FAST PASSIVE SOURCE LOCALIZATION IN RANGE-RATE WITH TILTED LINE ARRAYS

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ABSTRACT

We present a new approach for estimating the velocity of a source based on a model of differential phase change of the modes in a shallow-water environment. This approach is easily implementable and computationally fast compared to our previous approaches [1, 2, 3]. We use the velocity and bearing estimates of a strong interferer to suppress it in order to detect and localize weaker sources. Range-rate localization is particularly effective in discriminating sources located in the near-endfire region, where conventional beamforming has the poorest discrimination.

1. INTRODUCTION

Matched Field Processing (MFP) techniques localize sources in a shallow water environment by computing a replica vector that is based on channel modes with a given set of environmental parameters [4]. The resulting beamformer output produces a likelihood surface that shows peaks corresponding to the range and depth of the source. The surface also contains peaks at ambiguous ranges and depths, which makes it difficult to determine the true source and to distinguish the source from the interferer, particularly when there is uncertainty about the environmental parameters.

The discrimination ability of conventional beamforming (as well as of Minimum Variance Distortionless Response (MVDR) methods [5]) is best at broadside and the poorest at endfire. This is illustrated in Figure 1, which shows the bearing response produced by a 400-sensor horizontal array when the two sources, that are separated by five degrees, are located (a) in the broadside region, and (b) near-endfire. In this paper we localize in range-rate. Part of the motivation for this is that range-rate resolution is highest near-endfire, complementing the abilities of conventional beamformers, which have the poorest bearing resolution near-endfire.

Recent papers have discussed approaches to passive localization of moving sources in range-rate, using non-parametric particle filtering (sequential resampling techniques) [1, 2, 3]. An additional advantage of localizing in range-rate is that the estimation is less sensitive to wavenumber errors that are due to environmental uncertainties [1]. A recursive Bayesian state-space model, similar to that of Kalman filter, is used by the authors in [2, 3] to incorporate target dynamics. A method of sequential importance sampling (SIS) is proposed for updating the likelihood. The method is based on an approach to non-linear Bayesian state estimation that employs a discrete approximation to the state probability density function [6].

In this work we present a new method of estimating the velocity of the source. In comparison with the particle-filter approach, method is computationally fast, requires low SNR, and provides an excellent resolution in the endfire region. We also propose a method for estimating modal phases that makes possible effective dynamic spatial cancellation needed for detection and localization of a source in the presence of strong interference. Our previous approaches [1, 2, 3] assumed a vertical array geometry, while a new method, described here, is generalized to tilted array geometries (the calibration of which was considered in [7]), and, in particular, horizontal towed-array geometry.

2. SIGNAL MODEL

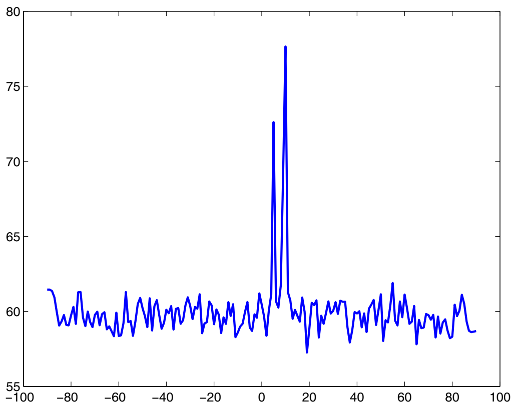
We assume a shallow-water environment, which leads to a signal model consisting of a superposition of modal response patterns. The signal replica vector $\underset{\sim}{s}_t$ is written as

$$\underset{\sim}{s}_t \propto \sum_{m=1}^{M} \underset{\sim}{\psi}_m \odot \underset{\sim}{\varphi}_m \cdot \psi_m(d) e^{-jk_m \cdot r_t}, \tag{1}$$

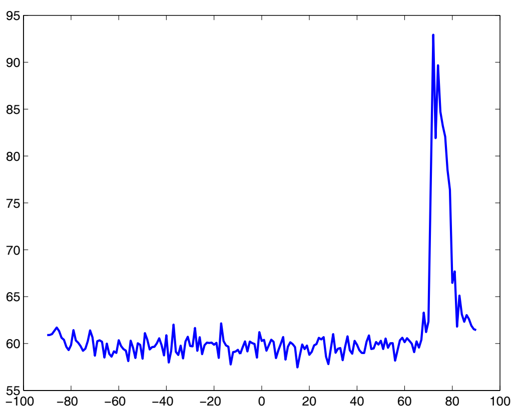
where:

- $\underset{\sim}{\psi}_m$ is the profile of the mth mode sampled at the depths for each sensor of the array,
- $\underset{\sim}{\varphi}_m$ is a vector similar to a "free-space" steering vector with the wavenumber replaced by the wavenumber of the mode, containing the phases across the array due to a source at bearing ϑ_s measured from the broadside direction,
- $\psi_m(d)$ is the profile of the mth mode sampled at the depth of the source,
- k_m is the vertical wavenumber for mth mode,

Submission to ICASSP 2005. This work was supported by the U.S. Office of Naval Research under Contract N00014-03-1-0830, and by NSERC Canada under Grant 262026.



