MOBILE LOCALIZATON METHOD BASED ON MULTIDIMENSIONAL SIMILARITY ANALYSIS

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ABSTRACT

A novel noise subspace based method is applied to the minimum localization system using time-of-arrival (TOA) measurements from three base stations (BS). Since the distance measurement between the mobile station (MS) and the BS bears analogy to the multidimensional similarity (MDS) between their coordinates, we express the MS coordinate as the linear combination of the BSs' coordinates, where the weight vector lies in the noise subspace of the MDS matrix. It is proved that this weight vector is the area coordinate of the MS when the triangle formed by the three BSs serves as the reference frame. Because the dimension knowledge of the localization problem is utilized to estimate the noise subspace and to mitigate the errors in TOA measurements, the proposed method is superior to the ordinary linear localization method in most of the enhanced quadrants of the area coordinates system.

1. INTRODUCTION

Recently, positioning of the mobile station (MS) in cellular systems attracted large interest and the number of applications based on the location information grew rapidly. Large-scale deployment of such applications usually requires methods for positioning that are accurate and simple enough to be used in mobile phones.

Many linear methods have been proposed to estimate the MS position in closed-form. The first and simplest of the existing methods is the Cell ID (CID) method where the position estimate is the coordinate of the serving BS. The second of the existing methods is described in [1], the MS position is calculated as the average (centroid) of the positions of all *N* BSs whose beacons the MS can decode. It is called UnWeighted Centroid (UWC) method. The CID method can be regarded as a special case of UWC with N=1. In [2], three approximated linear methods were proposed to estimate the position of the MS. The first, Path-Gain Weighted Centroid (PGWC) method, is based Jia Xu, Jun Tang, Ying-Ning Peng Department of Electronic Engineering Tsinghua University Beijing 100084, China ynpeng@mail.tsinghua.edu.cn

on signal strength, the second, Time Weighted Centroid (TWC) method, is based on time, and the third, PGWC+TA, is a hybrid between time and signal-strength methods. Finally, the third benchmark method uses circular multilateration of propagation delays. This is sometimes known as Time-of-Arrival (TOA) [3]. In our implementation of TOA localization, the ordinary method calculates the MS positions from the MS-BS*i* propagation delays using a linear and closed-form solution [4].

In this paper, we propose a simple but quite accurate localization method and compare it with the ordinary method by means of simulation. Our results show that in the noisy measurement environment, the proposed algorithm is superior to ordinary linear localization methods in most of the enhanced quadrants of area coordinates system. Unlike the enhanced quadrant-aware localization method in [5] and [6], the linear weight vector in the proposed method is derived from multidimensional similarity analysis.

Our proposed methods are very simple and do not require complicated calculations as opposed to some iterative methods [7]-[9]. The performance measures are coordinate bias and cumulative distribution function of the root mean square location error, and the evaluation is done for seven quadrants in the area coordinate system. Because the dimension knowledge of the localization problem is utilized to estimate the noise subspace and to mitigate the errors in range measurements, the proposed method is superior to the ordinary linear localization method in most of the enhanced quadrants.

2. LINEAR TOA LOCALIZATION

Consider the problem of MS location using range measurements from three BSs. Assume that the BSs locate at (x_i, y_i) , *i*=1,2,3, and the MS locates at (x_0, y_0) in the system of rectangular coordinates.

The MS position can be expressed as

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$
(1)

It is the linear combination of BSs' position, where the weight vector (s_1, s_2, s_3) is not unique. Introducing the following constraint

$$s_1 + s_2 + s_3 = 1 \tag{2}$$

Equation (1) becomes

$$\begin{bmatrix} 1\\ x_0\\ y_0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1\\ x_1 & x_2 & x_3\\ y_1 & y_2 & y_3 \end{bmatrix} \begin{bmatrix} s_1\\ s_2\\ s_3 \end{bmatrix}$$
(3)

From Cramer's determinant formula, we have

$$s_{1} = \frac{1}{s_{123}} \begin{vmatrix} 1 & 1 & 1 \\ x_{0} & x_{2} & x_{3} \\ y_{0} & y_{2} & y_{3} \end{vmatrix}$$
$$s_{2} = \frac{1}{s_{123}} \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{0} & x_{3} \\ y_{1} & y_{0} & y_{3} \end{vmatrix}$$
$$s_{3} = \frac{1}{s_{123}} \begin{vmatrix} 1 & 1 & 1 \\ x_{1} & x_{2} & x_{0} \\ y_{1} & y_{2} & y_{0} \end{vmatrix}$$

where

$$s_{123} = \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

It can be proved that the absolute value of s_1, s_2 and s_3 is the normalized area (with respect to the area of triangle formed by BS1, BS2 and BS3) of triangle formed by (MS, BS2, BS3), (MS, BS1, BS3) and (MS, BS1, BS2), respectively. Therefore, we call (s_1, s_2, s_3) area coordinates of the MS. The signs of (s_1, s_2, s_3) are determined by the enhanced quadrant in which the MS lies [5]. As shown in Fig.1, we have seven quadrants in area coordinate system, whereas four in rectangular coordinate system.

