MAXIMUM SIGNAL-TO-NOISE RATIO GPS ANTI-JAM RECEIVER WITH SUBSPACE TRACKING

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ABSTRACT

In this paper, we propose an anti-jam GPS receiver which suppresses interference by projecting the received signal on the noise subspace obtained via subspace tracking. The resulting interference-free signal is then processed by a beamformer, whose weight vector is obtained by maximizing the signal-to-noise ratio at the beamformer output. It is shown that the proposed receiver can effectively eliminate interfere and enhance the GPS signals at the receiver output.

1. INTRODUCTION

Global Positioning System (GPS) is a tool to determine position, velocity, and precise time worldwide by measuring the time-of-arrival of signals emitted from satellites. In addition to its original military purpose, GPS has found a wide range of civilian applications such as navigation, land surveying and mapping, and timing and synchronization for telecommunication networks.

For GPS applications, the main challenges are the vulnerability of the GPS receivers to strong interference and the multipath effects on receiver synchronization. GPS employs spread-spectrum (SS) signaling, which provides a certain degree of protection against interference. However, if the interfering signal's power exceeds the 30 dB processing gain offered via the spreading/dispreading of the GPS C/A signal, the receiver is unable to recover the navigation information conveyed in GPS signal. Therefore, the design of GPS receivers must mitigate the interference and combat its effect on the receiver's ability to synchronize with different satellites. Multipath, on the other hand, is caused by signal reflections and diffractions between the satellite and the GPS receiver. In GPS, the desired signal is the direct path signal. All other signals distort the desired signal and lead to ranging measurement errors.

In this paper, we propose an interference suppression scheme which combines subspace tracking and adaptive beamforming. Specifically, the received signal is first projected into its noise subspace. The resulting interference-free signal is then processed by a spatial filter, whose weights are determined by the maximum signal-to-noise ratio (MSNR) criterion. Computer simulations have shown that the proposed method is effective in combating strong interference and enhancing the GPS signal.

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2. SUBSPACE TRACKING INTERFERENCE SUPPRESSION

2.1. Signal Model

The GPS receiver is equipped with an M-element spatial array, as shown in Figure 1. The waveforms impinging on the array are those of the GPS signal and its multipath, interference, and noise. After down-conversion and chip-rate sampling, the received signal vector from the antenna array can be presented in discrete-time format as

$$\mathbf{x}(n) = \sum_{k=0}^{K} s_k(n) c_k(nT_s - \tau_k(n))\mathbf{a}_k + \sum_{l=1}^{L} u_l(n)\mathbf{d}_l + \mathbf{v}(n), \tag{1}$$

where

T_s	Nyquist sampling interval;
K	number of multipath components;
$s_k(n)$	kth signal component;
c_k	kth C/A-code sample;
$\tau_k(n)$	time-delay of the kth component;
a_k	spatial signature of the kth satellite multipath;
L	number of interferers;
$u_l(n)$	waveform of the lth interferer;
$\mathbf{d}_l$	spatial signature of the lth interferer;
$\mathbf{v}(n)$	additive white Gaussian noise sample vector.

Due to the weak cross-correlation of the C/A-codes, only one satellite is considered in Eq. (1). The subscript 0 is designated to the direct-path signal. Let $\mathbf{s}(n) \triangleq s_0(n)c_0(nT_s - \tau_0(n))\mathbf{a}_0$ denote the data vector across the array due to the direct-path signal. Then, Eq. (1) can be rewritten as

$$\mathbf{x}(n) = \mathbf{s}(n) + \mathbf{s_m}(n) + \mathbf{u}(n) + \mathbf{v}(n), \tag{2}$$

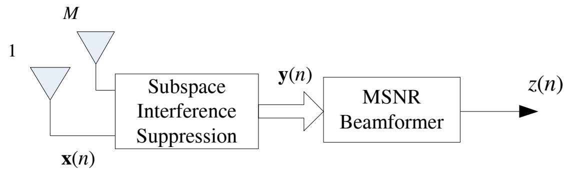

Fig. 1. Block diagram of the proposed GPS receiver.

where $\mathbf{s}_{\mathrm{m}}(n)$ denotes the contributions from K multipath reflections,

$$\mathbf{s}_{\mathrm{m}}(n) \triangleq \sum_{k=1}^{K} s_k(n) c_k(nT_s - \tau_k(n)) \mathbf{a}_k, \tag{3}$$

and $\mathbf{u}(n) \triangleq \sum_{l=1}^{L} u_l(n) \mathbf{d}_l$ is the compound interference vector.

