# SQUINT MODE SAR IMAGING WITH RANGE-WALK REMOVAL

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# ABSTRACT

In this paper, a new point of view is proposed to correct the range migration and focus the highly squinted SAR data. The procedure of range-migration correction is done not only in the Range-Doppler (R-D) domain and the 2-D frequency domain as usual, but also in the time domain. A modified Chirp Scaling (CS) algorithm is described based on the new point. A quantitative analysis and simulations show that the new algorithm can process the SAR data with a squint angle as large as 70 degree and a range swath of 50 km or wider.

#### **1. INTRODUCTION**

The difficulty for imaging of squinted SAR data is how to decouple the signals of range and azimuth due to the range migration and eliminate the difference of azimuth signal at various range cells. The azimuth phase history may vary greatly with range due to a large range swath or a short wave length of transmitted signal. Some algorithms have been proposed for processing the squint mode SAR data, such as the modified version of R-D algorithm[1][2], the Chirp Scaling (CS) algorithm[3], the Non-linear CS algorithm (NCS)[4][5], the Exact Transform Function algorithm (ETF)[6][7] and the algorithms of sub-aperture type[8]. In the algorithms mentioned above, the procedure of decoupling and focusing is totally done in the R-D domain or the 2-D frequency domain. To yield a good performance, the algorithms described in [4]-[7] use a high order polynomial as the phase model in the Doppler domain. The algorithm in [8] reduces the quantity of range migration by sub-apertures.

In this paper, a novel point of view is proposed to correct the range migration and decouple the signals of azimuth and range. The range migration can be divided into two parts, the linear part and the non-linear part. With the increase of the squint angle, the linear part, i.e. the range-walk becomes the dominant part of the range migration. As the range-walk is independent of range and uniform to the whole scene, it can be easily removed by a shear operation in the time domain. With the range-walk removed, the residual range migration is much small, the coupling between range and azimuth and the difference of azimuth signal at various range cells are also reduced. All of those make the squinted SAR data easy to be processed.

In next section, the range migration is analyzed in detail. In Section 3, a modified CS algorithm with rangewalk removal is described as the application of the new method. A performance comparison of the modified CS algorithm and the classic CS algorithm is presented in Section 4. Some simulation results are presented in Section 5 and the conclusion is in the Section 6.

## 2. THE ANALYSIS OF RANGE MIGRATION

The geometry of the squinted SAR is shown in Figure. 1.





The spacecraft moves along the X direction at an altitude of H with a constant velocity v. The radar antenna transmits and receives pulses in a squinted direction defined by the squint angle  $\theta$  and the look angle  $\Phi$ . Assuming that the closest slant distance from a point target to the SAR platform at zero squint angle is  $R=H/\cos\Phi$ , the instantaneous distance at slow time (azimuth time) t is:

$$r(t) = \sqrt{R^2 + [v(t + R \tan \theta / v)]^2} .$$
(1)  
=  $R / \cos \theta + r_t(t) + r_c(t)$ 

The range-walk, i.e. the linear part of (1), is expressed as [9]:

$$r_{L}(t) = v \cos\left(\frac{\pi}{2} - \theta\right) t = v \sin(\theta) t$$
 (2)

and the range-curvature is

$$r_{C}(t) = r(t) - r_{L}(t) - R/\cos\theta.$$
(3)

To show the shape of  $r_c$  under various squint angles, a simulation result of (3) in an aperture is given in Figure. 2 and the parameters used are listed in Table I.



Table I. Parameters for simulation of the range-curvature

Figure 2. Range-curvatures with various squint angles

There are 15 curves in Figure 2. They are the rangecurvatures with squint angles from 0 degree to 70 degree with an increase of 5 degree between each. It is obvious that the part of range-curvature in range-migration becomes less and less as the squint angle becomes larger

To give a quantitative analysis of the relationship of range-walk and the range-curvature, the total quantity of range-walk is defined as:

$$R_L = \sum_i [r_L(t_i) - \min(r_L)]$$
(4)

and the total quantity of range-curvature is defined as:

$$R_c = \sum_i r_c(t_i) \,. \tag{5}$$

where  $t_i$  is the slow time sample. The ratio of (4) to (5) is shown in Figure 3.



It can be seen from Figure 3 that the ratio becomes larger with the increase of squint angle, and the rangewalk becomes the dominant part of the range-migration.

