ALTERNATE SOURCE AND RECEIVER LOCATION ESTIMATION USING TDOA WITH RECEIVER POSITION UNCERTAINTIES

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ABSTRACT

The accuracy of source localization is sensitive to the knowledge of the receiver positions. In the presence of the receiver position error, a robust algorithm is necessary to improve performance. This paper presents an iterative algorithm for estimating alternately the location of an emitter and the positions of receivers using Time Difference of Arrival (TDOA) measurements, when the receivers have random position errors. The proposed solution is based on weighted least-squares (WLS) minimization, and does not have convergence problem. The estimated accuracy of emitter and receiver locations are approaching the CRLB under Gaussian noise with small receiver position error. The performance of the proposed algorithm is evaluated through simulations.

1. INTRODUCTION

Receiver location uncertainty can severely deteriorate source localization accuracy. Even relatively small uncertainty in receiver (sensor) locations could make considerable contributions to localization error. Thus, the problem of improving the estimated source location under receiver position errors is an important issue and has been addressed by many authors [1]-[7].

As such, in Direction of arrival (DOA) estimation, various self-calibration algorithms have been proposed to jointly estimate the source DOA and the sensor locations. Rockah and Schultheiss [1]-[2] utilized the CRLB to derive a DOA estimation method in the presence of sensor position uncertainty, with the knowledge that one sensor location and its direction to the second sensor is known exactly. Tseng *et al*. [3] proposed projection-rotation-scaling (PROS) method to estimate the steering vector in uncalibrated arrays which is used to improve the reliable source DOA estimates. Weiss and Friedlander [4] proposed an extension method on the MUSIC algorithm which alternately estimates the DOA and the senor locations until convergence is achieved. Flanagan and Bell [5] applied both (PROS) [3] and Weiss and Friedlander [4] technique to alternately estimate the signal DOA and sensor location when large sensor position errors are presence. Viberg and Swindlehurst [6] proposed the maximum a posteriori noise subspace fitting (MAP-NSF) method which estimates all parameters simultaneously by using a Bayesian method. However, only a few of these methods were developed to handle a source localization using TDOA in the presence of receiver position uncertainty.

Ho *et al*. [7] recently proposed a closed-form solution for source localization using TDOA with erroneous receiver positions. The method estimates only the source location and is not able to provide receiver location estimates that would be useful in calibration and subsequent estimation task. In this paper, a method that estimates at alternate the emitter location and receiver positions in the presence of random receiver position error is proposed. Both the source and receiver positions estimates are able to reach the CRLB for Gaussian noise if the receiver position errors are small.

The outline of this paper is as follows. Section 2 is the derivation of the proposed solution. Section 3 shows the simulation results. Finally, Section 4 is the conclusion.

2. THE PROPOSED SOLUTION

The proposed method consists of three steps as shown in Figure 1. The first step is to estimate the emitter location assuming the receiver positions are exact although in fact they have noise. In the second step, the estimated source location is used to reduce the noise in the receiver positions through estimation process. In the last step, the emitter location is estimated again using the improved receiver positions from the second step. The emitter location estimate will be more precise due to more accurate receiver positions. Step two and three can be repeated several times to obtain an even better source location. The three steps are described below separately.

2.1. Step 1: Source localization with receiver location errors

In the first step, we ignore the receiver position errors and assume that the receiver positions are correct. Then we apply the method from Chan and Ho [8] to estimate the source location. The Chan and Ho method employs a nuisance variable that allows emitter location to be solved efficiently, and utilizes the computed nuisance variable value to improve the accuracy of the emitter location. For the purpose of completeness and the derivations in step 2 process, we shall briefly summarize the Chan and Ho method below.

