NEAR FIELD IMAGING OF SUBSURFACE TARGETS USING WIDE-BAND MULTI-STATIC RELAX/CLEAN ALGORITHMS

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ABSTRACT

This paper presents two new imaging algorithms for detecting the positions of subsurface targets, e.g., land mines, using seismic waves. They are based on the CLEAN algorithm and its high resolution version RELAX. This paper will show that how the CLEAN and RELAX algorithms can be modified to work in a multi-static active array setup for detecting passive targets. Seismic surface waves reflected from various targets are collected at a receiving array and processed to locate the reflectors. The modified imaging algorithms will be demonstrated to work for experimental seismic data that includes mines and rocks (clutter).

1. INTRODUCTION

Detection of buried land mines and subsurface structures has been investigated at Georgia Tech in recent years using seismic/acoustic waves [1]. A seismic wave is launched from a source, and it travels through the soil and interacts with targets. The reflection from the targets can be measured by placing the sensors on the surface. The goal is to use these reflected waves to localize the buried targets. Waves that propagate in a medium can be divided into two main types, surface waves and body waves. Surface waves, such as Rayleigh waves can be measured by placing the sensors on the surface. The body waves, such as shear and pressure waves, are present inside the medium. The interaction of the Rayleigh wave, which carries about the 65 percent of energy will be utilized in the detection of buried targets.

The CLEAN algorithm was first used in radio astronomy. Later on, CLEAN and its high resolution version RELAX were modified to work for the case of wide band DOA estimation. Another version was also developed for aero-acoustic imaging, to detect the position of near field sources in passive sensing [2]. The same algorithm can be modified to work for the case of active detection of buried targets. Although a single source could be used, it is more robust to use an array of sources to have multiple looks at the same target. The problem is also a wide band, and can exploit the fact that seismic waves penetration depth is dependent upon the wave-length.

This paper will explore the link between the array models for passive and active sensing. After establishing the link, it will be shown how the RELAX/CLEAN algorithm can be modified to work for the multi-static array case. In the end, the new imaging algorithm will be applied to experimental data collected in a laboratory setting.

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2. RESPONSE MATRIX AND LINK BETWEEN ACTIVE AND PASSIVE SENSING

Using an active array system of N sources and N receivers, an $N \times N$ response matrix, $\mathbf{K}(\omega)$ can be formed after performing N separate transmit-receive operations. The system geometry is shown in Fig. 1. Measurements are made in the time domain, but processing is done in the frequency domain at a single frequency. A simplified response matrix model at a frequency ω can be given by [3],

$$\mathbf{K}(\omega) = \mathbf{H}_1(\omega)\mathbf{D}(\omega)\mathbf{H}_2(\omega) \tag{1}$$

where $\mathbf{H}_2(\omega)$ models the propagation from the transmitter array to the targets, $\mathbf{D}(\omega)$ the scattering matrix, and $\mathbf{H}_1(\omega)$ the propagation from targets back to the receive array. With no multiple scattering, $\mathbf{D}(\omega)$ is a diagonal matrix. The elements of the propagation matrices are given by the Green's function.

In passive sensing (wide-band) the received signal at frequency ω is given by

$$X(\omega) = \mathbf{A}(\omega)S(\omega) + B(\omega) \tag{2}$$

where $\mathbf{A}(\omega)$ is the steering vector matrix, $S(\omega)$ is signal from target, and $B(\omega)$ the noise matrix. For near field targets, the i^{th} column of the steering vector matrix (3-D) is

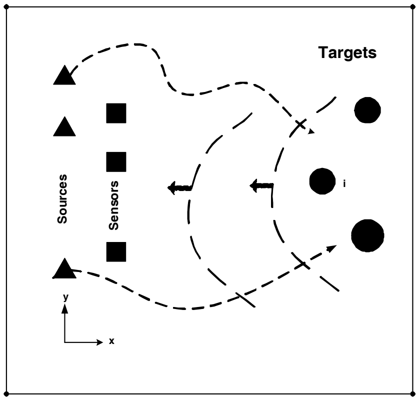


Fig. 1. Near Field Active Array Setup

given by [2],

$$a(r,\theta,\omega) = [\frac{1}{R_{i,1}} e^{j\frac{2\pi}{\lambda(\omega)}R_{i,1}}, ..., \frac{1}{R_{i,N}} e^{j\frac{2\pi}{\lambda(\omega)}R_{i,N}}] \quad (3)$$

where $R_{i,n}$ is the distance between the n^{th} sensor and the i^{th} target.

