration, and are based on either the QR or the singular value decomposition of $\mathbf{A}_k \Gamma^{\frac{1}{2}}$, depending on the matrix dimensions and condition.

The calculation of the response vector $\mathbf{a}(n,k)$ is a significant factor in the overall computation. We have implemented a table look-up method, in which the spatial and Dopper response vectors are pre-computed on a fine grid, and their Kronecker products are computed "on the fly" as needed to fill out the model matrices $\mathbf{A}_k$.

The EM algorithm has been known to exhibit slow convergence in high-dimensional problems, and the present case is no exception. We have implemented the ordered subsets (OS) EM algorithm [7] as an alternative to the standard EM algorithm. The speed-up factor obtained is roughly equal to the number of subsets used in the OS-EM algorithm, a result consistent with that of other reported applications.

5. Ground Moving Target Detection

Once the clutter scattering function is known, it can be used for the detection of ground moving targets. Suppose that there exists a hypothesized target track on the ground; this track could be generated by a "track-before-detect" system, beyond the scope of this paper. The target appears in each of the datacubes through a response vector $\mathbf{a}_k$ which is function of its instantaneous position and velocity relative to that of the radar platform.

Let $\mathbf{R}_k$ be the clutter covariance matrix for datacube k, confined to the range bin containing the hypothesized target at time k. Let $\mathbf{a}_k$ be the target response vector. The target is assumed to be fluctuating and has an unknown deterministic amplitude b_k for all $k = 1 \cdots K$. The hypothesis testing problem is

$$\text{H0: } \mathbf{z}_k \sim CN(0, \mathbf{R}_k) \qquad k = 1 \cdots K$$

$$\text{H1: } \mathbf{z}_k \sim CN(b_k \mathbf{a}_k, \mathbf{R}_k) \qquad k = 1 \cdots K$$

Since the target amplitude is unknown, this is a composite hypothesis testing problem and we adopt the Generalized Likelihood Ratio Test (GLRT) approach, determining the maximum-likelihood (ML) estimate of b_k under H1 and substituting it back into the likelihood function. We have that

$$\hat{b}_{k,ML} = \frac{\mathbf{a}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{z}_k}{\mathbf{a}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{a}_k} \tag{5.1}$$

which leads to a test statistic of the form

$$t = \sum_{k=1}^{K} \frac{|\mathbf{a}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{z}_k|^2}{\mathbf{a}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{a}_k} \quad . \tag{5.2}$$

Under hypothesis H0 the test statistic is subject a central χ^2 distribution with K complex degrees of freedom, and under H1 it is subject to non-central χ^2 distribution, with non-centrality paper equal to the signal-to-noise ratio (SNR)

$$\rho = \sum_{k=1}^{K} |b_k|^2 \mathbf{a}_k^{\mathrm{H}} \mathbf{R}_k^{-1} \mathbf{a}_k \quad . \tag{5.3}$$

The probabilities of false alarm and detection for such a detector are well-known.

We recognize that the use of estimated $\mathbf{R}_k$ in this detection problem will inevitably reduce detector performance, as is the case in all adaptive detectors. The performance of this "clairvoyant" detector provides an upper bound on the performance of all such adaptive detectors. A complete analysis of the detection performance with the estimated $\mathbf{R}_k$ appears intractable. The important point here is that the clutter model allows us to carry out the adaptive detection in situations where 1) the clutter is heterogeneous and therefore there is no "secondary data" from other range bins to estimate $\mathbf{R}_k$, and 2) the clutter covariance can be estimated from data at other platform positions and orientations.

6. Summary and Conclusions

A method for estimating the ground scattering function using wide-area surveillance radar data has been presented. The imaging problem was cast as one of structured covariance estimation with time-varying measurement models and illumination patterns. The EM algorithm was derived and implemented, and some of the computational issues arising from its implementation were discussed. Once the clutter scattering function is established, it can be incorporated into an algorithm for ground moving target detection, which uses data cubes collected over a period of time from a number of different platform positions and orientations.

The algorithms described here were developed as part of a large project in Knowledge Aided Sensor Signal Processing and Expert Reasoning (KASSPER), an ongoing initiative of the U.S. Defense Advanced Research Projects Agency (DARPA). Space does not allow us to include here details on the implementation and simulation results. For more information see [8].

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