Consider a scenario where an array of M sensors is used to determine the unknown emitter source location $\mathbf{u} = [x, y, z]^T$. Let $\mathbf{s}_i^o = [x_i^o, y_i^o, z_i^o]^T$ be the true receiver positions. The true distance between the source and the i^{th} sensor is

$$r_i^o = |\,\mathbf{u} - \mathbf{s}_i^o\,| = \sqrt{(x - x_i^o)^2 + (y - y_i^o)^2 + (z - z_i^o)^2}. \quad (1)$$

The TDOA of a signal received by the sensor pair i and 1 is t_{i1}, and the signal propagation speed is defined by c. Then the set of TDOA measurement equations are:

$$r_{i1}^o = ct_{i1} = r_i^o - r_1^o. \quad (2)$$

Let $\boldsymbol{\theta}_{1,e}^{(1)} = [x,\, y,\, z, r_1]^T$ be the unknown vector. Expressing (2) as $r_{i1}^o + r_1^o = r_i^o$, squaring both sides and substituting (1), it can be derived that the WLS solution of $\boldsymbol{\theta}_{1,e}^{(1)}$ is [8]

$$\boldsymbol{\theta}_{1,e}^{(1)} = (\mathbf{G}_{1,e}^{(1)T}\mathbf{W}_{1,e}^{(1)}\mathbf{G}_{1,e}^{(1)})^{-1}\mathbf{G}_{1,e}^{(1)T}\mathbf{W}_{1,e}^{(1)}\mathbf{h}_{1,e}^{(1)}, \quad (3)$$

where

$$\mathbf{h}_{1,e}^{(1)} = \begin{bmatrix} r_{21}^2 - l_{21} \\ \vdots \\ r_{M1}^2 - l_{M1} \end{bmatrix}, \mathbf{G}_{1,e}^{(1)} = -2\begin{bmatrix} (\mathbf{s}_2 - \mathbf{s}_1)^T & r_{21} \\ \vdots & \vdots \\ (\mathbf{s}_M - \mathbf{s}_1)^T & r_{M1} \end{bmatrix},$$

and $l_{i1} = \mathbf{s}_i^T \mathbf{s}_i - \mathbf{s}_1^T \mathbf{s}_1$, for $i = 2, ..., M$. The matrix $\mathbf{W}_{1,e}^{(1)}$ is the weighting matrix defined as

$$\mathbf{W}_{1,e}^{(1)} = \mathbf{B}_{1,e}^{(1)-T} \mathbf{Q}_t^{-1} \mathbf{B}_{1,e}^{(1)-1}, \tag{4}$$

where in (3), the true receiver positions $\mathbf{s}_i^o$ are replaced by the noisy value, $\mathbf{s}_i$. The matrix $\mathbf{B}_{1,e}^{(1)}$ is $2\,diag\{r_2, r_3, \cdots, r_M\}$ and the matrix $\mathbf{Q}_t$ is a known covariance matrix of c^2 times TDOA noise. The covariance matrix of $\boldsymbol{\theta}_{1,e}^{(1)}$ is given by [8]

$$cov(\boldsymbol{\theta}_{1,e}^{(1)}) = (\mathbf{G}_{1,e}^{(1)T} \mathbf{W}_{1,e}^{(1)} \mathbf{G}_{1,e}^{(1)})^{-1}. \tag{5}$$

Next, the estimated r_1 from $\boldsymbol{\theta}_{1,e}^{(1)}$ is used to improve the accuracy of emitter location. Let $\boldsymbol{\theta}_{2,e}^{(1)} = [(x-x_1)^2, (y-y_1)^2, (z-z_1)^2]^T$. By using another LS minimization based on (1) with i =1, we have [8]

$$\boldsymbol{\theta}_{2,e}^{(1)} = (\mathbf{G}_{2,e}^{(1)T} \mathbf{W}_{2,e}^{(1)} \mathbf{G}_{2,e}^{(1)})^{-1} \mathbf{G}_{2,e}^{(1)T} \mathbf{W}_{2,e}^{(1)} \mathbf{h}_{2,e}^{(1)}, \tag{6}$$

where

$$\mathbf{h}_{2,e}^{(1)} = \begin{bmatrix} \left(\theta_{1,e}^{(1)}(1) - x_1\right)^2 \\ \left(\theta_{1,e}^{(1)}(2) - y_1\right)^2 \\ \left(\theta_{1,e}^{(1)}(3) - z_1\right)^2 \\ \theta_{1,e}^{(1)}(4)^2 \end{bmatrix}, \mathbf{G}_{2,e}^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