2.2. Subspace Tracking Based Interference Suppression

Under the assumption that the GPS signals, interference, and noise are independent, the covariance matrix of the received signal becomes

$$\mathbf{R}_{xx} = E\{\mathbf{x}(n)\mathbf{x}^H(n)\} = \mathbf{R}_s + \mathbf{R}_u + \mathbf{R}_v, \tag{4}$$

where $E\{\cdot\}$ represents the statistical expectation, $(\cdot)^H$ denotes conjugate transpose, and $\mathbf{R}_s$, $\mathbf{R}_u$, and $\mathbf{R}_v$ are the covariance matrices of the GPS signals, the interference, and the noise, which are defined, respectively, as:

$$\mathbf{R}_s \triangleq E\{[\mathbf{s}(n) + \mathbf{s}_{\mathrm{m}}(n)][\mathbf{s}(n) + \mathbf{s}_{\mathrm{m}}(n)]^H\} \tag{5}$$

$$\mathbf{R}_u \triangleq E\{\mathbf{u}(n)\mathbf{u}^H(n)\} \tag{6}$$

$$\mathbf{R}_v \triangleq E\{\mathbf{v}(n)\mathbf{v}^H(n)\} = \sigma_v^2 \mathbf{I}_M, \tag{7}$$

where $\mathbf{I}_M$ is an $M \times M$ identity matrix.

The subspace tracking based GPS anti-jam receiver is motivated by the fact that in GPS, the desired GPS signals are well below the noise floor (usually 20 to 30 dB below the noise floor). As such, the total received signal power is dominated by the jamming signals. In this case, the covariance matrix $\mathbf{R}_{xx}$ is approximated as [1]

$$\mathbf{R}_{xx} \approx \mathbf{R}_v + \mathbf{R}_u. \tag{8}$$

By performing singular value decomposition (SVD) of $\mathbf{R}_{xx}$, we can effectively decompose the received signal into two subspaces:

$$\begin{aligned} \mathbf{R}_{xx} &= \sum_{i=1}^{M} \lambda_i \mathbf{e}_i \mathbf{e}_i^H \approx \sum_{i=1}^{L} \lambda_i \mathbf{e}_i \mathbf{e}_i^H + \sigma_v^2 \sum_{i=L+1}^{M} \mathbf{e}_i \mathbf{e}_i^H \\ &\triangleq \mathbf{U}_{\mathrm{I}} \mathbf{\Sigma}_{\mathrm{I}} \mathbf{U}_{\mathrm{I}}^H + \mathbf{U}_{\mathrm{V}} \mathbf{\Sigma}_{\mathrm{V}} \mathbf{U}_{\mathrm{V}}^H, \end{aligned} \tag{9}$$

where $\mathbf{\Sigma}_{\mathrm{I}} = \mathrm{diag}\{\lambda_1, \cdots, \lambda_L\}$ is an $L \times L$ diagonal matrix whose elements are the L largest eigenvalues. $\mathbf{U}_{\mathrm{I}}$ is an $M \times L$ matrix whose columns, eigenvectors $\mathbf{e}_1, \cdots, \mathbf{e}_L$ associated with L largest eigenvalues, span the *interference subspace*. $\mathbf{\Sigma}_{\mathrm{V}} = \sigma_v^2 \mathbf{I}_{M-L}$ contains the rest $M - L$ eigenvalues, which are assumed to be equivalent to σ_v^2, and the columns of the $M \times M - L$ matrix $\mathbf{U}_{\mathrm{V}}$ are the associated $M - L$ eigenvectors, which span the *noise subspace*. Note that vectors $\{\mathbf{d}_1, \cdots, \mathbf{d}_L\}$ also span the interference subspace, i.e.,

$$\mathrm{span}\{\mathbf{e}_1, \cdots, \mathbf{e}_L\} = \mathrm{span}\{\mathbf{d}_1, \cdots, \mathbf{d}_L\}. \tag{10}$$