From the analysis above, the range-curvature becomes flat and the quantity of the range-curvature is minor when the squint angle becomes large. So, if an algorithm, such as the classic CS algorithm proposed in [3], can process the SAR data at zero squint angle, it must be able to process the highly squinted SAR data with the range-walk removed. In the next section, a modified CS algorithm is described with the range-walk removed first in the time domain.

### 3. CHIRP SCALING WITH RANGE-WALK REMOVAL

The SAR signal from a point target can be expressed as:

$$pp(t,\tau) = A(t,\tau)\exp\{-j\pi k[\tau - \frac{2r(t)}{c}]^2\}\exp[-j\frac{4\pi r(t)}{\lambda}].$$
(6)

In (6), t and  $\tau$  are azimuth time and range time respectively, k is the chirp rate of the transmitted signal, c is the light speed and  $\lambda$  is the wave length.  $A(t, \tau)$  is a function responsible for all the variation of amplitude. r(t)takes of the same form as (1) and can be redefined as:

$$r(t) = \sqrt{R^2 + [v(t+t_s)]^2}, \ t_s = R \tan \theta / v.$$
 (7)

The range-walk can be removed by shearing the image with the slope of  $-v \times \sin\theta$ . With the range-walk removed and amplitude variation function omitted, (6)can be rewritten as:

$$pp'(\tau,t) = \exp\{-j\pi k [\tau - \frac{2r'(t)}{c}]^2 - j\frac{4\pi r(t)}{\lambda}\}, \quad (8)$$

where  $r'(t) = r(t) - r_L(t)$ . In the 2-D frequency domain, the signal can be extended up to the second order term of the range frequency and expressed as:

$$PP = \exp[j2\pi(\frac{f_r^2}{2Km} - f_r\tau_d + R\frac{f_a\tan\theta}{\nu} - \frac{2R}{\lambda}\sqrt{A})], \quad (9)$$

where  $f_a$  and  $f_r$  is the azimuth and range frequency and

$$\frac{1}{Km} = \frac{1}{K} - BR$$

$$B = \frac{2\lambda}{c^2 \sqrt{A}} \left[\cos^2 \theta - \frac{1}{A} - \frac{f_a \lambda \sin \theta}{Av} - \frac{(f_a \lambda \sin \theta)^2}{4Av^2}\right]. \quad (10)$$

$$\tau_d = 2RX/c$$

$$X(f_a) = \frac{1 + f_a \lambda \sin \theta / (2v)}{\sqrt{A}} + \sin \theta \tan \theta$$

$$A = 1 - \left(\frac{f_a \lambda}{2v}\right)^2$$

Taking the inverse Fourier transform of (9) about  $f_r$ , the SAR signal in R-D domain will be:

$$pP(\tau, f_a) = \exp[-j\pi Km(\tau - \tau_d)^2]$$

$$\exp[j2\pi R \frac{f_a \tan \theta}{v} - j4\pi R \sqrt{A}/\lambda]$$
(11)

As described in [3], a reference range can be chosen as  $R_{ref}$  and an arbitrary range can be expressed as  $R = R_{ref} + \Delta r$ . The corresponding delay can be expressed as:

$$\tau_d = 2X(R_{\rm ref}+\Delta r)/c = \tau_{\rm ref}+\Delta \tau$$
 The scaling factor will be:

$$\alpha = X(f_{dc}) / X(f_a), \qquad (12)$$

where  $f_{dc}$  is the center frequency of azimuth. The chirp scaling function is defined as:

$$ph_{cs} = \exp\left[-j\pi \frac{Km_{ref}(\alpha - 1)}{\alpha} \left(\tau - \frac{2R_{ref}X(f_a)}{c}\right)^2\right].$$
(13)

The parameter of  $Km_{ref}$  in (13) refers the value of Km at  $R_{ref}$ . Assuming that Km is independent of range and equals to  $Km_{ref}$ . The chirp scaling process can be done by multiplying (13) with (11) and the result in the 2-D frequency domain will be

$$PP = \exp[j\pi \frac{\alpha f_r^2}{Km} - j2\pi f_r \tau_s], \qquad (14)$$
$$\times \exp[j2\pi (R \frac{f_a \tan \theta}{v} - 2\frac{R}{\lambda}\sqrt{A})]$$

where  $\tau_s = \tau_{ref} + \alpha \Delta \tau$  is the reshaped delay curve at *R*.