and $\mathbf{W}_{2,e}^{(1)}$ is the weighting matrix defined as

$$\mathbf{W}_{2,e}^{(1)} = \mathbf{B}_{2,e}^{(1)-T} cov(\boldsymbol{\theta}_{1,e}^{(1)}) \mathbf{B}_{2,e}^{(1)-1}, \tag{7}$$

where $\mathbf{B}_{2,e}^{(1)} = 2diag\{(x-x_1), (y-y_1), (z-z_1), r_1\}$ and $cov(\boldsymbol{\theta}_{1,e}^{(1)})$ is defined in (5). Hence, the position estimate $\mathbf{u}^{(1)} = [x, y, z]^T$ is

$$\mathbf{u}^{(1)} = \mathbf{P} \left[\begin{array}{ccc} \sqrt{\theta_{2,e}^{(1)}(1)} & \sqrt{\theta_{2,e}^{(1)}(2)} & \sqrt{\theta_{2,e}^{(1)}(3)} \end{array} \right]^T + \mathbf{s}_1, \tag{8}$$

where $\mathbf{P} = diag\{sgn(\theta_{1,e}^{(1)}(1) - x_1), sgn(\theta_{1,e}^{(1)}(2) - y_1), sgn(\theta_{1,e}^{(1)}(3) - z_1)\}$, and is used to remove the sign ambiguity of the square roots in (8). Additional details of this method can be found in [8].

2.2. Step 2: Estimation of receiver locations

The receiver locations are estimated based on the knowledge of the estimated emitter position from Section 2.1. The solution method employs nuisance variables that allow receiver locations to be solved efficiently, and improves the estimated receiver position accuracy using the estimated values of the nuisance parameters.

First, we assume the source position $\mathbf{u}^{(1)}$ is noise free, where $\mathbf{u}^{(1)}$ is from Section 2.1,

$$\mathbf{u}^{(1)} = [x^{(1)}, y^{(1)}, z^{(1)}]^T. \tag{9}$$

The available receiver positions are noisy and are represented by

$$\mathbf{s}_i = \mathbf{s}_i^o + \Delta \mathbf{s}_i, \tag{10}$$

where $\Delta \mathbf{s}_i$ is the receiver position error and is assumed to be a zero mean random vector with a certain density function.

Upon rewriting the TDOA measurement equation (2) as $r_{i1}^o + r_1^o = r_i^o$, squaring both sides and substituting the square of (1) at $i = 1$, the TDOA measurement equation becomes

$$r_{i1}^{o2} + 2 r_{i1}^o r_1 = \mathbf{s}_i^{oT} \mathbf{s}_i^o - \mathbf{s}_1^{oT} \mathbf{s}_1^o - 2(\mathbf{s}_i^o - \mathbf{s}_1^o)^T \mathbf{u}. \tag{11}$$

By representing r_{i1}^o in terms of $r_{i1} - c\Delta t_{i1}$, for $i = 2, 3, ..., M$ and r_1^o in terms of $r_1^{(1)} - \Delta r_1^{(1)}$ and then rearranging (11),it then becomes

$$2 r_i \Delta c t_{i1} + 2 r_{i1} \Delta r_1^{(1)} = r_{i1}^2 + 2 r_{i1} r_1^{(1)} + 2\mathbf{u}^{(1)T} \mathbf{s}_i^o - 2\mathbf{u}^{(1)T} \mathbf{s}_1^o - l_{i1}^o, \tag{12}$$

where

$$l_{i1}^o = \mathbf{s}_i^{oT} \mathbf{s}_i^o - \mathbf{s}_1^{oT} \mathbf{s}_1^o, \tag{13}$$

and the second order error terms are ignored.

It can be seen that (12) is a set of linear equations with respect to $\mathbf{s}_i^o$ and l_{i1}^o. Thus, the nonlinear equations can be transformed into linear equations by considering $\mathbf{s}_i^o$ and l_{i1}^o as independent variables. Then, they can be solved by the LS method. Next, l_{i1}^o will be eliminated through the use of another LS minimization.

