TRANSMISSION DIVERSITY SMOOTHING FOR MULTI-TARGET LOCALIZATION

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ABSTRACT

A new method for target localization by radar or sonar systems based on spatially coded signal transmission is proposed. Recently, it has been shown that spatially-coded signal transmission allows obtaining virtual sensors. We show that for any array geometry, these virtual sensors are grouped into subarrays with identical structures. The transmission diversity, embedded in the spatially-coded signal model, is used to spatially smooth the signal covariance matrix in order to enable using eigenstructure-based methods for multiple coherent target localization. Unlike conventional spatial smoothing and forward-backward averaging, the proposed Transmission Diversity Smoothing (TDS) algorithm is not limited to uniform or symmetric arrays, and does not decrease the array aperture. The performance of the algorithm implemented with MUSIC is tested using simulations and compared to the spatial smoothing method. The results show that the TDS algorithm is consistent and outperforms the spatial smoothing method.

1. INTRODUCTION

Orthogonal signal transmission for radars and active sonars has recently been proposed in [1] and [2]. Space-time coding of the transmitted signal was presented and its properties was analyzed. The main advantages of this new configuration include: 1) digital beamforming of the transmitted beams, 2) extension of the array aperture by virtual sensors, resulting in narrower beams, 3) virtual spatial tapering of the extended array aperture, resulting in lower sidelobes, 4) higher angular resolution, 5) larger number of targets which can be detected and localized, and 6) lower spatial transmitted peak power density.

In many radar and sonar applications, the received signals from different directions are fully correlated. One well known reason of coherency is multipath. In active radar and sonar systems, the received echo signals from different targets are also considered as coherent. The reason is that although the phases of the echo signals vary between the different pulses or snapshots due to the Doppler effect, optimal target parameters estimation requires Doppler filtering of the received signal before targets directions estimation. The signal phases after Doppler processing are almost constant. Moreover, the amplitude of the echo signal is usually constant during transmission of several snapshots. Therefore, the echo signals from different targets are considered as coherent. This implies that eigenstructure-based methods, such as MUSIC and ESPRIT cannot be directly implemented for multi-target localization, because these methods fail in scenarios in which the received signals are fully correlated.

In order to "decorrelate" the signals in the data covariance matrix, Evans et al. [3] proposed a preprocessing technique referred to as spatial smoothing. Several authors [4] - [5] investigated this method, combined with forward-backward averaging. The drawback of this approach is the reduction of the effective array aperture length, resulting in lower resolution and accuracy. An alternative "decorrelation" method is redundancy averaging [6], [7]. In [8], it is shown that this preprocessing method induces bias in the DOA estimates.

In this paper, we show that the virtual sensors, obtained from uncorrelated signal transmission, are grouped into subarrays with identical structures. This enables spatial smoothing of the signal covariance matrix, followed by eigenstructure-based methods for multiple coherent target localization. Unlike conventional spatial smoothing, the proposed Transmission Diversity Smoothing (TDS) algorithm is not limited to uniform arrays, does not assume spatially white noise, and does not decrease the array aperture. Moreover, for an array of M elements, the number of targets which can be localized using the TDS algorithm is $M-1$, while with spatial smoothing, combined with forward-backward averaging, the maximum number of targets is $2M/3$.

2. SPATIALLY CODED SIGNAL MODEL

Consider an M-element antenna array transmitting M narrowband signals. The samples of baseband equivalent signals are denoted by the vectors $\mathbf{s}[n] = [s_1[n], \ldots, s_M[n]]^T$, $n = 1, \ldots, N$ where $\{s_m[n]\}_{n=1}^N$ denotes the transmitted signal by the mth element and n represents the time index. The correlation matrix of $\mathbf{s}[n]$ is given by