From (3), we have the following orthogonal property,

$$\begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix} \begin{bmatrix} x_1 - x_0 & y_1 - y_0 \\ x_2 - x_0 & y_2 - y_0 \\ x_3 - x_0 & y_3 - y_0 \end{bmatrix} = 0$$
(4)

which means that the vector $[s_1, s_2, s_3]$ lies in the orthogonal subspace spanned by vectors $[x_1 - x_0 \quad x_2 - x_0 \quad x_3 - x_0]$ and $[y_1 - y_0 \quad y_2 - y_0 \quad y_3 - y_0]$. If the vector $[s_1, s_2, s_3]$ is estimated from TOA measurements, the MS position can be calculated from (4). In [5], the absolute value of area coordinate of MS is estimated by using Heron formula, but the sign is determined by the enhanced quadrant information. When this information is not known, it may be determined from an original estimation of MS position. However, when the MS locates close to the axes of the reference triangle, the

ambiguity of the enhanced quadrant will degrade the location performance.

In the following two sections, we introduce the multidimensional similarity (MDS) matrix and establish the relationship between the vector $[s_1, s_2, s_3]$ and the null space of the MDS matrix. Based on this relationship, we can calculate the vector $[s_1, s_2, s_3]$ from the null subspace of the MDS matrix without additional quadrant information.

3. MULTIDIMENSIONAL SIMILARITY ANALYSIS

Assume that the distance between the *i*-th BS and the MS is r_i , between the *i*-th BS and the *j*-th BS is d_{ij} , *i*, *j*=1,2,3. Note that $d_{ii} = 0$ and $d_{ij} = d_{ji}$. Define a symmetric matrix as

$$\mathbf{D} = \frac{1}{2} \begin{bmatrix} 2r_1^2 & r_1^2 + r_2^2 - d_{12}^2 & r_1^2 + r_3^2 - d_{13}^2 \\ r_2^2 + r_1^2 - d_{21}^2 & 2r_2^2 & r_2^2 + r_3^2 - d_{23}^2 \\ r_3^2 + r_1^2 - d_{31}^2 & r_3^2 + r_2^2 - d_{32}^2 & 2r_3^2 \end{bmatrix}$$
(5)

Denote the element of matrix **D** by D_{ij} , *i*, *j*=1,2,3. From cosine formula, we have

$$D_{ij} = r_i^2 + r_j^2 - d_{ij}^2$$

= $2r_i r_j \cos \theta_{ij}$
= $2[x_i - x_0 \quad y_i - y_0] \begin{bmatrix} x_j - x_0 \\ y_j - y_0 \end{bmatrix}$

where θ_{ij} is the angle between the vectors $\begin{bmatrix} x_i - x_0 & y_i - y_0 \end{bmatrix}$ and $\begin{bmatrix} x_j - x_0 & y_j - y_0 \end{bmatrix}$. Therefore, we have

$$\mathbf{D} = \begin{bmatrix} x_1 - x_0 & y_1 - y_0 \\ x_2 - x_0 & y_2 - y_0 \\ x_3 - x_0 & y_3 - y_0 \end{bmatrix} \begin{bmatrix} x_1 - x_0 & x_2 - x_0 & x_3 - x_0 \\ y_1 - y_0 & y_2 - y_0 & y_3 - y_0 \end{bmatrix}$$
(6)

It can be seen that the matrix **D** is semi-positive with the rank equaling to 2 if vectors $\begin{bmatrix} x_1 - x_0 & x_2 - x_0 & x_3 - x_0 \end{bmatrix}$ and $\begin{bmatrix} y_1 - y_0 & y_2 - y_0 & y_3 - y_0 \end{bmatrix}$ is not correlated, and D_{ij} , the element of matrix **D**, is the correlation between the vectors $\begin{bmatrix} x_i - x_0 & y_i - y_0 \end{bmatrix}$ and $\begin{bmatrix} x_j - x_0 & y_j - y_0 \end{bmatrix}$, *i*, *j*=1,2,3. Because D_{ij} measures the similarity between two vectors, matrix **D** is also called multidimensional similarity matrix [10].