Let $\boldsymbol{\theta}_{1,p}^{(1)} = [l_{21}^o, ..., l_{M1}^o, \mathbf{s}_1^{oT}, ..., \mathbf{s}_M^{oT}]^T$ be the unknown vector. From (10) and (12), we have an error vector

$$\boldsymbol{\epsilon}_{1,p}^{(1)} = \mathbf{h}_{1,p}^{(1)} - \mathbf{G}_{1,p}^{(1)} \boldsymbol{\theta}_{1,p}^{(1)}, \tag{14}$$

where

$$\mathbf{h}_{1,p}^{(1)} = \begin{bmatrix} r_{21}^2 + 2 r_{21} r_1^{(1)} \\ \vdots \\ r_{M1}^2 + 2 r_{M1} r_1^{(1)} \\ \mathbf{s}_1 \\ \vdots \\ \mathbf{s}_M \end{bmatrix},$$

$$\mathbf{G}_{1,p}^{(1)} = \begin{bmatrix} \mathbf{I}_{(M-1,M-1)} & \begin{bmatrix} 2\mathbf{u}^{(1)T} & -2\mathbf{u}^{(1)T} & & \mathbf{0}^T \\ \vdots & & \ddots & \\ 2\mathbf{u}^{(1)T} & \mathbf{0}^T & & -2\mathbf{u}^{(1)T} \end{bmatrix} \\ \mathbf{O}_{(3M,M-1)} & \mathbf{I}_{(3M,3M)} \end{bmatrix},$$

where $\mathbf{0}$ is a 3x1 column vector of zero and $\mathbf{I}$ is a identity matrix, and $\mathbf{O}$ is a zeros matrix. Since $\mathbf{G}_{1,p}^{(1)}$ is a square matrix of size 4M-1, minimizing the second norm of $\boldsymbol{\epsilon}_{1,p}^{(1)}$ yields

$$\boldsymbol{\theta}_{1,p}^{(1)} = \mathbf{G}_{1,p}^{(1)-1} \mathbf{h}_{1,p}^{(1)}. \tag{15}$$

The error is composed of two parts, $\boldsymbol{\epsilon}_{1,p}^{(1)} = [\boldsymbol{\epsilon}_{1,p,a}^{(1)T}, \boldsymbol{\epsilon}_{1,p,b}^{(1)T}]^T$. The first part is from left hand side of (12). The second part is from the receiver position measurement (10). Thus, we have

$$\boldsymbol{\epsilon}_{1,p,a}^{(1)} = c\mathbf{B}_{1,p}^{(1)} [\Delta \mathbf{t}] + \mathbf{D}_{1,p}^{(1)} \Delta r_1^{(1)}, \tag{16}$$

and

$$\boldsymbol{\epsilon}_{1,p,b}^{(1)} = \begin{bmatrix} \Delta \mathbf{s}_1^T & \dots & \Delta \mathbf{s}_M^T \end{bmatrix}^T, \tag{17}$$

where $\mathbf{B}_{1,p}^{(1)} = 2diag\{r_2, ..., r_M\}$, $\mathbf{D}_{1,p}^{(1)} = 2[r_{21}, \dots, r_{M1}]^T$, and $\Delta r_1^{(1)}$ is the error of the range from the reference receiver to the estimated emitter from Section 2.1. By using Taylor series to expand (1) at the true values with $i = 1$, we obtain

$$\Delta r_1^{(1)} = \frac{-1}{r_1^{(1)}} \left(\mathbf{u}^{(1)} - \mathbf{s}_1\right)^T \Delta \mathbf{s}_1,$$

where the second and higher order error terms are ignored. Hence $\boldsymbol{\epsilon}_{1,p,a}^{(1)}$ can be rewritten as

$$\boldsymbol{\epsilon}_{1,p,a}^{(1)} = \mathbf{C}_{1,p,a}^{(1)} \Delta \boldsymbol{\Psi}, \tag{18}$$

where

$$\mathbf{C}_{1,p,a}^{(1)} = \left[\overbrace{2\mathbf{B}_{1,p}^{(1)}}^{M-1} \quad \overbrace{\frac{-2}{r_1^{(1)}} \begin{bmatrix} r_{21} \\ \vdots \\ r_{M1} \end{bmatrix} \left(\mathbf{u}^{(1)} - \mathbf{s}_1\right)^T}^{3} \quad \overbrace{\mathbf{0} \cdots \mathbf{0}}^{3(M-1)} \right],$$