Even though the spatial signatures $\mathbf{d}_l$, $l = 1, \cdots, L$ of the interferers are usually unknown at the receiver, the interference subspace can be explicitly obtained by first computing an estimate of $\mathbf{R}_{xx}$, then performing SVD. For large arrays, however, the eigendecomposition imposes heavy computational burdens on the receiver. Therefore, such method may not be suitable for real time processing of GPS signals. An alternative approach is to use the so called *subspace tracking* techniques [2]. Such techniques are especially suitable for GPS because the power of the interferer is much stronger than that of the GPS signal, and the number of interferers is limited which, in turn, limits the dimension of the interference subspace. Subspace tracking estimates the interference subspace recursively on a sample-by-sample basis and, thus, avoids explicit calculation of the matrix $\mathbf{R}_{xx}$. For the anti-jam GPS receiver, we use the projection approximation subspace tracking with deflation (PASTd) method proposed in [3]. The PASTd algorithm first estimates the most dominant eigenvector and the projection of the received data onto this eigenvector is then removed from the received data. Now the second dominant eigenvector becomes the most dominant one in the updated data vector and it can be extracted as well. Repeating this procedure, all desired eigencomponents can be estimated sequentially. The PASTd algorithm is summarized in Table 1.

Table 1. The PASTd Algorithm

Choose an initial $\mathbf{e}(0)$ properly
For $n = 1, 2, \cdots$ Do
$\quad \mathbf{x}_1(n) = \mathbf{x}(n)$
$\quad$ For $l = 1$ to L Do
$\qquad \alpha_l(n) = \mathbf{e}_l^H(n-1)\mathbf{x}_l(n)$
$\qquad \gamma_l(n) = \beta\gamma_l(n-1) + \lvert\alpha_l(n)\rvert^2$
$\qquad \mathbf{e}_l(n) = \mathbf{e}_l(n-1) + \frac{[\mathbf{x}_l(n) - \mathbf{e}_l(n-1)\alpha_l(n)]\alpha_l^*(n)}{\gamma_l(n)}$
$\qquad \mathbf{x}_{l+1}(n) = \mathbf{x}_l(n) - \mathbf{e}_l(n)\alpha_l(n)$

Usually, the initial $\mathbf{e}(0)$ can be chosen from an identity matrix. In the above algorithm, $\mathbf{e}_i$ is an estimate of the lth eigenvector and γ_l is the corresponding eigenvalue, and $0 < \beta \leq 1$ is the forgetting factor.

Once the interference subspace is available, the noise subspace can be obtained from the orthogonal projection of the interference subspace, which is given by

$$\mathbf{U}_{\mathrm{I}}^{\perp} = \mathbf{I}_M - \mathbf{U}_{\mathrm{I}}(\mathbf{U}_{\mathrm{I}}^H \mathbf{U}_{\mathrm{I}})^{-1} \mathbf{U}_{\mathrm{I}}^H, \tag{11}$$

where $(\cdot)^{1}$ denotes matrix inverse. Therefore, columns of $\mathbf{U}_{\mathrm{I}}^{\perp}$ span the noise subspace. The projection of $\mathbf{x}(n)$ onto $\mathbf{U}_{\mathrm{I}}^{\perp}$ yields

$$\mathbf{y}(n) = \mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{x}(n) = \mathbf{U}_{\mathrm{I}}^{\perp}[\mathbf{s}(n) + \mathbf{s}_{\mathrm{m}}(n)] + \mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{v}(n), \quad (12)$$

which only contains contributions from the GPS components and noise.

3. MSNR BEAMFORMER

From the above discussion, we know that by projecting the received data onto the noise subspace, the interfering signals are completely suppressed. After the suppression of the interference, the GPS signal is still far below the noise floor. In order to synchronize the receiver with the satellite, which is usually achieved by cross-correlating the received data with a locally generated C/A-code and identifying the maximum value, the GPS signal must be enhanced. To this end, we design a filter such that the output of the filter achieves the maximum signal-to-noise ratio (MSNR).

