# EFFICIENT IMPLEMENTATION OF A SPECTRUM ANALYZER FOR FIXED POINT ARCHITECTURES

*Antonio Valdessalici, Giacomo Frassi, Alberto Bellini*

AIDA srl, University of Parma,
Parco Area delle Scienze, 181/A, I-43100 Parma, ITALY
valdes@www.aidasrl.it, frassi@www.aidasrl.it, alberto.bellini@unipr.it

## ABSTRACT

This paper presents a novel and efficient algorithm to compute fractional band spectrum digital decomposition without using Fast Fourier Transformation (FFT). The algorithm is optimized for fixed point architectures in order to reduce the required computational power. It is based on lattice coupled all-pass IIR digital filters, down-sampling and recursive techniques. Hence the proposed algorithms is capable of computing up to third band spectrum analysis on low cost processors. Though fixed-point arithmetic is used the spectrum analysis may be performed at very low frequency, irrespectively of the sampling frequency, and very narrow band pass filters can be realized. The latter features are critical to achieve with traditional IIR structures.
The proposed algorithm is patent pending.

## 1. INTRODUCTION

Band spectrum decomposition is a wide spread technique for signal analysis in several civil and professional applications. As an example the European standards for professional acoustic and vibrational measurements require this band spectrum decomposition for audio signal in order to monitor harmonics contents and energy. The recommendations prevent from using FFT in professional measurements instruments, therefore the use of time domain digital filter is a mandatory option. Power consumption and cost are constrained also, because of the portable requirements of the instruments. It turns out that IIR filters and fixed-point processor are the best trade-off. However a stable and narrow band IIR at low frequencies is a very critical task for fixed-point arithmetic. The latter is required for vibrational analysis, that requires usually the very narrow band filters centered at frequencies very far from the sampling frequency. Instability, quantization errors, limit cycles are among the major issue to face even with floating point architectures.
The proposed algorithm relies on the lattice coupled all-pass IIR digital filter to overcome the above limitations. This kind of filter is designed to reduce digital quantization error problems, and thanks to an efficient use of its complementary properties and to a recursive process the computational cost is remarkably reduced. It turns out that narrow band digital filters at vibrational frequencies can be implemented efficiently on low cost digital processors.

## 2. BAND SPECTRUM DECOMPOSITION

European standards for acoustic professional instrumentations require IIR digital filters to compute the fractional band spectrum decomposition.
Octave and third octave filter banks are used to compute an accurate spectrum analysis. In order to obtain an acoustic decomposition of the spectrum, the range of audible frequencies (typically from 20Hz to 20kHz) is divided in several parts (usually octaves or fractions of octave) following specific rules [1]. A more selective analysis can be obtained dividing each octave in a few subbands with all the above specified constraints.
Usually 11 band pass filters, whose width is an octave, are used to compute the band spectrum decomposition of an audio signal. The center band frequency of each filter is the double of the center band frequency of the previous filter, and all the filters have contiguous borders. This choice is mimics hearing characteristics, as the sensitivity of ears is a logarithmic function of frequency.
Usually the center band frequency of the last band pass filter is at 16kHz, and all the others are consequently defined.

## 3. A NOVEL ARCHITECTURE FOR OCTAVE FILTER BANKS

As detailed in the previous sections the use of IIR filters is mandatory, though quantization and rounding errors can produce instability and noise, especially for fractional octave filters, where the width is narrow. The use of fixed-point processors and of very low center frequencies enhances these effects.
Lattice filters can partially solve the above issues, since they are intrinsically robust versus quantization errors. They are a common solution for fixed point implementations [2], [3], optimal in terms of limit cycles

and quantization errors [4]. The proposed algorithm is a further improvements towards the implementation of an efficient and robust spectrum analyzer. The basic block for the implementation of band spectrum decomposition is a lattice digital filter. Then exploiting the lattice coupled all-pass filter structure, and recursive decimation [5] a full band spectrum decomposition is obtained. Beyond a single lattice coupled all-pass filter only an anti-aliasing is required to limit signal bandwidth. In fact the lattice coupled all-pass filter realized a couple of filter with the same cutoff frequency, fig. 1, at the cost of about one single lattice filter.