and $\Delta \boldsymbol{\Psi} = [c\Delta t_{21}, ..., c\Delta t_{M1}, \Delta \mathbf{s}_1^T, ..., \Delta \mathbf{s}_M^T]^T$ is the vector that contains the TDOA measurement noise and receiver position noise, and $\mathbf{0}$ is a Mx1 column vector of zero. Thus

$$\boldsymbol{\epsilon}_{1,p}^{(1)} = \begin{bmatrix} \boldsymbol{\epsilon}_{1,p,a}^{(1)} \\ \boldsymbol{\epsilon}_{1,p,b}^{(1)} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{1,p,a}^{(1)} \\ \cdots\cdots \\ \mathbf{O} \quad \mathbf{I} \end{bmatrix} \Delta \boldsymbol{\Psi} = \mathbf{C}_{1,p}^{(1)} \Delta \boldsymbol{\Psi} \tag{19}$$

where $\mathbf{O}$ is a (3M)x(M-1) zero matrix and $\mathbf{I}$ is a 3M identity matrix. By substituting $\mathbf{h}_{1,p}^{(1)} = \mathbf{h}_{1,p}^{(1)o} + \Delta\mathbf{h}_{1,p}^{(1)}$, $\mathbf{G}_{1,p}^{(1)} = \mathbf{G}_{1,p}^{(1)o} + \Delta\mathbf{G}_{1,p}^{(1)}$, $\boldsymbol{\theta}_{1,p}^{(1)} = \boldsymbol{\theta}_{1,p}^{(1)o} + \Delta\boldsymbol{\theta}_{1,p}^{(1)}$ and ignoring the second order error terms, (15) can be reduced to

$$\Delta\boldsymbol{\theta}_{1,p} = \mathbf{G}_{1,p}^{(1)-1}\boldsymbol{\epsilon}_{1,p}^{(1)}. \tag{20}$$

From the weighted LS theory, the estimate of $\boldsymbol{\theta}_{1,p}^{(1)}$ has a covariance matrix given by

$$cov(\boldsymbol{\theta}_{1,p}^{(1)}) = \mathbf{G}_{1,p}^{(1)-1}\mathbf{C}_{1,p}^{(1)}\mathbf{Q}\mathbf{C}_{1,p}^{(1)T}\mathbf{G}_{1,p}^{(1)-T}, \tag{21}$$

where

$$\mathbf{Q} = E[\Delta\boldsymbol{\Psi}\Delta\boldsymbol{\Psi}^T] = \begin{bmatrix}\mathbf{Q}_t & \mathbf{O}^T \\ \mathbf{O} & \mathbf{Q}_p\end{bmatrix}, \tag{22}$$

and $\mathbf{Q}_t$ and $\mathbf{Q}_p$ are the known covariance matrices of c^2 times TDOA noise and receiver position noise, respectively.

Because of the relationship between $\mathbf{s}_i$ and l_{i1} as shown in (13), the estimated values of l_{i1} can be used to improve the receiver position estimates by forming

$$\boldsymbol{\epsilon}_{2,p}^{(1)} = \mathbf{h}_{2,p}^{(1)} - \mathbf{G}_{2,p}^{(1)}\boldsymbol{\theta}_{2,p}^{(1)}, \tag{23}$$

where

$$\mathbf{h}_{2,p}^{(1)} = \begin{bmatrix}\theta_{1,p}^{(1)}(1) \\ \vdots \\ \theta_{1,p}^{(1)}(M-1) \\ \theta_{1,p}^{(1)}(M)^2 \\ \vdots \\ \theta_{1,p}^{(1)}(4M-1)^2\end{bmatrix}, \mathbf{G}_{2,p}^{(1)} = \begin{bmatrix}\mathbf{1}^T & -\mathbf{1}^T & & \mathbf{0}^T \\ & & \ddots & \\ \mathbf{1}^T & \mathbf{0}^T & & -\mathbf{1}^T \\ \cdots & \cdots & \cdots & \cdots \\ & \mathbf{I}_{(3M,3M)} & & \end{bmatrix},$$

$$\boldsymbol{\theta}_{2,p}^{(1)} = \begin{bmatrix} (\mathbf{s}_1 \odot \mathbf{s}_1)^T & \cdots & (\mathbf{s}_M \odot \mathbf{s}_M)^T \end{bmatrix}^T, \tag{24}$$

$\mathbf{0}$ is a 3x1 vector of zeros and $\odot$ represents the Schur product.

