MULTITARGET TRACKING USING A NEW SOFT-GATING APPROACH AND SEQUENTIAL MONTE CARLO METHODS

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ABSTRACT

In this paper we propose an extension of the soft-gating approach for measurement-to-target assignment for multitarget tracking. Given the latest observation and a set of multitarget particles, the proposed method combines efficient m-best 2-D data assignment and sampling methods to compute a feasible measurement-to-target assignment with an associated probability for each particle. The particles containing the multitarget states and the association vectors can then be used to recursively estimate the posterior distribution of the targets using sequential Monte Carlo methods. Computer simulations demonstrate the robustness and effectiveness of the proposed method for data association and multitarget tracking.

1. INTRODUCTION

Multitarget tracking (MTT) [3, 4, 5] is an essential requirement for surveillance systems using one or multiple sensors to monitor the environment. Typical applications can be found in navigation, air trafic control, and military surveillance systems. Classical approaches, such as the Joint Probabilistic Data Association Filter (JPDAF) and its numerous derivatives [4, 6], have demonstrated their ability in tracking multiple targets by considering all or part of the combinations of measurement-to-target assignments. However these methods generally suffer from two drawbacks. Firstly, as the number of measurements increases for the multitarget scenario, the computational intensity for data association becomes formidable. To alleviate the computational load, one may use a gating approach [4, 6] to eliminate unlikely measurement-to-target pairings. Doing so, however, may lead to incorrect assignment especially when targets are closely spaced. Secondly, as Kalman Filter (KF) type algorithms, including the extended Kalman Filter (EKF), [4, 5, 6] are usually used for target state estimation, these classical approaches are subject to failure in target tracking if the data models are highly nonlinear and non-Gaussian.

In this paper we propose a new measurement-to-target assignment approach that combines the m-best 2-D data assignment algorithm [4, 6] and sampling methods, such as the soft-gating approach [1, 2] for data association, given the latest observation and a set of multitarget state particles. This method takes advantage of the efficiency of the m-best 2-D data assignment method, and is compatible with a sequential Monte Carlo (SMC) framework [7]. These particles, comprising the multitarget states and the association vectors, can be used to recursively estimate the posterior distribution function of the targets, using SMC [8, 9, 10] which are able to perform well in the situations where the KF type algorithms fail.

This paper is organised as follows. Section 2 presents a general state-space model for the MTT problem, and describes the derivation of the necessary probability distributions. Section 3 presents the proposed data association method. Simulation results are given in Section 4, followed by the conclusions in Section 5.

2. DATA MODEL

Let $\boldsymbol{x}_t$ be a combined target state vector for K targets, i.e., $\boldsymbol{x}_t = [\boldsymbol{x}_{1,t}^T, ..., \boldsymbol{x}_{k,t}^T, ..., \boldsymbol{x}_{K,t}^T]^T$, where each target independently [11] follows a dynamic model given by

$$\boldsymbol{x}_{k,t} = \boldsymbol{f}_k(\boldsymbol{x}_{k,t-1}, \boldsymbol{v}_{k,t}),\ k \in \{1, ..., K\}, \tag{1}$$

where $\boldsymbol{x}_{k,t}$ denotes the state vector of the kth target, and $\boldsymbol{f}_k(\cdot)$, which models the motion of the target, can be a linear or nonlinear function. The noise $\boldsymbol{v}_{k,t}$ is a zero-mean random variable with a fixed and known covariance matrix $\boldsymbol{\Sigma}_v$.