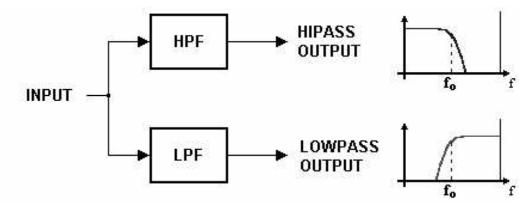

Fig 1 Lattice coupled all-pass structure.

In the following the procedure used to obtain a standard band spectrum decomposition is detailed. The first anti-aliasing low pass filter should have a cut-off frequency of 22.626kHz, that is the upper limit of the highest band, centered at 16kHz. Then the first band pass filter is implemented by a lattice coupled all-pass filter. The band pass filter is realized as a cascade of a high pass and a low pass filter at different cutoff frequencies. Thanks to the decimation process the same filter is recursively used to realize all the band-pass filters.

As an example, in order to obtain a band pass filter with cutoff frequencies [$f_0/2$, $f_0$], the input signal must be processed by a low-pass lattice filter with cutoff frequency $f_0$. Then the signal is decimated (one sample every two samples is kept) and re-processed by the same lattice filter with half of the sampling frequency. So doing the real cutoff frequency of the filter is halved, changing the sampling rate [5]. The use of coupled all-pass lattice filters produces both low-pass and high-pass signals at the same cut-off frequency. It turns out that the output of the high-pass path is the desired band-pass filter, while the low pass path is the anti-aliasing filter for the next cycle of the recursive decimation process, fig. 2.

Antialiasing conditions constraint the choice of $f_0$:

$$f_0 < f_s/4$$

where $f_s$ is the sampling frequency of the acquired signal.

A successful decimation process halves the sampling rate of the input signal to $f_s/2$, and this requires that the signal bandwidth is included in the range [0, $f_s/4$]. The latter condition is enforced by an anti-aliasing filter.

As already detailed in the proposed recursive process the low-pass path of the lattice filter plays two roles: anti-aliasing filter and shape of the band pass filter.

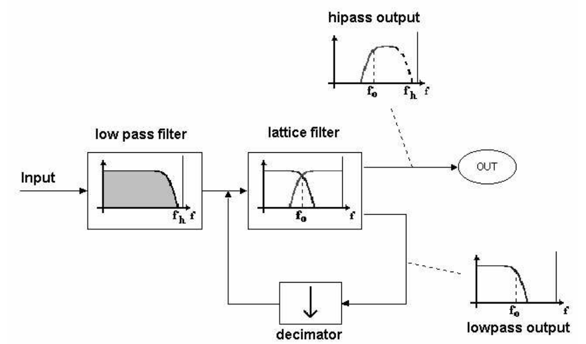

Fig. 2 Block diagram of recursive octave band decomposition.

Therefore a octave band decomposition is obtained by applying recursively a single coupled all-pass lattice filter, as reported in fig. 2, provided that center frequency is chosen carefully. The cut-off frequency $f_h$ of the initial anti-aliasing filter (shadowed block in fig. 2) is at 22.627kHz, and the lattice filter is characterized by:

$$f_0 = 0.471375 \cdot (f_s/2)$$

when the initial sampling frequency of the input signal is at $f_s$=48kHz. The signal produced by the anti-aliasing filter is fed to the lattice filter at cut-off frequency $f_0$. The low-pass path is recursively reported at the input of the same filter after the decimation process, and the high-pass path (discarded at the first loop) is the band-pass filter.

Usually the audio spectrum is divided into 11 octave bands, where the lower one ranges from 11Hz to 22Hz. The proposed algorithm can easily realize those filters, and even filters down to less than 1 Hz. The same results is impossible for a traditional IIR filter. The availability of band pass filters at very low frequencies is an attractive option for vibrational frequency analysis. Several applications exist where acoustic and vibrational spectrum analysis must be performed with the same instrument. The proposed method is well fitted to this problem, the only requirement is a few more recursive loops. After each loop the signal is decimated, and a narrower band is isolated at lower frequencies.

Another attractive feature of the proposed algorithm is the reduced computational power. In fact each time that the input signal is decimated, the total number of samples of the signal are halved, but no loss of information occurs. Therefore at each loop the computational cost halves. It turns out that the computational power increases as a logarithmic function with the number of bands.