$$\mathbf{R_s} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{s}[n]\mathbf{s}^H[n] = \begin{bmatrix} 1 & \beta_{12} & \cdots & \beta_{1M} \\ \beta_{21} & 1 & \cdots & \beta_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{M1} & \beta_{M2} & \cdots & 1 \end{bmatrix}, \tag{1}$$

in which β_{ij} is the correlation coefficient between the ith and jth signals. The phases of $\{\beta_{ij}\}_{i,j=1}^M$ control the transmitted beam direction of the coherent component of the transmitted signal. In the case of orthogonal transmitted signals ($\{\beta_{ij}\}_{i\neq j} = 0$), the correlation matrix is an identity matrix: $\mathbf{R_s} = \mathbf{I}_M$, i.e. omni-directional transmission. In common radar systems, coherent signals are transmitted by the array and therefore the rank of $\mathbf{R_s}$ is equal to one.[1]

In the presence of a single target at direction θ with no multipath, the received signal at the mth element of the array located at $\mathbf{x}_m \triangleq \left[x_m^{(1)}, x_m^{(2)}\right]^T$ (see Fig. 1), is given by:

$$y_m[n] = \alpha \sum_{i=1}^{M} A_{mi}(\theta) s_i[n] + w_m[n], \quad m = 1, \ldots, M \quad n = 1, \ldots, N \tag{2}$$

where α denotes the complex amplitude of the received signal, $w_m[n]$ is the additive noise at the mth element, and $A_{mi}(\theta) = exp(-jw_c\tau_{mi}(\theta))$ describes the total array response of the signal, transmitted by the ith element and received by the mth element, where w_c is the carrier frequency. The total accumulated phase from the ith transmitting element to the mth receiving element for the far-field case can be written as

$$\omega_c\tau_{mi}(\theta) = \omega_c(\underbrace{\tau_m(\theta)}_{receive} + \underbrace{\tau_i(\theta)}_{transmit}) = \mathbf{k}^T(\theta)(\mathbf{x}_m + \mathbf{x}_i) , \tag{3}$$

where $\mathbf{k}(\theta) = \frac{2\pi}{\lambda}[\sin\theta, \cos\theta]^T$ and λ stands for the signal wavelength.

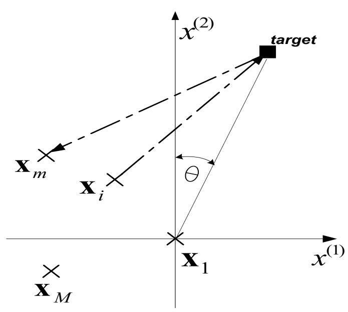

Figure 1: Array configuration

Let $a_m(\theta)$ denote the spatial response of the mth element. Then, the mith element of the array response matrix, $A_{mi}(\theta)$, can be decomposed as:

$$\begin{aligned}[\mathbf{A}(\theta)]_{mi} = A_{mi}(\theta) = exp(-jw_c(\tau_m(\theta) + \tau_i(\theta))) = \\ = a_m(\theta)a_i(\theta), \quad m, i = 1, \ldots, M .\end{aligned} \tag{4}$$

Note that the elements of $\mathbf{A}(\theta)$ depend on θ through all possible combinations of delays in transmit and receive modes. In fact, $A_{im}(\theta)$ is the array response for transmit from the ith element and receive by the mth element. Hence, the array response matrix can be written as

$$\mathbf{A}(\theta) = \mathbf{a}(\theta)\mathbf{a}^T(\theta) , \tag{5}$$

[1]The different elements transmit the same signal with phase shifts for beam steering.

where $\mathbf{a}(\theta)$ is the transmitted or the received array response vector. In matrix notation, Eq. (2) can be stated as:

$$\mathbf{y}[n] = \alpha\mathbf{A}(\theta)\mathbf{s}[n] + \mathbf{w}[n], \quad n = 1, \ldots, N , \tag{6}$$

where $\mathbf{y}[n]$, $\mathbf{s}[n]$ and $\mathbf{w}[n]$ are vectors of the received signal, the transmitted signal and the additive noise, respectively.