In this paper, a single sensor is employed, but the entire framework can be readily extended for multiple sensors. Let $\boldsymbol{y}_t = [\boldsymbol{y}_{1,t}^T, ..., \boldsymbol{y}_{m,t}^T, ..., \boldsymbol{y}_{M_t,t}^T]^T$ be an observation received by the sensor with M_t measurements. The mth measurement may originate from a true target or clutter. To distinguish between the measurements due to targets and clutter, an association vector $\boldsymbol{\alpha}_t = [\alpha_{1,t}, ..., \alpha_{m,t}, ..., \alpha_{M_t,t}]^T$ is defined. It is assumed that the measurement-to-target assignment is always on a one-to-one basis. That is, at a given time t a measurement (target) can always be assigned to only one target (measurement). If $\boldsymbol{y}_{m,t}$ is associated with the kth target, $\alpha_{m,t} = k$, then $\boldsymbol{y}_{m,t}$ can be expressed as

$$\boldsymbol{y}_{m,t} = \boldsymbol{g}_m(\boldsymbol{x}_{k,t}, \boldsymbol{w}_{m,t}), \tag{2}$$

where $\boldsymbol{g}_m(\cdot)$, the mth observation model, may be a linear or nonlinear function, $\boldsymbol{w}_{m,t}$, mutually independent of $\boldsymbol{v}_{k,t}$, is also a zero-mean random variable with a fixed and known covariance matrix $\boldsymbol{\Sigma}_w$. While a true target may exist, it may not be detected when, for example, the probability of target detection P_D is low, and its measurement may not be received by the sensor, leading to data loss. On the contrary, $\alpha_{m,t}$ is set to zero if $\boldsymbol{y}_{m,t}$ originates from clutter, whose distribution is assumed uniform over the surveillance region [4, 6]. From this point onward, it is assumed that all targets share the same evolution model, and that all measurements share the same observation model.

Following the framework in [1, 2, 12], we adopt the assumption that $\boldsymbol{\alpha}_t$ is a stochastic variable that is dependent on the current multitarget state and observations. We may jointly estimate $\{\boldsymbol{x}_t, \boldsymbol{\alpha}_t\}$ by sequentially estimating the posterior distribution $p(\boldsymbol{x}_t, \boldsymbol{\alpha}_t|\boldsymbol{y}_{0:t})$ using SMC methods, also known as particle filters [8, 13, 14]. We may formulate a SMC framework in terms of prediction and update equations for N particles [1, 2, 12] as follows

$$\begin{aligned}(\boldsymbol{x}_t^{(i)}, \boldsymbol{\alpha}_t^{(i)}) &\sim q(\boldsymbol{x}_t, \boldsymbol{\alpha}_t|\boldsymbol{x}_{t-1}^{(i)}, \boldsymbol{\alpha}_{t-1}^{(i)}, \boldsymbol{y}_t), \\ &= p(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{t-1}^{(i)})\, q(\boldsymbol{\alpha}_t|\boldsymbol{x}_t^{(i)}, \boldsymbol{y}_t),\end{aligned} \tag{3}$$

$$w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p(\boldsymbol{y}_t|\boldsymbol{x}_t^{(i)}, \boldsymbol{\alpha}_t^{(i)}) p(\boldsymbol{\alpha}_t^{(i)}|\boldsymbol{x}_t^{(i)}) p(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{t-1}^{(i)})}{q(\boldsymbol{x}_t, \boldsymbol{\alpha}_t|\boldsymbol{x}_{t-1}^{(i)}, \boldsymbol{\alpha}_{t-1}^{(i)}, \boldsymbol{y}_t)}, \tag{4}$$

where $i \in \{1, ..., N\}$ and $\sum_{i=1}^{N} w_t^{(i)} = 1$. The terms $p(\boldsymbol{x}_t^{(i)}|\boldsymbol{x}_{t-1}^{(i)})$ and $q(\boldsymbol{\alpha}_t|\boldsymbol{x}_t^{(i)}, \boldsymbol{y}_t)$ in (3) are the combined dynamic prior in (1) for K targets and the association proposal, respectively, whereas the terms $p(\boldsymbol{y}_t|\boldsymbol{x}_t^{(i)}, \boldsymbol{\alpha}_t^{(i)})$ and $p(\boldsymbol{\alpha}_t^{(i)}|\boldsymbol{x}_t^{(i)})$ in (4) are the combined likelihood for M_t measurements in (2) and the association prior, respectively. Details about these priors can be found in [1, 2, 12].