The procedure described above is suitable for octave band decomposition, but can be extended to fractional octave band decomposition without major changes. Then the selectivity of the spectrum analysis can be an option.

As an example third octave band decomposition can be realized, using three coupled allpass lattice filters applying recursive decimation with the same tips already detailed. Fig. 3 reports the block diagram of the third octave band decomposition: 33 band pass filters are obtained.

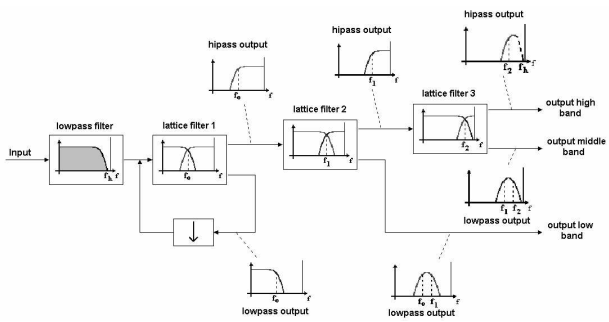


Fig. 3 Third octave band decomposition.

The lattice filter 1 plays the same role as the lattice filter in fig. 2: its low pass path works as anti aliasing filter for the decimation process, and as the upper limit of band pass filters. The high pass path fixes the lower limit for the next lattice filters, and propagates the signal to them. Lattice filters 2 and 3 in a similar fashion produce the band pass filters for each octave.
In summary exploiting recursive decimation process and three coupled allpass lattice filters a third octave band analysis is obtained, that is fully compliant with European recommendations [1]. Of course as before vibrational analysis is still possible at the cost of additional loops. After each loop the cut-off frequencies are halved, while the computational cost is halved too.

## 4. IMPLEMENTATION OF THE SYSTEM AND EXPERIMENTAL RESULTS

The proposed algorithms were implemented on a fixed point BF533 DSP.
At first common filters structure were benchmarked with the adopted processors, in order to compare computational cost of IIR filters and of coupled allpass lattice filters. As expected the lattice structure is the best for a fixed point implementation, since it is very robust towards coefficient quantization noise [2].
Five IIR structures were implemented and compared, monitoring two main parameters: computational cost, and the output audio quality. Each structure was used for a third order band pass filter at the same cut-off frequency in assembly language at a sampling frequency of 48 kHz.
The Direct Form II on 16 bit fixed point processors requires only 15 execution cycles. However the width of the filter band cannot be narrow. The lower band pass filter ranges from 8kHz to 12kHz. Moreover an annoying noise is perceived at the output, because of the limited number of resolution bits.
As for Direct Form II in floating point emulation the results are as follows. Two version of this filter were realized, a non-standard version using 16 bit for mantissa and 16 for the exponent (16.16 format), and a standard version with 24 bit for mantissa and 8 bit for the exponent (24.8 format). The same filter requires 395 execution cycles for the 16.16 format and 503 for the 24.8 format. The audio quality of these filters is better than the 16 bit DF II, and 24.8 format is slightly better in terms of noise that 16.16 format. However below 2 kHz.
As for Direct Form II decomposed in second order sections (SOS) for 32-bit fixed point processors the results are as follows. This structure works properly down to a band pass filter centered at 1kHz (707Hz to 1414Hz). The same filter requires 91 execution cycles and the audio quality is appreciable.
Eventually the coupled allpass lattice structure was considered. The noise at filter output is decreased, and 165 execution cycles are needed. This structure works properly also down to a pass band filter centered at 1kHz (without downsampling).
In the following a comparative analysis for complementary couple is made neglecting DFII 16 bit structure for its unacceptable quality. It turns out that the lattice is the cheapest one thanks to the coupled allpass lattice structure.
In summary the lattice structure is the best in terms of audio quality and in terms of computational cost. The chart of figure 4 reports the execution cycles required by the third order single filter under test (dark grey) and by the complementary couple filters.
Then the whole procedure described in the previous section was implemented on the BF533 at 270 MHz. In the debug environment a statistical profiling tool is available that allows to estimate the computational cost of the spectrum analyzer, fig. 5. It turns out that the third octave band spectrum analyzer (including energy parameter computations, I/O interfaces, and overheads) exploits 35 % of the whole DSP processing power for one single channel at a sampling rate of 48 kHz. The processor at 270 MHz runs in the main idle cycle for 65% of execution time, as reported in the second column of fig. 5.
In order to realize filters compliant with the filter masks of European standards [1], the following filter orders are used. Referring to a third octave spectrum analysis, fig. 3, the lattice filter 1 is a $15^{th}$ order, the lattice filter 2 is a $11^{th}$ order, and the lattice filter 3 is a $7^{th}$ order one. Referring to an octave spectrum analysis, fig. 2, the lattice filter 1 is a $7^{th}$ order one, with a remarkable reduction of the requested computational cost.
The realized algorithm divides the input signal in slots of 2048 samples. So doing at a sampling rate of 48 kHz, about 23 slots per second are processed. Therefore the throughput of the band decomposition algorithm is of about 23 decompositions per second. Tab. 1 reports the *measured* execution cycles required to compute band