4. SUBSPACE BASED LOCALIZATION

Let the eigenvalue decomposition of **D** be $\mathbf{D} = \mathbf{U}\Sigma\mathbf{U}^T$ where $\Sigma = diag(\lambda_1, \lambda_2, \lambda_3)$, $\mathbf{U} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$ and $\lambda_1 \ge \lambda_2 > \lambda_3$, $\begin{bmatrix} \\ \end{bmatrix}^T$ denotes transposition. Because the rank of matrix **D** equals to 2, we have

$$\lambda_3 = 0$$

and

$$\mathbf{u}_3^T \mathbf{D} \mathbf{u}_3 = \mathbf{0}$$

From (6), we have

$$\mathbf{u_{3}^{T}} \begin{bmatrix} x_{1} - x_{0} & y_{1} - y_{0} \\ x_{2} - x_{0} & y_{2} - y_{0} \\ x_{3} - x_{0} & y_{3} - y_{0} \end{bmatrix} = 0$$
(7)
Comparing (4) with (7) yields
$$\begin{bmatrix} s_{1} & s_{2} & s_{3} \end{bmatrix} = \alpha \mathbf{u_{3}^{T}}$$

where

$$\alpha = \frac{1}{sum(\mathbf{u}_3)}$$

is sum of the elements of vector \mathbf{u}_3 . From (7), a novel subspace based position estimation can be given by

$$x_0 = \frac{\mathbf{u}_3^{\mathsf{T}}}{sum(\mathbf{u}_3)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$
(8)

$$y_0 = \frac{\mathbf{u}_3^{\mathsf{T}}}{sum(\mathbf{u}_3)} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$
(9)

5. SIMULATION RESULTS

In this section, we compare the proposed location method with the ordinary method, which gives the position estimation as [4]

$$\begin{bmatrix} x_{0} \\ y_{0} \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} x_{2} - x_{1} & y_{2} - y_{1} \\ x_{3} - x_{1} & y_{3} - y_{1} \end{bmatrix}^{-1} \begin{bmatrix} x_{2}^{2} + y_{2}^{2} - x_{1}^{2} - y_{1}^{2} + r_{1}^{2} - r_{2}^{2} \\ x_{3}^{2} + y_{3}^{2} - x_{1}^{2} - y_{1}^{2} + r_{1}^{2} - r_{3}^{2} \end{bmatrix}$$
(10)

As shown in Fig.1, three base stations locate at (0, 0), (2500, 4330) and (5000, 0), and the range measurement error is Gaussian distributed with zero mean and standard deviation of 30, all the units are meter. To compare the performance of the above two methods, seven mobile stations are fixed in different quadrant of area coordinate system.

Table 1 gives the comparison results of the root mean square location error. They are obtained from 300 runs at each MS position. It can be seen that the improvement of the proposed method is significant in quadrant 2, 4, 5 and 7. Because the root mean square location error is not enough to reflect the distribution of location error, we also plot in Figs.2-5 the estimation bias of x and y coordinates obtained in each run.



Fig.1 Position of mobile stations and base stations. There is one mobile station in each quadrant of area coordinate system.

Table 1: Comparison of root mean square location error in different quadrant of area coordinate system. (m)

Quadrant	Proposed	Ordinary
	method	method
1	66.03	66.07
2	22.30	41.18
3	24.69	22.25
4	26.87	42.29
5	38.35	49.15
6	23.49	23.44
7	40.78	50.36

Comparison of location error in different quadrant of area coordinate system is illustrated in Figs.2-5. The left two columns of these figures are the location bias of x and y coordinates obtained from the proposed method and the ordinary method, respectively, and the right column is the cumulative distribution function (CDF) of the root mean square location error (solid line: the proposed method, dotted line: the ordinary method). Though the location error distributes differently in each quadrant, it can be seen that the improvement of the proposed method is significant in most of quadrants, except quadrant 3.

6. CONCLUSION

This paper establishes the relationship between the weight vectors of the enhanced quadrant-aware method and the proposed dimension-aware method, which estimate the rectangular coordinates of the mobile station by linear combination of that of the base stations. Unlike the former method, the linear weight vector is estimated from multidimensional similarity analysis and the additional enhanced quadrant information is not required in the proposed method. Simulation results shown that the proposed method performs better than the ordinary method in most of quadrants in area coordinate system.



Fig.2 Comparison of location error in quadrants 1 and 2.



Fig.3 Comparison of location error in quadrants 3 and 4.



Fig.4 Comparison of location error in quadrants 5 and 6.



Fig.5 Comparison of location error in quadrant 7.

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