TARGET DETECTION VIA FUSED SONAR WAVEFORMS

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ABSTRACT

A common model for sonar clutter is of the transmitted signal convolved with a colored Gaussian process relating to the sea-bottom profile. This environment becomes strongly non-Gaussian if there are multiple realizations: the univariate statistics remain Gaussian, but the joint probability density function (pdf) is not. It turns out that the gains versus a Gaussian-assumption can be substantial.

1. INTRODUCTION

We consider reverberation in active sonar to be the reflection of the transmitted signal from bottom, surface and from distributed matter. In this paper we are specifically interested in the Gaussian model [5] since it is commonly applied, since it is an intuitive and simplified version of [2], and since it is tractable.

Under the Gaussian model for clutter, we model the clutter as the convolution ($\star$) of the transmitted signal $s(t)$ with a (low-pass Gaussian) "bottom-profile" process $b(t)$. Intuitively, $b(t)$ ought to be relatively constant with time (not that $b(t)$ is nearly constant, but rather that the profile is repeatable); again, intuitively, a wise detection procedure should examine the return more than once, in order that there be some estimation — and subtraction of the effect — of $b(t)$. Thus, in this paper, we consider the "two-look" situation

$$\begin{aligned} r_1(t) &= A_1 s_1(t-\tau_d)e^{j2\pi f_d t} + \nu_1(t) + b_1(t)\star s_1(t) \\ r_2(t) &= A_2 s_2(t-\tau_d)e^{j2\pi f_d t} + \nu_2(t) + b_2(t)\star s_2(t) \end{aligned} \quad (1)$$

in which A_i are complex Gaussian amplitudes, τ_d and f_d are the delay/Doppler pair associated with the target and $\nu_i(t)$ are additive white complex Gaussian noises, and in which the first terms are missing if the target is absent. It may be beneficial that the interrogating waveforms $s_1(t)$ and $s_2(t)$ be of different types [3], since it is known that different waveforms can have complementary characteristics with regard to range and range-rate [1]. Note that in (1) some generality is admitted in that the two clutter-generating processes $b_1(t)$ and $b_2(t)$ are allowed to vary; but for the problem to be interesting they must be strongly dependent.

Model (1) informs the paper. It may appear a trivial Gaussian problem for which some version of the matched filter is optimal; and indeed if $b_1(t)$ and $b_2(t)$ were jointly Gaussian, this would be so. However, let us consider the case that $b_2(t) = b_1(t)e^{j\theta}$ for $\theta \sim \mathcal{U}(0, 2\pi)$: the two noises becomes uncorrelated, and the corresponding Gaussian assumption detector combines matched filter energies. The issue is that despite the Gaussian appearance of the problem, it is not Gaussian.

In the following Section 2 we shall abstract from (1) a simple bivariate detection problem and demonstrate the non-Gaussianity directly. In Section 3 we shall look at several versions of (1). In Section 4 we give examples of the performances of the optimal (or approximately optimal) detectors, and compare them with the matched filter that would arise from an assumption of Gaussianity. A more detailed version of this paper is available in [4].

2. DISCUSSION OF SCALAR CASE

Let us consider the simplification of (1) in which all is scalar. The hypothesis test is

$$\begin{aligned} H: \quad & y_1 = \nu_1 + be^{j\theta_1} \\ & y_2 = \nu_2 + be^{j\theta_2} \\ K: \quad & y_1 = s_1 + \nu_1 + be^{j\theta_1} \\ & y_2 = s_2 + \nu_2 + be^{j\theta_2} \end{aligned} \quad (2)$$

in which

- b is complex normal with variance $2\sigma_b^2$ and zero mean
- θ_1 and θ_2 are iid uniform (0,2π)
- ν_1 and ν_2 are iid complex normal with variance $2\sigma^2$ and zero mean
- s_1 and s_2 are known
- y_1 and y_2 are the observations

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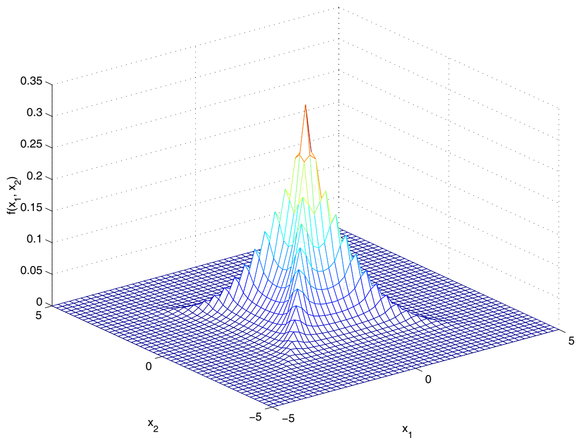

Fig. 1. Joint distribution of x_1 and x_2 (the real parts of y_1 and y_2 in (2)) for $\sigma = 0.1$ and $\sigma_b = 1$.