spectrum decomposition of a slot of 2048 samples of the input signal, and the execution cycles per second required in order to obtain a continuous band spectrum analysis with 23.4375 decompositions per second. The two columns refer respectively to octave band and third octave band decomposition.

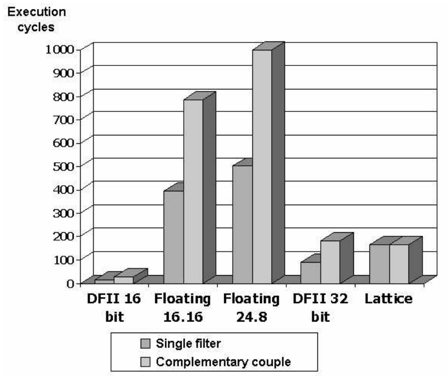


Fig 4 Execution cycles of the sample filter for different IIR filter structures.

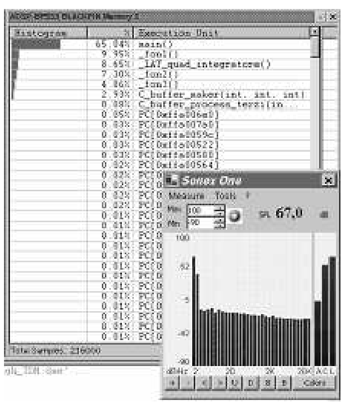

Fig. 5 Statistical profiling of the implemented spectrum analyzer, and snapshoot of output interface.

| | Octave band | Third octave band |
|---|---|---|
| Execution cycles for 2048 samples | ~ 600 k cycles | ~ 2.68 Mcycles |
| Execution cycles per second. | ~ 14 Mcycles/s | ~ 63 Mcycles/s |

Tab. 1 Performances of the proposed algorithm for one single slot, and for continuous (23.4375 slots per second) analysis.

As before the results of tab. 1 are referred to 270 MHz. Operating the processor at its maximum frequency of 600 MHz, a real-time third octave band decomposition can be realized for 4 channels at the same time.
The same performances would be impossible with other filter topologies. As an example tab. 2 reports a comparison in terms of execution cycles required to compute a band spectrum decomposition of a 2048 samples buffer.

Then a prototype version of the spectrum analyzer with the proposed algorithm was implemented on a BF533 for vibrational analysis too. The spectrum analysis ranges from 0.7Hz up to 20kHz, but no theoretical limitations exists for lower frequencies. Currently the bottleneck is the capability of the ADC. The same results would be impossible with already existing algorithms.

| | Octave band | Third octave band |
|---|---|---|
| LATTICE | 599200 | 2682000 |
| DFII 32 bit | 870000 | 4100000 |
| Floating 24.8 | 4807000 | 22660000 |

Tab. 2 Comparison of different filters for a band spectrum decomposition of a 2048 samples buffer.

## 5. CONCLUSIONS

This paper presents a novel algorithm for the implementation of fractional octave band decomposition on fixed-point processors. The algorithm is optimized for computational cost and allows very narrow band analysis at vibrational frequency, down to less than 1 Hz. It is based on the use of recursive decimation and on coupled allpass lattice filters.
A comparison of several filter structure is made, that emphasizes the superior features of the adopted lattice structure.
A prototype of a third octave spectrum analyzer was realized using a BF533 processor. Experimental results confirm that the proposed algorithm is well fitted to fixed-point processor and fractional octave band decomposition.

## 6. ACKNOWLEDGMENT

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