In an active array system, if a signal (an impulse) is applied at transmitter (source) n, the received scattering is the n^{th} column of $\mathbf{K}(\omega)$, and is given by

$$R_n(\omega) = \mathbf{H}_1(\omega)\mathbf{D}(\omega)H_{2n}(\omega) + B_n(\omega) \quad (4)$$

where $H_{2n}(\omega)$ is the n^{th} column of $\mathbf{H}_2(\omega)$ and $B_n(\omega)$ is the noise vector.

For the single target case, the matrix $\mathbf{H}_1(\omega)$ in (4) has only one column which is given by the illuminating Green's vector

$$g(\mathbf{y}_t, \mathbf{x}, \omega) = [G(\mathbf{y}_t, \mathbf{x}_1, \omega), G(\mathbf{y}_t, \mathbf{x}_2, \omega)..., G(\mathbf{y}_t, \mathbf{x}_N, \omega)] \quad (5)$$

where $\mathbf{y}_t$ is the position of the target and $\mathbf{x}$ is the position of the sensors (receivers). The Green's function G for a 3-D space is

$$G(r, r', \omega) = \frac{1}{|r - r'|} e^{j\frac{2\pi}{\lambda(\omega)}|r-r'|} \quad (6)$$

Thus for the single target case, the scattering received at the n^{th} column of the response matrix, corresponding to a pulse from the n^{th} transmitter is

$$R_n(\omega) = g(\mathbf{y}_t, \omega)\xi(\omega)H_{2n}(\omega) + B_n(\omega) \quad (7)$$

where $\xi(\omega)$ is the scattering coefficient of the target and $\xi(\omega)H_{2n}(\omega)$ represents the scattered energy from the target. Equation (7) is analogous to (2), with $\xi(\omega)H_{2n}(\omega)$ interpreted as the target signal; the steering vectors as given by (3) and (5) have the same form, thus proving the link between near field active and passive sensor systems.

3. WIDEBAND MULTI-STATIC RELAX/CLEAN ALGORITHMS

We will start by defining a multi-static response matrix at frequency ω. A simplified model is

$$\mathbf{K}(\omega) = \mathbf{H}_1(\mathbf{p}, \omega)\mathbf{S}(\omega) \quad (8)$$

where we have replaced $\mathbf{D}(\omega)\mathbf{H}_2(\omega)$ by the target signal $\mathbf{S}(\omega)$, and $\mathbf{p}$ is the position vector that we are trying to estimate. Using the l frequencies in the range where Rayleigh wave has enough energy [4], a least-squares criterion can be formed [2]

$$G = \sum_{l=1}^{L} [\|\mathbf{K}(\omega_l) - \mathbf{H}_1(\mathbf{p}, \omega_l)\mathbf{S}(\omega_l)\|_F]^2 \quad (9)$$

where $\|.\|_F$ denotes the Euclidean norm (Frobenius norm), since $\mathbf{K}(\omega)$ is a matrix. Another version of multilook RELAX (M-RELAX) was also proposed in [5] which treats each look direction independently and then sums them to form the final solution. However, in this paper all the look directions are combined in a single response matrix. The positions of targets which are embedded in propagation model $\mathbf{H}_1(\omega)$ can be estimated as a solution of this minimization problem. However, it is also necessary to estimate the target signal. To minimize G, first we fix the position and solve for target signals at each of the l frequencies. If there are N transmitter (sources), then there will be N versions of these target signals. The least-squares estimate for each target signal is given by

$$\hat{\mathbf{S}}(\omega) = (\mathbf{H}_1(\mathbf{p}, \omega)\mathbf{H}_1(\mathbf{p}, \omega))^{-1}\mathbf{H}_1(\mathbf{p}, \omega)\mathbf{K}(\omega) \quad (10)$$

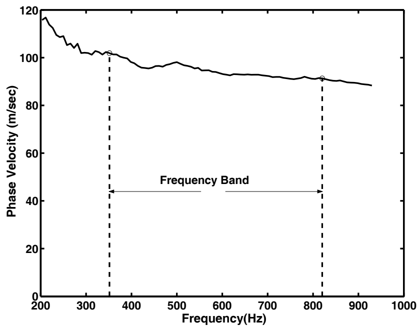
Then we substitute this estimate into (9), and form the following minimization problem

$$\hat{\mathbf{p}} = \arg\min_{\mathbf{p}} \sum_{l=1}^{L} \|[\mathbf{I} - \frac{\mathbf{H}_1(\mathbf{p}, \omega_l)\mathbf{H}_1^H(\mathbf{p}, \omega_l)}{\mathbf{H}_1^H(\mathbf{p}, \omega_l)\mathbf{H}_1(\mathbf{p}, \omega_l)}]\mathbf{K}(\omega_l)\|_F^2 \quad (11)$$