Minimizing the weighted second norm of $\boldsymbol{\epsilon}_{2,p}^{(1)}$ yields

$$\boldsymbol{\theta}_{2,p}^{(1)} = (\mathbf{G}_{2,p}^{(1)T}\mathbf{W}_{2,p}^{(1)}\mathbf{G}_{2,p}^{(1)})^{-1}\mathbf{G}_{2,p}^{(1)T}\mathbf{W}_{2,p}^{(1)}\mathbf{h}_{2,p}^{(1)}, \tag{25}$$

where $\mathbf{W}_{2,p}^{(1)}$ is the weighting matrix given by $E[\boldsymbol{\epsilon}_{2,p}^{(1)}\boldsymbol{\epsilon}_{2,p}^{(1)T}]^{-1}|_{\boldsymbol{\theta}_{2,p}=\boldsymbol{\theta}_{2,p}^o}$ and $\boldsymbol{\theta}_{2,p}^o$ is the true solution of $\boldsymbol{\theta}_{2,p}$. The error vector is resulted from the difference between the results from the first stage, $\boldsymbol{\theta}_{2,p}^{(1)}$, and the true receiver position,

$$\boldsymbol{\epsilon}_{2,p}^{(1)} = \mathbf{B}_{2,p}^{(1)}\Delta\boldsymbol{\theta}_{1,p}^{(1)}, \tag{26}$$

where

$$\mathbf{B}_{2,p}^{(1)} = \begin{bmatrix}\mathbf{I}_{(M-1,M-1)} & \mathbf{O}_{(M-1,3M)} \\ \mathbf{O}_{(3M,M-1)} & \begin{bmatrix} 2\theta_{1,p}^{(1)}(M) & & 0 \\ & \ddots & \\ 0 & & 2\theta_{1,p}^{(1)}(4M-1)\end{bmatrix}\end{bmatrix}.$$

From (19), (20) and (26), $\boldsymbol{\epsilon}_{2,p}^{(1)}$ becomes

$$\boldsymbol{\epsilon}_{2,p}^{(1)} = \mathbf{C}_{2,p}^{(1)}\Delta\boldsymbol{\Psi}, \tag{27}$$

where $\mathbf{C}_{2,p}^{(1)} = \mathbf{B}_{2,p}^{(1)}\mathbf{G}_{1,p}^{(1)-1}\mathbf{C}_{1,p}^{(1)}\Delta\boldsymbol{\Psi}$. The weighting matrix $\mathbf{W}_{2,p}^{(1)}$ is then expressed as

$$\mathbf{W}_{2,p}^{(1)} = \left(\mathbf{C}_{2,p}^{(1)}\mathbf{Q}\mathbf{C}_{2,p}^{(1)T}\right)^{-1}. \tag{28}$$

The position estimate $\mathbf{s}^{(1)} = [\mathbf{s}_1^T, ..., \mathbf{s}_M^T]^T$ is given by the square root of $\boldsymbol{\theta}_{2,p}^{(1)}$

$$\mathbf{s}^{(1)} = \mathbf{P}\begin{bmatrix} \sqrt{\boldsymbol{\theta}_{2,e}^{(1)}(M)} & \cdots & \sqrt{\boldsymbol{\theta}_{2,e}^{(1)}(4M-1)} \end{bmatrix}^T, \tag{29}$$

where $\mathbf{P} = diag\{sgn(\theta_{1,e}^{(1)}(M)), \ldots, sgn(\theta_{1,e}^{(1)}(4M-1))\}$, and is used to remove the sign ambiguity of the square root operations in (29). Error in the location estimate can be found by squaring and taking differential of (29) as,

$$\Delta\mathbf{s}^{(1)} = \mathbf{B}_{3,p}^{(1)-1}\Delta\boldsymbol{\theta}_{2,p}^{(1)} = \mathbf{B}_{3,p}^{(1)-1}\mathbf{G}_{2,p}^{(1)\dagger}\mathbf{C}_{2,p}^{(1)}\Delta\boldsymbol{\Psi}_{2,p}^{(1)}, \tag{30}$$