In the case of L targets scenario, Eq. (6) is modified to:

$$\mathbf{y}[n] = \sum_{l=1}^{L} \alpha_l \mathbf{A}(\theta_l)\mathbf{s}[n] + \mathbf{w}[n], \quad n = 1, \ldots, N . \tag{7}$$

The noise vectors $\{\mathbf{w}[n]\}_{n=1}^N$ are assumed to be independent, zero-mean, complex Gaussian with known covariance matrix $\mathbf{R_w}$. With no loss of generality, we can assume that $\mathbf{R_w} = \sigma_w^2\mathbf{I}_M$, where $\mathbf{I}_M$ is an identity matrix of size M. If this assumption is not satisfied, the model in Eq. (7) can be pre-whitened.

Our goal is to estimate the target directions $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_L]^T$ from the measurements $\{\mathbf{y}[n]\}_{n=1}^N$ in the presence of unknown complex amplitudes $\alpha_1, \ldots, \alpha_L$.

3. EQUIVALENT SPATIALLY CODED MODEL

In [1] and [2], it is shown that the sufficient statistics for estimation of $\boldsymbol{\theta}$ from the measurements $\{\mathbf{y}[n]\}_{n=1}^N$ is given by:

$$\boldsymbol{\eta}_m \triangleq \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{y}[n]s_m^*[n], \quad m = 1, \ldots, M , \tag{8}$$

which is obtained by matching the observed data to the mth signal, $\{s_m[n]\}_{n=1}^N$. The sufficient statistics matrix is defined as

$$\mathbf{E} = [\boldsymbol{\eta}_1, \ldots, \boldsymbol{\eta}_M] = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{y}[n]\mathbf{s}^H[n] . \tag{9}$$

For orthogonal signals, the mth column of the matrix $\mathbf{E}$ represents the array measurement of the signal transmitted by the mth element. It can be shown that the sufficient statistics $\{\boldsymbol{\eta}_m\}_{m=1}^M$ are dependent for non-orthogonal signals. Independent sufficient statistics, $\{\tilde{\boldsymbol{\eta}}_m\}_{m=1}^M$, can be obtained by matching the measurements to the transformed signals as follows:

$$\tilde{\boldsymbol{\eta}}_m \triangleq \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{y}[n]\tilde{s}_m^*[n], \quad m = 1, \ldots, M , \tag{10}$$

where $\tilde{s}_m[n]$ is the mth element of the transformed signal vector $\tilde{\mathbf{s}}[n]$, given by

$$\tilde{\mathbf{s}}[n] \triangleq \boldsymbol{\Lambda}^{-1/2}\mathbf{U}^H\mathbf{s}[n] , \tag{11}$$

in which $\mathbf{U}$ and $\boldsymbol{\Lambda}$ are the matrices of eigenvectors and eigenvalues of the signal correlation matrix, $\mathbf{R_s}$, respectively. In matrix notation, the independent sufficient statistics is given by

$$\tilde{\mathbf{E}} \triangleq [\tilde{\boldsymbol{\eta}}_1, \ldots, \tilde{\boldsymbol{\eta}}_M] = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathbf{y}[n]\tilde{\mathbf{s}}^H[n] = \mathbf{E}\mathbf{U}\boldsymbol{\Lambda}^{-1/2} . \tag{12}$$