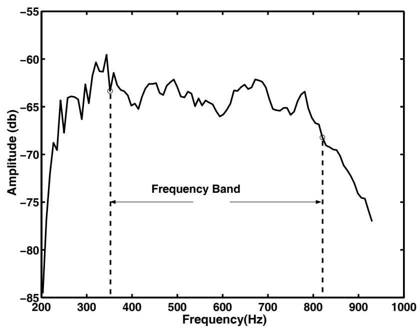
The RELAX/CLEAN algorithm can the be used to perform this minimization, see [2] for details of the algorithm. If there are N sources and receivers, then for a single source (one column of response matrix) the Euclidean norm for a vector of size $(1 \times N)$ is calculated (as used in the original algorithm). However, we extend it for N sources, so that the multi-static response matrix can be used. The only change is that in this case, the Euclidean norm (Frobenius norm) of a matrix of size $(N \times N)$ will be calculated in the solution of (11).

4. PROCESSING OF THE EXPERIMENTAL DATA

The algorithm has been applied to experimental data obtained in a laboratory setting [1]. The data collection scenario consists of inert landmines buried in a large sandbox.



(a)



(b)

Fig. 2. Estimated Rayleigh wave parameters (a) Phase Velocity and (b) Amplitude versus frequency.

In this case, an anti-personal mine (AP) and an anti-tank (AT) mine were buried in the sand. The AP mine is buried at a depth of 1.5 cm and the bigger AT mine is buried at 5 cm. There are 8 sources (shakers), spaced 15 cm apart. A radar-based sensor is used as a receiver, which is capable of measuring soil displacements as small as 1 nm. There are 51 receiving points, 2 cm apart. The estimated amplitude and phase velocity of the Rayleigh wave as a function of frequency [4] are shown in Figs. 2(a) and (b). The plots also show the frequency range of the Rayleigh wave which is then chosen for processing. The penetration depth of a seismic wave depends on its wavelength, with lower frequencies penetrating deep, and vice versa. Since we have two different targets at different depths, it is important that the processing range used should cover both higher and lower frequency bands. The frequency range used is 60 discrete frequencies covering 350 Hz – 820 Hz.

Since the setup has a 2-D geometry, the propagation model uses a 2-D Green's function with the velocity values obtained from Fig. 2. The model is given by

$$G(r, r^{'}, \omega) = \frac{i}{4} H_0^{(1)}\left(\frac{\omega}{v(\omega)}|r - r^{'}|\right) \tag{12}$$

where $H_0^{(1)}$ is the Hankel function of first kind and order zero. The RELAX algorithm requires a prior estimate of the number of targets, which can be obtained by using the singular values of the response matrix [3]. For the RELAX algorithm the number of targets used is 2, and the algorithm was run for 10 iterations, but it seems to converge in just few iterations, with location estimates obtained as the minimizer of (11). The target location estimates, and also the coordinates of targets obtained in each iteration are shown in Figs. 3(a) and (b). The grid size used in the search is (60×100) cm, with a step size of 1 cm. The algorithm was run for 60 frequencies, and the minimum was found within this search area. The results for the CLEAN algorithm are shown in Figs. 4(a) and (b). CLEAN is an iterative algorithm, in which at each iteration some fraction (0.1) of the previously estimated signal is subtracted out. The results indicate that algorithm tends to diverge if run for too many iterations, e.g., after only five iterations. At each iteration the algorithm gives the coordinate of strongest target, and then removes all (RELAX) or some portion (CLEAN) of this target contribution. Therefore, at the next iteration the algorithm finds the next strongest target (if one exists). This causes an alternating behavior in the coordinate estimate as shown in Figs. 3(b) and 4(b), respectively.