(a) at the broadside region



(b) at the endfire

Fig. 1. Bearing response with sources (a) at 85 and 90 degrees and (b) at 5 and 10 degrees; SNR=-15dB, SIR=-10dB, N=400.

- r_t is the range between the first sensor of the array and the source at time t,
- $\odot$ represents the Hadamard (element by element) vector product.

For a tilted linear array, the phases in the steering vector $\underline{\varphi}_m$ are given by

$$\underline{\varphi}_m[n] = \exp(jk_m(n-1)\Delta x \cos\gamma \sin\vartheta_s),$$

where Δx is the sensor spacing and γ is the tilt angle of the array ($\gamma = 0$ for a horizontal, $\gamma = \pi/2$ for a vertical array).

The resulting signal vector $\underline{s}_t$ is a superposition of multiple modes of propagation on each sensor of the array. It is a column vector of length N, where N is the number of sensors. It is convenient to rewrite Eqn. 1 in matrix form:

$$\underline{s}_t = \mathbf{H}(\vartheta_s, d)\underline{x}_t, \tag{2}$$

where:

- $\mathbf{H}$ is the $N \times M$ mode matrix, the mth column of which is given by a modal response vector $\underline{\psi}_m \odot \underline{\varphi}_m \cdot \psi_m(d)$,
- $\underline{x}_t = e^{-jk_m \cdot r_t}$ is the vector of the initial phases for each mode.

The motion of a source can be described using a state-space model with the state equation

$$\underline{x}_t = \cdot\mathbf{A}(v_s\Delta t) \cdot \underline{x}_{t-1}, \tag{3}$$

where $\mathbf{A} = e^{-jk_m v_s \Delta t}$ is a diagonal state-transition matrix corresponding to the differential change in range for a hypothesized velocity of the source, v_s. Finally, the data is modeled by the measurement equation

$$\underline{y}_t = a_t\mathbf{H}(\vartheta_s, d)\underline{x}_t + \underline{n}_t, \tag{4}$$

where $\underline{n}_t$ is additive zero-mean complex Gaussian noise and a_t is an additional random Gaussian distributed amplitude, due to the fact that we are performing narrowband post-FFT processing on a broadband source (we consider the case where the source spectrum is broad compared to the FFT bin width).

3. SVD BASED METHOD FOR PASSIVE LOCALIZATION

We wish to estimate velocity based on the the differential phase change of the modal amplitudes. This is similar in spirit to Doppler processing. However, we have the added challenge of estimating velocity in the presence of the random source amplitudes a_t, which remove phase coherence between successive data snapshot vectors. Therefore we do not attempt to model the absolute phase of the wavefront; instead we model the relative phase between modes.

A minimum mean-square error estimate of $a\underline{x}$, based on $\underline{y}$ (see equation 4), can be obtained as follows:

$$\widehat{a_t\underline{x}_t} = \mathbf{R}_{(a\underline{x})\underline{y}}\mathbf{R}_{\underline{yy}}^{-1}\underline{y}_t \tag{5}$$

Assuming that $a_t\underline{x}_t \sim CN[E\{a_t\underline{x}_t\}, \sigma_s^2\mathbf{I}]$, where σ_s^2 is the variance of a_t, we find that the covariance matrix of the data vector is given by

$$\mathbf{R}_{\underline{yy}} = \sigma_s^2\mathbf{H}\mathbf{H}^T + \sigma_n^2\mathbf{I}, \tag{6}$$

and the cross-covariance is given by

$$\mathbf{R}_{(a\underline{x})\underline{y}} = \sigma_s^2\mathbf{H} + E\{(a\underline{x})_t\underline{n}_t^T\} = \sigma_s^2\mathbf{H} \tag{7}$$

Substituting 6 and 7 into 5, we obtain:

$$\begin{aligned}\widehat{a_t\underline{x}_t} &= \sigma_s^2\mathbf{H}^T(\sigma_s^2\mathbf{H}\mathbf{H}^T + \sigma_n^2\mathbf{I})^{-1}\underline{y}_t \\ &= \mathbf{H}^T(\mathbf{H}\mathbf{H}^T + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I})^{-1}\underline{y}_t.\end{aligned} \tag{8}$$