where $\mathbf{G}_{2,p}^{(1)\dagger} = (\mathbf{G}_{2,p}^{(1)T}\mathbf{W}_{2,p}^{(1)}\mathbf{G}_{2,p}^{(1)})^{-1}\mathbf{G}_{2,p}^{(1)T}\mathbf{W}_{2,p}^{(1)}$ and $\mathbf{B}_{3,p}^{(1)} = 2diag\{\mathbf{s}_1^o, ..., \mathbf{s}_M^o\}$. Hence, the position covariance matrix is

$$cov(\mathbf{s}^{(1)}) = \mathbf{B}_{3,p}^{(1)-1}\mathbf{G}_{2,p}^{(1)\dagger}\mathbf{C}_{2,p}^{(1)}\mathbf{Q}\mathbf{C}_{2,p}^{(1)T}\mathbf{G}_{2,p}^{(1)\dagger T}\mathbf{B}_{3,p}^{(1)-T}. \tag{31}$$

2.3. Step 3: Re-estimation of emitter location

In this step, we estimate the emitter position again using the receiver position estimates from Section 2.2. By assuming that the estimated receiver position from Section 2.2 is correct, then the Chan and Ho [8] method is used to re-estimate the emitter location. Therefore, Equations (3), (6), (8) are used to find the location of the emitter.

CRLB often used as the benchmark for the performance of an unbiased estimator. The details of the CRLB that considers the receiver positions uncertainty can be found in [7]. We briefly summarize the CRLB below. Let $\mathbf{v} = \left[\mathbf{r}^T, \mathbf{s}^T\right]^T$ be the vector that contains the TDOA measurements $[r_{21}, r_{31}, \cdots, r_{M1}]^T$ and the noisy sensor positions $\left[\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_M^T\right]^T$. If $p(\mathbf{v}|\boldsymbol{\theta})$ is the probability density function of $\mathbf{v}$ parameterized on the unknown vector $\boldsymbol{\theta}$, where $\boldsymbol{\theta} = \left[\mathbf{u}^T, \mathbf{s}^T\right]^T$, then the CRLB is given by [7]

$$CRLB = \left\{-E\left[\begin{pmatrix}\frac{\partial^2 \ln p(\mathbf{v}|\boldsymbol{\theta})}{\partial\mathbf{u}\partial\mathbf{u}^T}\end{pmatrix} \quad \begin{pmatrix}\frac{\partial^2 \ln p(\mathbf{v}|\boldsymbol{\theta})}{\partial\mathbf{u}\partial\mathbf{s}^T}\end{pmatrix} \\ \begin{pmatrix}\frac{\partial^2 \ln p(\mathbf{v}|\boldsymbol{\theta})}{\partial\mathbf{s}\partial\mathbf{u}^T}\end{pmatrix} \quad \begin{pmatrix}\frac{\partial^2 \ln p(\mathbf{v}|\boldsymbol{\theta})}{\partial\mathbf{s}\partial\mathbf{s}^T}\end{pmatrix}\right]\right\}^{-1}. \tag{32}$$

3. SIMULATIONS

This section provides the comparison of the performance of the proposed solution method with that of the Chan and Ho method [8] which does not take the receiver position errors into account and the CRLB [7]. The positions of the receivers are {(0,0,600),(400,0,0),(0,500,0),(350,200,100),(-100,-100,-100),(120,140,150),(60,70,300)}. TDOA measurement vector is obtained by adding Gaussian noise vector with the correlation matrix $\mathbf{R}$ to the true values, where $\mathbf{R}$ is set to σ_t^2 in the diagonal elements and $0.5\sigma_t^2$ otherwise [8]. Hence, $\mathbf{Q}_t = c^2\mathbf{R}$. The receiver position noise is independent at different coordinates and receivers and is set to zero mean Gaussian white noise. The receiver position noise and TDOA noise are independent. The emitter is located at (2000,1900,1700). In the simulation, the TDOA noise power, σ_t^2, is fixed at $0.001/c^2$ and the results are generated by varying the receiver position noise power, σ_p^2. Each receiver has different noise powers and they are $\sigma_p^2[1, 1, 1, 1, 75, 100, 25]$.