Unlike the soft-gating approach [1, 2] that *sequentially* samples the elements of $\boldsymbol{\alpha}_t^{(i)}$, we propose to jointly sample the elements of $\boldsymbol{\alpha}_t^{(i)}$. We use the 2-D assignment algorithm discussed in Section 3 to compute the m-best associations, and then define a discrete proposal distribution over these assignments with the corresponding proposal probabilities given by the assignment algorithm, i.e., $\{\boldsymbol{\alpha}_t^{(i)}, u(\boldsymbol{\alpha}_t^{(i)})\}$. Substituting the particles $\{\boldsymbol{\alpha}_t^{(i)}, u(\boldsymbol{\alpha}_t^{(i)})\}_{i=1}^{N}$ into (4) yields

$$w_t^{(i)} \propto w_{t-1}^{(i)} \frac{p(\boldsymbol{y}_t|\boldsymbol{x}_t^{(i)}, \boldsymbol{\alpha}_t^{(i)}) p(\boldsymbol{\alpha}_t^{(i)}|\boldsymbol{x}_t^{(i)})}{u(\boldsymbol{\alpha}_t^{(i)})}. \tag{5}$$

3. M-BEST 2-D MEASUREMENT-TO-TARGET ASSIGNMENT ALGORITHM

The 2-D assignment algorithm [4, 6] is an intuitive method for solving classical assignment problems, which includes the data association problem for MTT applications, given that the assignment is always on a one-to-one basis. However, this single-scan approach to data association may not provide reliable performance, leading to track loss and improperly partitioned measurements into tracks and false alarms. Therefore, determining the m-best solutions becomes especially important, since the hard irrevocable decisions that the best-solution approaches make can be mitigated using the m-best assignment algorithm [4, 6].

The measurement-to-target assignment problem can be cast as a constrained optimisation problem [4, 6] that maximises a function $C(\cdot)$ as follows

$$\boldsymbol{a}_t^{(j)} = \arg \max_{\boldsymbol{a}_t \in \boldsymbol{\mathcal{A}}^{(j)}} \Big\{ C(\boldsymbol{\alpha}_t, \boldsymbol{x}_t, \boldsymbol{y}_t) \Big\}, \tag{6}$$

where $\boldsymbol{a}_t^{(j)}$, the jth best solution in the feasible solution space $\boldsymbol{\mathcal{A}}^{(j)}$, can be easily mapped to $\boldsymbol{\alpha}_t$[1], subject to

$$\sum_{k=0}^{K} a_{l,k}^{(j)} = 1, \quad l \in \{1, ..., M_t\}, \tag{7}$$

$$\sum_{l=1}^{M_t} a_{l,k}^{(j)} = 1, \quad k \in \{1, ..., K\}, \tag{8}$$

$$\boldsymbol{\mathcal{A}}^{(j)} \in \boldsymbol{\mathcal{A}} - \bigcup_{p=1}^{j-1} \boldsymbol{a}_t^{(p)}, \tag{9}$$

where $\boldsymbol{\mathcal{A}}$ is the space for *all* feasible assignments. The constraints in (7) and (8) ensure that the feasible solution is a one-to-one measurement-to-target assignment, and (9) ensures that the jth best solution does not overlap with the other solutions. The l, kth elements of $C(\boldsymbol{\alpha}_t, \boldsymbol{x}_t, \boldsymbol{y}_t)$ is the probability of assigning $\boldsymbol{y}_{l,t}$ to target k, given by

$$c_{l,k} = \begin{cases} 0, & \text{if } k = 0, \\ \log \left\{ \frac{P_D\, p(\boldsymbol{y}_{l,t}|\boldsymbol{x}_t, \alpha_l = k)}{p(\boldsymbol{y}_{l,t}|\alpha_l = 0)} \right\}, & \text{if } \log(\cdot) > 0, \\ -\infty, & \text{otherwise}, \end{cases} \tag{10}$$