Let $\mathbf{w}$ be the $M \times 1$ weight vector. Then, the output of the filter is given by

$$\begin{aligned} z(n) &= \mathbf{w}^H\mathbf{y}(n) \\ &= \mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}[\mathbf{s}(n) + \mathbf{s}_{\mathrm{m}}(n)] + \mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{v}(n), \end{aligned} \quad (13)$$

and the filter $\mathbf{w}$ is determined from

$$\begin{aligned} \mathbf{w}_{\mathrm{MSNR}} &= \max_{\mathbf{w}} \frac{E\{|\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}[\mathbf{s}(n) + \mathbf{s}_{\mathrm{m}}(n)]|^2\}}{E\{|\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{v}(n)|^2\}} \\ &= \max_{\mathbf{w}} \frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_s\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}}. \end{aligned} \quad (14)$$

Equation (14) indicates that the determination of beamformer $\mathbf{w}$ requires the power of the GPS signal (contains both the power from the direct-path signal and the contributions from its multipath components) at the receiver. Generally, the calculation of the GPS signal power requires some *a priori* knowledge of the satellite. For example, as indicated in [4], if the location of the satellite is known, then the GPS signal power can be computed as follows. Assume that the satellite is located at the angle (θ, ψ). For GPS, the temporal autocorrelation of the transmitted signal, which is essentially the autocorrelation function of the Gold code denoted as $R_c(\tau)$, is known. The $M \times M$ matrix $\mathbf{R}_s$ is calculated in the absence of interference and noise. Let $\eta(m)$ denote the phase shift for the GPS satellite at the angle (θ, ψ) to the mth antenna and let $\mathbf{r} \triangleq \begin{bmatrix} e^{j\eta(1)} & \cdots & e^{j\eta(M)} \end{bmatrix}^T$. Then, $\mathbf{R}_s$ is given by $\mathbf{R}_s = R_c(0)\mathbf{r}\mathbf{r}^H$.

On the other hand, if the satellite location information is not available at the receiver, we take an alternative approach to solve the above maximization problem. From Eq. (12) we know that $\mathbf{y}(n)$ only contains contributions from the GPS signals and noise. Due to the weakness of the GPS signals, the output of the projection is dominated by noise. Note that

$$\begin{aligned} \frac{E\{|\mathbf{w}^H\mathbf{y}(n)|^2\}}{E\{|\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{v}(n)|^2\}} &= \frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_{xx}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}} \\ &= \frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_s\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}} + 1, \end{aligned} \quad (15)$$

which shows that the beamformer $\mathbf{w}$ that maximizes

$$\frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_s\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{H\perp}\mathbf{w}} \quad (16)$$

also maximizes

$$\frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_{xx}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}}. \quad (17)$$

Therefore, the filter $\mathbf{w}$ can be found by solving

$$\mathbf{w}_{\mathrm{MSNR}} = \max_{\mathbf{w}} \frac{\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_{xx}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}}{\sigma_v^2\mathbf{w}^H\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}}. \quad (18)$$

Thus, the optimum $\mathbf{w}$ is the eigenvector corresponding to the dominant eigenvalue of the following generalized eigenvalue problem:

$$\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{R}_{xx}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w} = \mu\mathbf{U}_{\mathrm{I}}^{\perp}\mathbf{U}_{\mathrm{I}}^{\perp H}\mathbf{w}, \quad (19)$$

where μ denotes the dominant eigenvalue, which is also the maximum SNR.

In practice, $\mathbf{R}_{xx}$ is replaced by its sample estimate, which can also be computed recursively [5].

4. SIMULATIONS

A linear uniform array consisting of $M = 7$ sensors with half-wavelength spacing is used in the simulation.