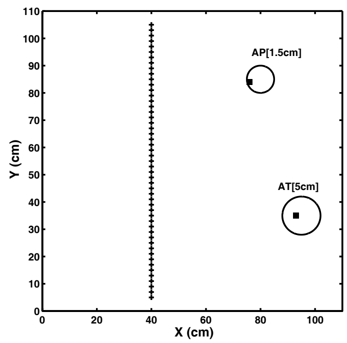
Another interesting result is obtained when the landmine is surrounded by buried rocks. A small AP mine was buried at the depth of 1.5 cm surrounded by some rocks. Figure 5 shows the mine and rock locations with sand removed. The image can be formed of the surface as a function of position $\mathbf{p}$ as

$$\mathbf{SURF}(\mathbf{p}) = \sum_{l=1}^{L} \left\| \left[\mathbf{I} - \frac{\mathbf{H}_1(\mathbf{p}, \omega_l)\mathbf{H}_1^H(\mathbf{p}, \omega_l)}{\mathbf{H}_1^H(\mathbf{p}, \omega_l)\mathbf{H}_1(\mathbf{p}, \omega_l)} \right] \mathbf{K}(\omega_l) \right\|_F^2$$

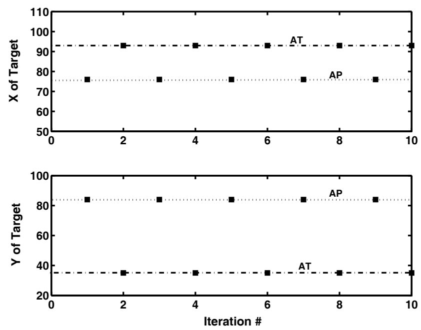
The minimizer of this function (over a grid) is the location estimate. The inverse of this surface is plotted in Fig. 6, at the first iteration of the RELAX algorithm. The peak location is at the coordinates (36,46) cm, which is the correct estimate of the target's location, even in the presence of rocks. The way the seismic waves behave with rocks and man made targets is different, with waves reflected more from the latter. This is very promising to detect landmines, as compare to other techniques.

5. CONCLUSIONS

In this paper, we modified the RELAX and CLEAN algorithms to work for the detection of passive buried targets using seismic waves. In addition, we have shown how the methods work for a multi-static active array system. Finally, the algorithm was successfully applied to experimental data obtained in a laboratory setup.

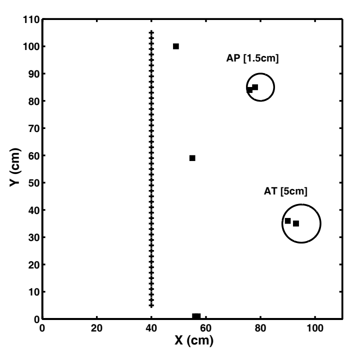


(a)

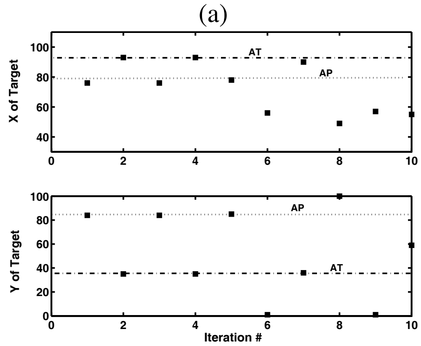


(b)

Fig. 3. RELAX (a) Location Estimates (b) Target Coordinates.



(a)



(b)

Fig. 4. CLEAN (a) Location Estimates (b) Target Coordinates.

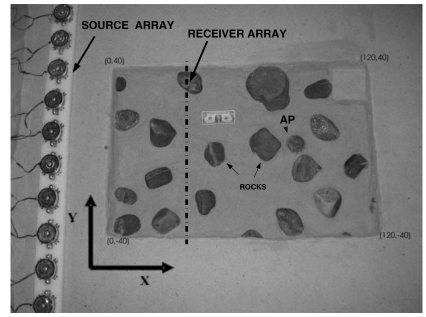


Fig. 5. Experimental Setup

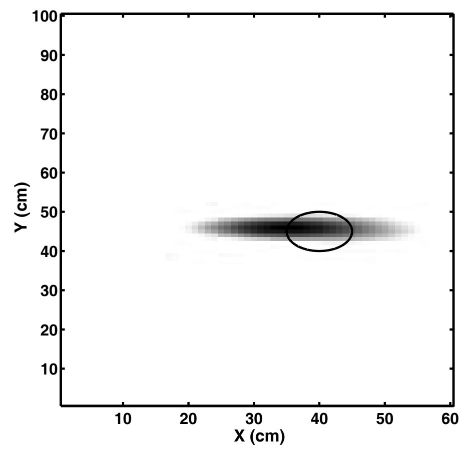


Fig. 6. Location Estimates

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