$l \in \{1, ..., M_t\}$. Accordingly, this problem can be reformulated [4, 6], subject to these constraints as

$$\boldsymbol{a}_t^{(j)} = \arg\max \sum_{k=0}^{K} \sum_{l=l}^{M_t} c_{l,k}\ a_{l,k}^{(j)}. \tag{11}$$

[1]The mapping from $\boldsymbol{a}_t^{(j)}$ to $\boldsymbol{\alpha}_t$ is easy, i.e., if $a_{l,k}^{(j)} = 1$, then $\alpha_{l,t}^{(j)} = k$. Otherwise, $\alpha_{l,t}^{(j)} = 0$.

Parameters	Values
$\mathbf{\Sigma}_v$	$\text{diag}([5 \times 10^{-4}, 5 \times 10^{-4}])$
$\mathbf{\Sigma}_w$	$\text{diag}([0.0001, 25])$
T	300
N	500

Table 4.1. Parameters for computer simulation.

N	the best	the m-best	Soft-gating
50	58.1 %	54.5 %	61.1 %
100	20.3 %	17.8 %	35.0 %
200	8.4 %	8.0 %	10.5 %
500	2.9 %	2.7 %	5.6 %
1000	2.6 %	2.4 %	2.4 %
5000	2.6 %	2.4 %	2.4 %
10000	2.3 %	2.2 %	2.2 %

Table 4.2. Comparison of the average probability of misassignment between the best solution, the m-best solution, and the soft-gating algorithms for different values of N in 50 independent runs.

Given the set of m-best solutions[2] and their associated probabilities $\{u(\boldsymbol{\alpha}_t^{(j)})\}_{j=1}^m$, where $u(\boldsymbol{\alpha}_t^{(j)}) \propto \exp(\sum_{l,k \in \boldsymbol{a}_t^{(j)}} c_{l,k})$ and $\sum_{j=1}^m u(\boldsymbol{\alpha}_t^{(j)}) = 1$, one can form the point-mass proposal distribution function for $\boldsymbol{\alpha}_t$ from which an association vector $\boldsymbol{\alpha}_t$, with associated probability $u(\boldsymbol{\alpha}_t)$, can be sampled, i.e.,

$$\boldsymbol{\alpha}_t \sim q(\boldsymbol{\alpha}_t|\boldsymbol{x}_t, \boldsymbol{y}_t) = \sum_{j=1}^{m} u(\boldsymbol{\alpha}_t^{(j)})\delta(\boldsymbol{\alpha}_t - \boldsymbol{\alpha}_t^{(j)}). \quad (12)$$

4. COMPUTER SIMULATIONS

In this section, we evaluate the performance of the proposed MTT algorithm on a challenging synthetic tracking problem. Fig. 1 depicts $K = 3$ tracks for $T = 300$ scans, each evolving according to a near constant velocity model [11] with parameters summarised in Table 4.1. The surveillance region in this simulation is $[-2000, 2000]^2$. The observations generated comprise target and clutter measurements, where the spatial density of clutter is controlled by the expected number of clutter measurement, $\Lambda_C = 5$. In addition, the probability of target detection is set to $P_D = 0.5$. The multitarget states are initialised around their true mean with large variances. The state and observation noises in (1) and (2) are assumed to be white Gaussian and their covariance matrices are shown in Table 4.1. It can be seen

[2]The value of m can be determined by checking whether $u(\boldsymbol{\alpha}_t^{(m)})$ is significant, say 1%, when compared with the culmulative probability $\sum_{j=1}^m u(\boldsymbol{\alpha}_t^{(j)})$.