Using the Woodbury's identity (Sherman-Morrison formula), equation 8 can be rewritten as follows:

$$\widehat{a_t \mathbf{x}_t} = (\mathbf{I} + \frac{\sigma_s^2}{\sigma_n^2}\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T \underline{\mathbf{y}}_t \tag{9}$$

It is clear from 9 that in the limit that σ_n^2 is much bigger than σ_s^2, the estimate $\widehat{a_t \mathbf{x}_t}$ can be approximated as

$$\widehat{a_t \mathbf{x}_t} = \mathbf{H}^T \underline{\mathbf{y}}_t \tag{10}$$

We use state equation 3 to bring an estimate $\widehat{a_t \mathbf{x}_t}$ back to the time $t = 0$:

$$\widehat{a_t \mathbf{x}_0} = \mathbf{A}^{-t} \cdot \widehat{a_t \mathbf{x}_t}, \tag{11}$$

where $\mathbf{A}^{-t}$ is the inverse of $\mathbf{A}^t$.

Let $\widehat{\mathbf{X}}$ be a matrix, each column of which is an estimate $\widehat{a_t \mathbf{x}_0}$, obtained as follows

$$\widehat{\mathbf{X}} = \left[\widehat{a_0 \mathbf{x}_0}, \mathbf{A}^{-1}\widehat{a_1 \mathbf{x}_1}, \mathbf{A}^{-2}\widehat{a_2 \mathbf{x}_2}, ..., \mathbf{A}^{-(t-1)}\widehat{a_{t-1}\mathbf{x}_{t-1}}\right].$$

This matrix would be a rank-1 outer product of $\mathbf{x}_0$ and the sequence of source amplitudes $\{a_t\}$, if only the source of interest was present. Then an estimate of the initial mode amplitudes is $\hat{\mathbf{x}}_0 = \mathbf{u}_{max}$, where $\mathbf{u}_{max}$ is the left singular vector corresponding to the biggest singular value in the singular value decomposition (SVD) of $\widehat{\mathbf{X}}$.

To produce an ambiguity surface based on hypothesized bearing and velocity, we first compute the mode matrix $\mathbf{H}(\vartheta_h)$ and the state transition matrix $\mathbf{A}(v_h)$ for each hypothesized values of bearing and velocity. Then, the ambiguity surface function $M(\vartheta_h, v_h)$ can be obtained by match filtering the estimated replica vector with the data:

$$M(\vartheta_h, v_h) = \sum_{t=1}^{K} \mathbf{w}_t^T \underline{\mathbf{y}}_t, \tag{12}$$

where

$$\mathbf{w}_t = \mathbf{H}(\vartheta_h)\mathbf{A}^t(v_h)\hat{\mathbf{x}}_0$$

is the weight vector.

Formula 12 requires finding an inner product across a dimension equal to the number of sensors (an inner product in array space). As the number of sensors in the array can be very large, this procedure can be computationally intensive.

Equation 12 can be rewritten in the following equivalent form:

$$\begin{aligned} M(\vartheta_h, v_h) &= \sum_{t=1}^{K} \left[\mathbf{H}(\vartheta_h)\mathbf{A}^t(v_h)\hat{\mathbf{x}}_0\right]^T y_t \\ &= \sum_{t=1}^{K} \left[\mathbf{A}^t(v_h)\hat{\mathbf{x}}_0\right]^T \mathbf{H}(\vartheta_h)^T y_t \end{aligned} \tag{13}$$

Defining vector $\tilde{\mathbf{x}}_t(v_h) = \mathbf{A}^t(v_h)\hat{\mathbf{x}}_0$, we obtain

$$M(\vartheta_h, v_h) = \sum_{t=1}^{K} \tilde{\mathbf{x}}_t^T(v_h)\left[\mathbf{H}^T(\vartheta_h)\underline{\mathbf{y}}_t\right]. \tag{14}$$

The procedure for calculating an ambiguity surface, described by 14, requires finding an inner product across the number of propagating modes (an inner product in mode space). Finding an inner product in mode space is much less computationally intensive than that in array space, as the number of propagating modes is much smaller than the number of sensors in the array. In our case, with $M = 9$ and $N = 400$, computations are reduced by a factor of about 44.

Ambiguity surfaces produced by the SVD-based method are shown on the Figure 2 for the case when the source is observed in the presence of interference. The source and the interferer have velocities -25 m/s and -4 m/s respectively, where the minus sign means that both source and interferer are moving toward the array. The initial bearing of the source and interferer are 85 and 90 degrees respectively. Figure 2 (a) shows an ambiguity surface when the signal-to-interference ratio (SIR) is -2 dB and Figure 2 (b) shows it when the SIR is -10 dB. We can see that SIR of -10dB is not sufficient for the source to be visible. With low SIR, only the velocity of the interferer v_i and its bearing ϑ_i can be estimated. Thus, the SVD-based method is able to localize a source in near-endfire directions, but, in the presence of strong interference with SIR=-10dB, this ability is degraded. We suggest an approach to suppressing this interference in the following section.