To focus the image in range direction, the match filter expressed as

$$ph_{mrrc} = \exp\left[-j\pi \frac{\alpha f_r^2}{Km_{ref}} + j4\pi f_r R_{ref} (X - 1/\cos\theta)/c\right] (15)$$

can be employed and the result will be:

$$PP = \exp\left[-j\frac{4\pi f_r R}{\cos\theta \times c} + j2\pi \left(R\frac{f_a \tan\theta}{v} - 2\frac{R}{\lambda}\sqrt{A}\right)\right].$$
(16)

In R-D domain, a range variant match filter expressed as

$$ph_{ma} = \exp\left[-j2\pi R\left(\frac{f_a \tan\theta}{v} - \frac{2\sqrt{A}}{\lambda}\right)\right]$$
(17)

can be employed to focus to SAR data in azimuth. In addition, the residual phase term should be also removed in the R-D domain. A shear operation may be employed to recover the geometry relationship of the image. The procedure of the modified CS with range-walk removal can be illustrated by Figure 4.

#### 4. COMPARISON WITH THE CLASSIC CHIRP SCALING ALGORITHM

For the algorithms of chirp scaling type, a key point is to eliminate or minimize the variation of Km with range. In the classic CS algorithm, Km is regarded as a constant of  $Km_{ref}$ . In the modified CS algorithm discussed above, Km is still regarded as a constant with the value of  $Km_{ref}$ , but the difference of Km at various range cells is much reduced by removing the range-walk in time domain. To show this, the band-unified difference of Km between the reference range  $R_{ref}$  and an arbitrary range R is defined as:

$$\sigma_{Km}(R) = \frac{1}{b_{az}} \sum_{f_a} [Km(R, f_a) - Km(R_{ref}, f_a)]^2,$$
(18)



where  $b_{az}$  is the band width of azimuth. The simulation results of (18) with the parameters in Table I by the modified CS algorithm and the classic CS algorithm are shown in Figure 5 to give a comparison.



Figure 5. The variation of Km with range under different squint angle. The dotted line is  $\sigma$  of the classic CS algorithm and the solid line is  $\sigma$  of the new algorithm.

From the six subplots in Figure 5, the variation of Km from the classic CS algorithm is much greater than that of the modified CS algorithm.

Figure 6 gives a more detailed plot of (18) by the new algorithm under different squint angle. It can be seen that the difference of Km at various range cells decreases as the squint angle becomes large. So, from the view of Km, the algorithm would perform better as the squint angle become larger.



Figure 6. The variation of *Km* under various squint angles from the new algorithm

### **5. SIMULATIONS**

Some simulation results are presented in this section to show the validity of the modified CS algorithm in processing the squinted SAR data. There are three pointtargets in the sense at the same azimuth direction. There is a distance of 50km between the center target and the two at each side in slant range direction. The parameters for simulation are listed in Table II.

Table II. Parameters for Simulation					
PRF	10,000Hz Aperture Time		1.6384s		
Pulse Band- Width	200MHz	Sampling Frequency	240 MHz		
Chirp Rate	20MHz/us	Pulse Width	10us		
Wave Length	0.03m	Velocity	7,540m/s		

The slant range of the center target is 700km, the near one is 650km and the far one is 750km. The theoretical resolution of range is 0.75m and the theoretical resolution of azimuth is 0.7892m, 0.8500m and 0.9107m of the three targets at zero squint angle. The simulation results of the three targets are listed in Table III (N-Near; C-Center, F-Far).

The azimuth resolution in Table III is defined as:

$$R_{az} = R_{sim}/R_{th},$$

where  $R_{az}$  is the value in the table,  $R_{sim}$  is the resolution of simulation result and  $R_{th}$  is the theoretical resolution.

	Squint Angle (deg)	Range Resolu tion (m)	Azimuth Resolution (resolution cells)	Range PSLR (dB)	Azimuth PSLR (dB)
N	30	0.751	1.010	-13.54	-12.89
	50	0.751	1.010	-13.54	-13.15
	70	0.751	1.011	13.53	-13.32
С	30	0.751	1.010	-13.54	-13.39
	50	0.751	1.010	-13.54	-13.46
	70	0.751	1.010	-13.54	-13.73
F	30	0.751	1.009	-13.54	-12.93
	50	0.751	1.011	-13.54	-13.17
	70	0.750	1.010	-13.54	-12.96

### Table III Simulation Results

#### 6. CONCLUSION

In the new algorithm and the method proposed in this paper, the information of the azimuth signal from time domain and Doppler domain is considered jointly and satisfactory results are obtained. In addition, the structure of the new algorithm is rather simple with respect to the NCS and the ETF, and also much more compact than that of the sub-aperture algorithms.

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