In the first simulation, we investigate the convergence of the PASTd algorithm. In this experiment, there is one jammer added to the received signal and we set the signal-to-noise ratio (SNR) to -10 dB and signal-to-interference ratio (SIR) to -30 dB. This lead to the interference-to-noise ratio (INR) of 20 dB. Figure 2 shows that after 250 samples, the eigenvalue converges to the true INR of 20 dB.

In the next experiment, we examine the interference suppression performance of the proposed GPS receiver. There are two jammers located at 30° and 60°, and the satellite is at 10°. Figure 3 shows that the receiver can successfully generate high gain toward the satellite direction, while placing deep nulls at the jammer locations. By using the proposed GPS receiver, the desired GPS signal is enhanced, whereas the jammers are suppressed.

Finally, we investigate the proposed receiver's synchronization capability. Generally, the synchronization can be achieved by cross-correlating the received signal with the locally generated C/A-code [6]. When the receiver synchronizes with the satellite, there is a maximum correlation. In the simulation, we considered three different scenarios to illustrate the receiver's performance. We set $SNR = -30$ dB and $SIR = -40$ dB. In the first case, the received signal is directly correlated with the C/A-code and the resulting normalized cross-correlation is shown in Figure 4(a). It is noted that without any processing, the synchronization fails. If only the interference suppression is applied, the receiver is able to synchronize with the satellite after the cross-correlation, but the noise contribution remains significant, as shown in 4(b). With the proposed receiver, however, the noise can be drastically reduced. This is shown in 4(c).

5. CONCLUSIONS

In this paper, we have considered the problem of interference cancellation in GPS. Specifically, a GPS receiver combining the subspace interference suppression and MSNR beamforming is proposed. Through computer simulations, we have shown that the proposed receiver is capable of providing high gains for the desired GPS signal while suppressing the strong interferers.

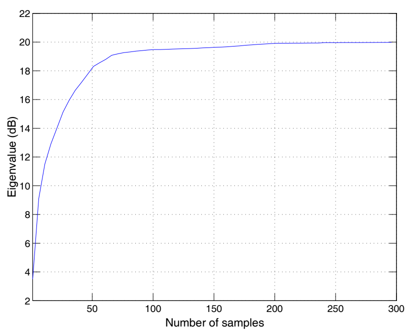

Fig. 2. Convergence of the eigenvalue.

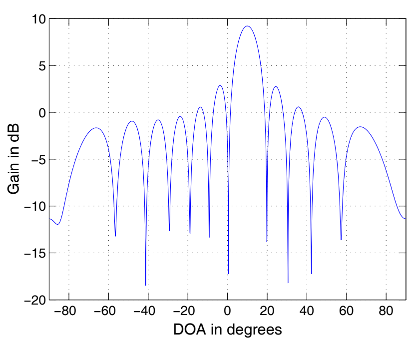

Fig. 3. Beampattern of the proposed GPS receiver.

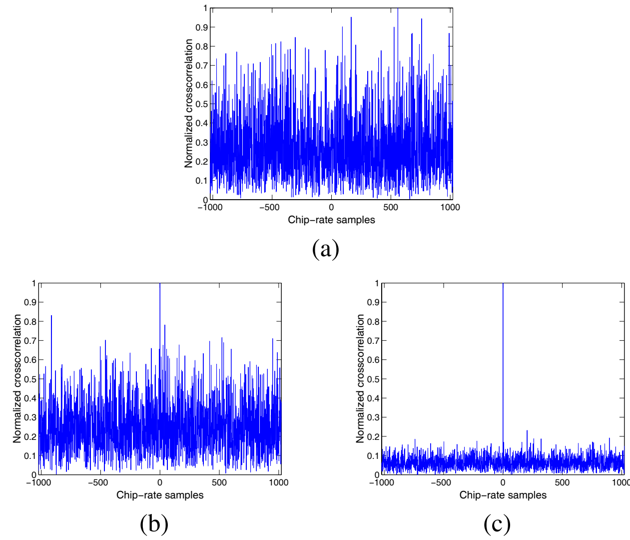

Fig. 4. Normalized cross-correlation. (a) Without interference suppression and beamforming; (b) With interference suppression but without beamforming; (c) Proposed receiver.

6. REFERENCES

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