4. INTERFERER SPATIAL CANCELLATION

One approach discussed in [3] is to employ a dynamically changed projection operator based on "particles" derived from sequential importance sampling.

Taking into account the interferer, the measurement equation 4 can be written as

$$\underline{\mathbf{y}}_t = a(t)\mathbf{H}(\vartheta_s)\mathbf{x}_t + b(t)\mathbf{H}(\vartheta_i)\mathbf{x}_t + \mathbf{n}_t, \tag{15}$$

where $b(t)$ is random Gaussian distributed amplitude due to the interferer and ϑ_i is the bearing of the interferer.

To suppress the interference the data vector is modified as follows:

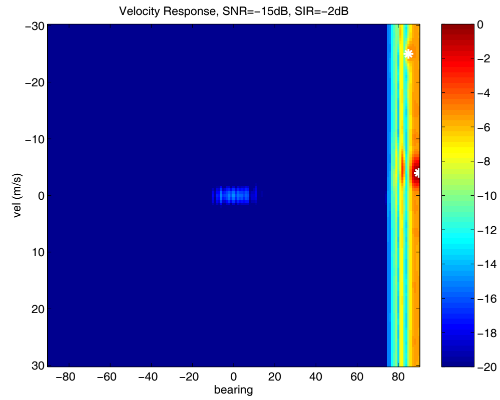
$$\underline{\mathbf{y}}_t \rightarrow (\mathbf{I} - \mathbf{P}_{s_t})\,\underline{\mathbf{y}}_t, \tag{16}$$

where $\mathbf{P}_{s_t}$ is the projection operator and $\mathbf{I}$ is identity matrix. The projection operator is recomputed for each time t as:

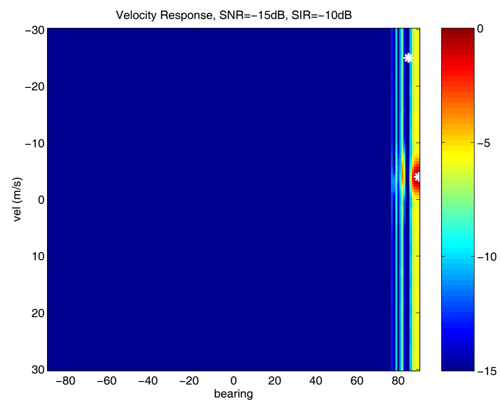
$$\mathbf{P}_{s_t} = \frac{s_t s_t^\dagger}{s_t^\dagger s_t}, \tag{17}$$

where

$$s_t = \mathbf{H}(\vartheta_i)x_t. \tag{18}$$



(a) SIR=-3dB



(b) SIR=-10dB

Fig. 2. Velocity-bearing ambiguity surfaces, obtained with the SVD based method, with (a) weak and (b) strong interference; SNR=-15dB, N=400.

Here x_t can be obtained for $\mathbf{A}^t(v_i)$ using the SVD based method presented in the previous section.

Figure 3 shows the ambiguity surface after the interference depicted in Figure 2 (b) (the strong interference case, SIR=-10 dB) has been suppressed in this manner.

5. CONCLUSIONS

The procedure which we present in this paper allows us to implement effective detection and localization of a source in a shallow water environment in the presence of a strong interferer. It employs low dimensional inner-products for each bin of the ambiguity surface, and is computationally faster than the previously investigated particle-filtering based approach. This procedure also has relatively low SNR requirements (as low as -15 dB per-element). The method can accommodate vertical, horizontal or general tilted array geometries.

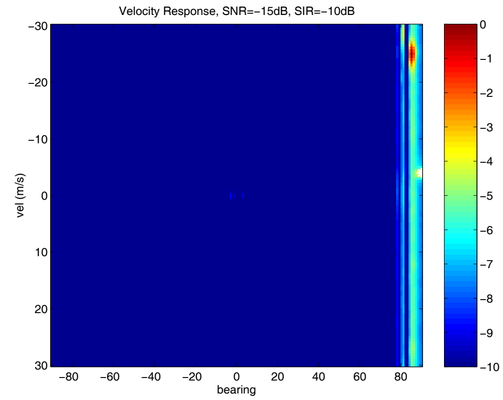


Fig. 3. Ambiguity surface in bearing and velocity, obtained with the SVD-based method after interferer is cancelled, SNR=-15dB, SIR=-10dB, N=400.

6. REFERENCES

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