Figure 2 compares the proposed method and the Chan and Ho method. It can be seen that the proposed method performs better than the Chan and Ho method even for only one iteration. At receiver position noise power 0.0001, our method gives approximately 1 dB better than the Chan and Ho method in the first iteration and 3 dB after 9 iterations and give the same results after 21 iterations. In another word, after 9 or 21 iterations, the proposed method is 1.99 times better than the Chan and Ho method. The proposed method yields better results when the receiver position noise is less than TDOA noise. From extensive simulation results, we observed that, in general, the proposed method performs best when the receiver position noise and TDOA noise are comparable.

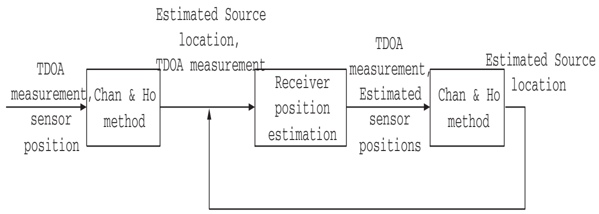

Fig. 1. Diagram of the proposed method.

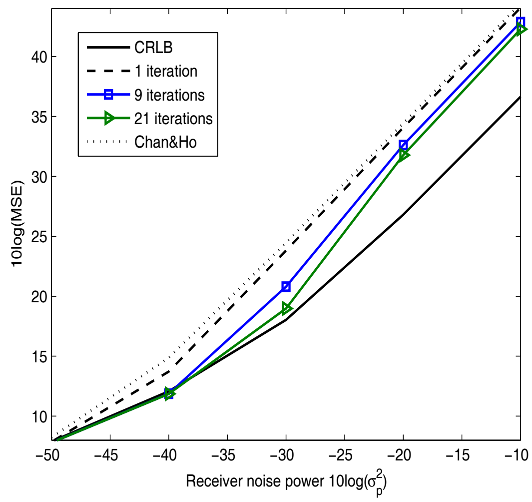

Fig. 2. Accuracy of emitter location estimate of the proposed method.

Figure 3 depicts the comparison of the average MSE of the receivers between the proposed method and the CRLB. The proposed method achieves the CRLB bound when the receiver position noise is moderate.

The proposed method has higher computational complexity than the existing method [7]. The cost is about twice time higher than that in [7] in each iteration. It provides, however, estimated receiver location which can be used for subsequence estimation task. Although the proposed algorithm is iterative, it does not require initial solution guesses and convergence is insensitive to the noise power in the receivers.

4. CONCLUSIONS

This paper developed an iterative solution for estimating at alternate the emitter location and the receiver positions based on TDOA measurements, where the receivers have random position errors. The solution method employs nuisance variables that allow both emitter and receiver locations to be solved efficiently. Additional processing is then applied by using the nuisance variable values to improve the estimation accuracy of emitter and receiver positions. The method requires only several WLS minimizations and no divergence behavior has been observed. Simulation shows that the estimation accuracy of the proposed source location method approaches the CRLB for Gaussian noise in small error region, and is better than the Chan and Ho [8] method that does not take the receiver position noise into account.

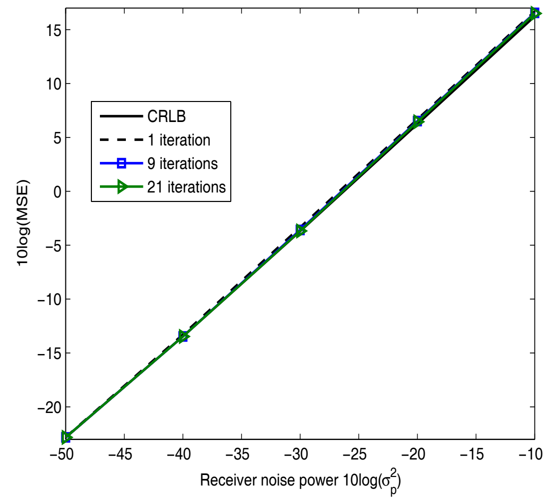

Fig. 3. Accuracy of the receiver position estimates of the proposed method.

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