The sufficient statistics are obtained by a matched filter: temporal matching the measurement vectors to different signal subspace components $\tilde{\mathbf{s}}[n]$. In fact, $\tilde{\boldsymbol{\eta}}_m$ represents the measurement of the mth component of $\tilde{\mathbf{s}}[n]$ by the array. Hence, the matching procedure results in M^2 virtual sensors instead of M actual sensors in the conventional configuration. An equivalent model for the sufficient statistics in matrix form can be obtained by substitution of (7) into (12):

$$\tilde{\mathbf{E}} = \sqrt{N}\sum_{l=1}^{L}\alpha_l \mathbf{A}(\theta_l)\mathbf{U}\mathbf{\Lambda}^{1/2} + \mathbf{V} \,, \tag{13}$$

where $\mathbf{V} \triangleq [\mathbf{v}_1, \ldots, \mathbf{v}_M] = \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\mathbf{w}[n]\tilde{\mathbf{s}}^H[n]$. It can be shown that the columns of $\mathbf{V}$ are zero-mean, i.i.d. with covariance matrix $\sigma_w^2\mathbf{I}$. By substitution of (5) into (13), the model for the mth column of the matrix $\tilde{\mathbf{E}}$ can be stated by

$$\tilde{\boldsymbol{\eta}}_m = \sqrt{N}\sum_{l=1}^{L}\mathbf{a}(\theta_l)\phi_{lm} + \mathbf{v}_m, \;\; m = 1, \ldots, M \,, \tag{14}$$

where ϕ_{lm} denotes the mth element of the row vector $\alpha_l\mathbf{a}^T(\theta_l)\mathbf{U}\mathbf{\Lambda}^{1/2}$. Obviously, the row vector $\alpha_l\mathbf{a}^T(\theta_l)\mathbf{U}\mathbf{\Lambda}^{1/2}$ carries information on the target DOA's. However, in order to obtain a model which enables smoothing of the covariance matrix, this information will be ignored. Therefore, the dependence of ϕ_{lm} on the θ_l is omitted.

4. TRANSMISSION DIVERSITY SMOOTHING

In this section, the proposed smoothing algorithm based on the model described in (14) is presented. Eq. (14) can be rewritten as:

$$\tilde{\boldsymbol{\eta}}_m = \sqrt{N}\mathbf{F}(\boldsymbol{\theta})\boldsymbol{\phi}_m + \mathbf{v}_m, \;\; m = 1, \ldots, M \tag{15}$$

where $\mathbf{F}(\boldsymbol{\theta}) \triangleq [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)]$ and $\boldsymbol{\phi}_m \triangleq [\phi_{1m}, \ldots, \phi_{Lm}]^T$. As stated in Section 3, the equivalent model for the sufficient statistics consists of M^2 virtual sensors. We divided these sensors into M sub-arrays of size M with identical structure. The measurements from the mth subarray is represented by the mth column of $\tilde{\mathbf{E}}$ Hence, the covariance matrix of the mth sub-array can be obtained as:

$$\mathbf{R}_{\tilde{\boldsymbol{\eta}}_m} = N\mathbf{F}(\boldsymbol{\theta})E\left[\boldsymbol{\phi}_m\boldsymbol{\phi}_m^H\right]\mathbf{F}^H(\boldsymbol{\theta}) + \underbrace{E\left[\mathbf{v}_m\mathbf{v}_m^H\right]}_{\sigma_w^2\mathbf{I}} \,. \tag{16}$$

Obviously, the rank of the signal subspace is equal to one, because $rank\left(E(\boldsymbol{\phi}_m\boldsymbol{\phi}_m^H)\right) = 1$. $\mathbf{R}_{\tilde{\boldsymbol{\eta}}_m}$ represents the autocorrelation matrix of the response to the mth component of the transmitted signal, $\tilde{s}_[n]$. In the proposed method, these matrices are smoothed, and therefore it is referred to as Transmission Diversity Smoothing (TDS).