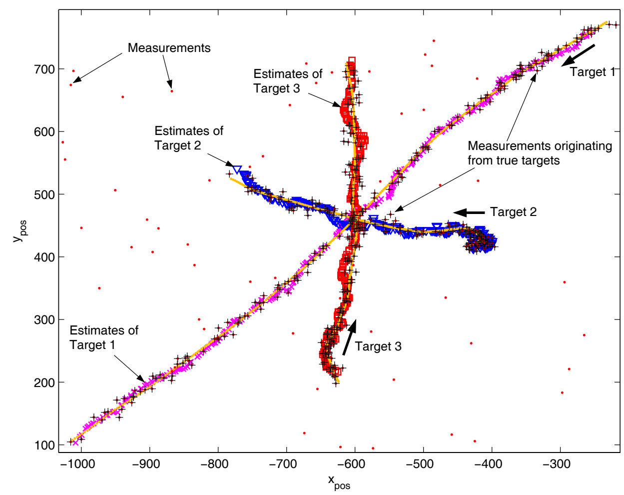

Fig. 1. A comparison between the true tracks and their estimates.

that not only are the tracks closely tracked by the algorithm, they are also unambiguously and successfully resolved by the proposed algorithm after they cross each other.

We also compared the misassignment of the best solution approach [4, 6, 12], the soft-gating algorithm [2], and the proposed method for data association, based on the same scenario shown in Fig. 1. A misassignment occurs when a measurement, be it originated from a true target or clutter, is incorrectly assigned to a detected target. For example, if a clutter measurement or a measurement originating from another target is assigned to a target, a misassignment occurs. As shown in Table 4.2 these methods have a comparable performance for large N, but when N is small the proposed method outperforms the other methods.

Finally, the performance of the proposed method and the other two approaches as a function of different numbers of particles N is compared in terms of the Root Mean Square Error (RMSE), defined as $RMSE_l = \sqrt{\frac{1}{KT}\sum_{t=1}^T ||\boldsymbol{x}_t - \hat{\boldsymbol{x}}_t^l(N)||^2}$, where $RMSE_l$ is the error for the lth independent run, and $\hat{\boldsymbol{x}}_t^l(N)$ is a posterior mean estimate of $\boldsymbol{x}_t$ for the lth run with N particles. For each value of N, a total of 20 independent runs were used with the same synthesised tracks but different observations. According to Fig. 2, the RMSE decreases as N increases, at the expense of an increased computational load.

5. CONCLUSIONS

In this paper we presented a new method for data association for multiple target tracking. Combining an efficient data assignment algorithm and sampling techniques, the proposed

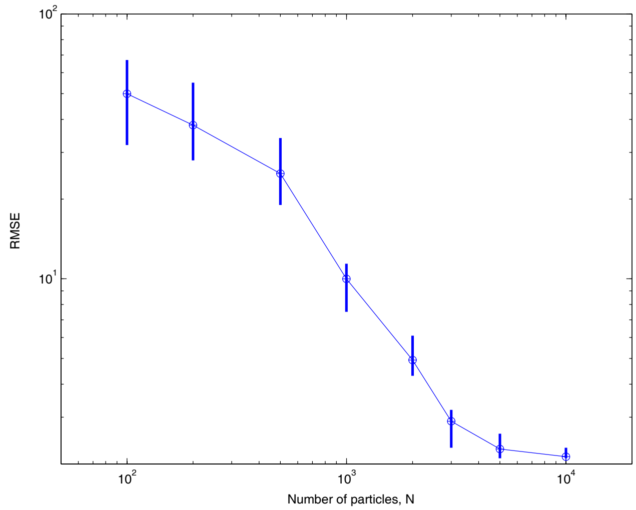

Fig. 2. RMSE evaluated for different values of N. Each vertical line on the curve represents the 1-σ error bars of the RMSE at a particular value of N.

method fits into the sequential Monte Carlo framework for recursively and jointly estimating the measurement-to-target assignment and multitarget states, given the latest observation. The computer simulations and performance evaluation showed that the proposed algorithm performed well in its intended areas, and also outperformed its predecessors.

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