By averaging these matrices over m, we obtain

$$\begin{aligned}\bar{\mathbf{R}}_{\tilde{\boldsymbol{\eta}}} &= \tfrac{1}{M}\sum_{m=1}^{M}\mathbf{R}_{\tilde{\boldsymbol{\eta}}_m} \\ &= N\mathbf{F}(\boldsymbol{\theta})E\left[\tfrac{1}{M}\sum_{m=1}^{M}\boldsymbol{\phi}_m\boldsymbol{\phi}_m^H\right]\mathbf{F}^H(\boldsymbol{\theta}) + \sigma_w^2\mathbf{I} \\ &= N\mathbf{F}(\boldsymbol{\theta})E\left[\tfrac{1}{M}\mathbf{\Phi}\mathbf{\Phi}^H\right]\mathbf{F}^H(\boldsymbol{\theta}) + \sigma_w^2\mathbf{I} \,,\end{aligned} \tag{17}$$

where $\mathbf{\Phi} \triangleq [\boldsymbol{\phi}_1, \ldots, \boldsymbol{\phi}_M]$. It can be shown that if the matrix $\mathbf{F}(\boldsymbol{\theta})$ and the signal autocorrelation matrix, $\mathbf{R_s}$ are full-rank, then the matrix $\mathbf{\Phi}\mathbf{\Phi}^H$ has full rank. This enables using eigenstructure-base methods such as MUSIC for this problem. Note that unlike the spatial smoothing, the array aperture is not reduced. This fact results in higher DOA estimation performance.

5. SIMULATION RESULTS

In this section, the localization performance using the proposed TDS algorithm with orthogonal signal transmission is evaluated and compared to the conventional model using the spatial smoothing algorithm. The MUSIC algorithm is used in both cases for target localization. A Uniformly Linear Array (ULA) of $M = 5$ sensors with half a wavelength spacing is used. The scenario includes two targets ($L = 2$), which are located at $\theta_1 = 0°$ and $\theta_2 = 15°$, respectively. In the spatial smoothing algorithm, the number of sensors in each subarray is $Ns = 4$.

In Fig. 2, the performances of the first target localization, θ_1, using different methods, are presented in terms of root-mean-square error (RMSE). It can be seen that the TDS algorithm with orthogonal transmitted signals outperforms the other configurations in which the transmitted signals are coherent. However, the CRB is not achieved by this configuration, because in the smoothing procedure, we dropped the information on $\boldsymbol{\theta}$ carried by ϕ_{lm} in Eq. (14). The TDS algorithm with the coherent signals fails, because the rank of the signal covariance matrix does not increase in the averaging process.

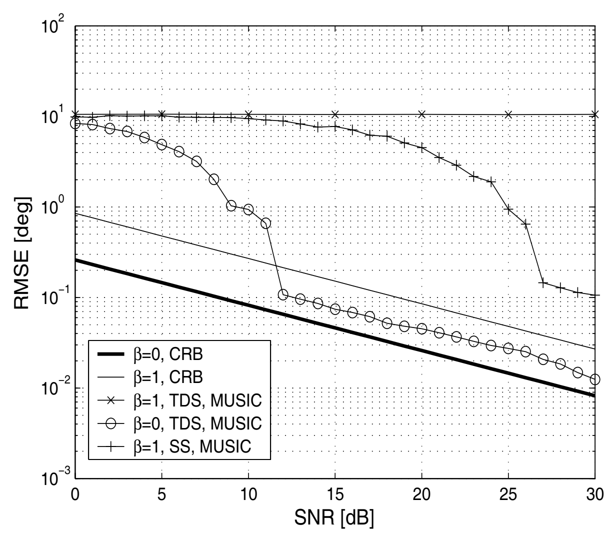

Figure 2: Performance of the TDS-MUSIC algorithm with orthogonal ($\beta = 0$) and coherent ($\beta = 1$) signals, compared to the spatial smoothing-MUSIC and CRB.

Fig. 3 illustrates the TDS-MUSIC spectrum using orthogonal and coherent signals, and with spatial smoothing, where $SNR = 15\ dB$. The TDS-MUSIC algorithm using orthogonal transmitted signals localizes the two targets in higher accuracy than the spatial smoothing-MUSIC, while TDS-MUSIC with coherent transmitted signals fails.

The performance of the TDS-MUSIC with orthogonal transmitted signals for non-uniform linear array from Fig. 4 is shown in Fig. 5. As already discussed, the TDS is not limited to any array geometry. The spatial smoothing algorithm cannot be implemented in this case, because it requires a ULA.

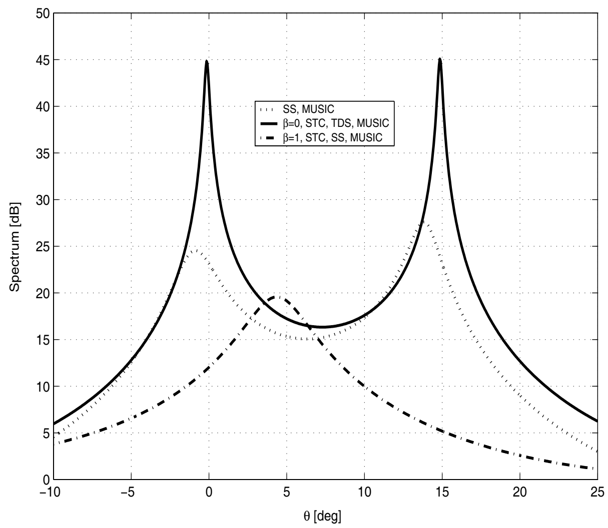

Figure 3: Spatial spectrum of TDS-MUSIC with $\beta = 0$ and $\beta = 1$ compared to the spatial smoothing-MUSIC; $M = 5$, $L = 2$, $\theta_1 = 0°$, $\theta_2 = 15°$, $SNR = 15\ dB$.

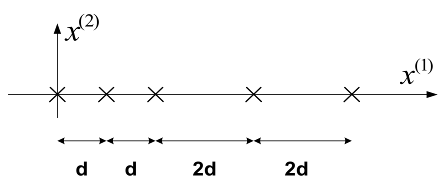

Figure 4: Non-uniform linear array; $d = \lambda/2$

6. CONCLUSIONS

In this paper, a new approach for target localization in active radar and sonar systems using spatially coded signal transmission was presented. The proposed TDS algorithm spatially smooths the received signal correlation matrix using transmission diversity, and it enables to use eigenstructure-based techniques for multi-target localization. The method decorrelates the recived signals which may be reflected paths from different directions or the signal echo fom different targets. Unlike conventional spatial smoothing or forward-backward averaging, the TDS does not require a uniform or symmetric array. In fact, no assumption of far-field targets or spatially white noise was employed. Moreover, it does not decrease the array aperture. The maximum number of targets which can be localized using the TDS method is $M - 1$, where M is the number of array elements.

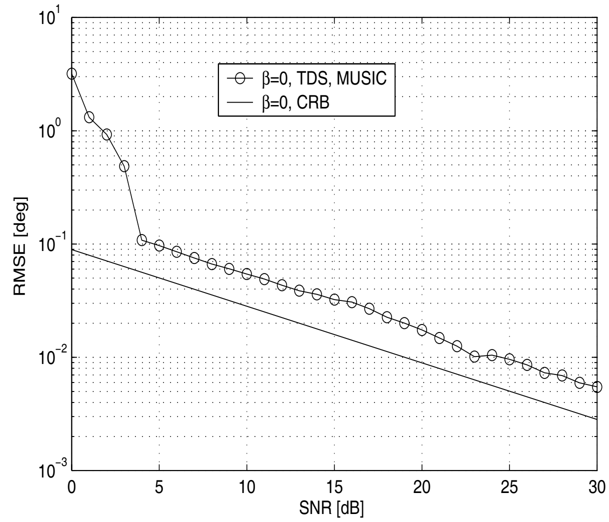

Figure 5: Performance of the TDS-MUSIC algorithm with orthogonal ($\beta = 0$) signals using non-uniform array, compared to the CRB.

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