Clearly $\Re(y_1)$ and $\Im(y_1)$, the real and imaginary parts of y_1, are jointly Gaussian (and independent); the same holds for $\Re(y_2)$ and $\Im(y_2)$. But let $x_1 = \Re(y_1)$ and $x_2 = \Re(y_2)$: figure 1 shows $f(x_1, x_2|H)$ for $\sigma = 0.1$ and $\sigma_b = 1$ by numerical integration. Obviously, $f(x_1, x_2|H)$ is not a bivariate Gaussian distribution. Intuition suggests that the matched filter

$$T_{mf} = \Re(s_1^* y_1 + s_2^* y_2) \tag{3}$$

statistic, which would be optimal were the noise Gaussian, is wasteful of information. At high CNR the detector

$$T_{sd} = \left(|y_1|^2 - |y_2|^2\right)^2 - \left(|y_1 - s_1|^2 - |y_2 - s_2|^2\right)^2 \tag{4}$$

that takes advantage of the common length of b should be preferable ("sd" denotes subtracted distance). But there is no guarantee that (4) should be close to optimal; we find that it is not.

3. OPTIMAL JOINT DETECTOR

This section derives and discusses the optimal or approximated optimal detectors for joint detection of two waveforms in four cases, with the last one a discrete-time version of (1). It is supposed that

- $\vec{b}$ is a vector that is complex normal with covariance $2R_b$ and zero mean
- θ_1 and θ_2 are iid uniform (0,2π)
- ϕ_1 and ϕ_2 are iid uniform (0,2π)
- $\vec{\nu}_1$ and $\vec{\nu}_2$ are iid complex normal with covariance $2\sigma^2 I$ and zero mean
- $\vec{s}_1$ and $\vec{s}_2$ are known vectors
- $\vec{y}_1$ and $\vec{y}_2$ are vectors of the observations
- Dimension of these vectors is L, and all vectors are column vectors. When $L = 1$, $R_b = \sigma_b^2$.

In the first two cases the clutter is $be^{j\theta_1}$ and $be^{j\theta_2}$; in cases III & IV the clutter is $(b \otimes s_1)e^{j\theta_1}$ and $(b \otimes s_2)e^{j\theta_2}$; this latter, with $\otimes$ denoting element-by-element multiplication, is as will be explained later a proxy for (1).

3.1. Case I: Clutter not related to signal, known signal

The hypothesis test is

$$\begin{aligned} H: \quad & \vec{y}_1 = \vec{\nu}_1 + \vec{b}e^{j\theta_1} \\ & \vec{y}_2 = \vec{\nu}_2 + \vec{b}e^{j\theta_2} \\ K: \quad & \vec{y}_1 = \vec{s}_1 + \vec{\nu}_1 + \vec{b}e^{j\theta_1} \\ & \vec{y}_2 = \vec{s}_2 + \vec{\nu}_2 + \vec{b}e^{j\theta_2} \end{aligned} \tag{5}$$

Some algebra (see [4]) leads us to the optimal test statistic

$$\begin{aligned} T_{opt\ I} = & \frac{1}{\sigma^2}\Re\left(\vec{y}_1^H (I - \Sigma^{-1})\vec{s}_1 + \vec{y}_2^H (I - \Sigma^{-1})\vec{s}_2\right) \\ & + \log I_0\left(\frac{1}{\sigma^2}\left|(\vec{y}_1 - \vec{s}_1)^H \Sigma^{-1} (\vec{y}_2 - \vec{s}_2)\right|\right) \\ & - \log I_0\left(\frac{1}{\sigma^2}\left|\vec{y}_1^H \Sigma^{-1} \vec{y}_2\right|\right) \end{aligned} \tag{6}$$

where

$$\Sigma = \sigma^2 R_b^{-1} + 2I \qquad \alpha = \theta_1 - \theta_2 \tag{7}$$

and in which $I_0(\cdot)$ is zero-order modified Bessel function of the first kind. Some similarity in form to the intuitive (4) is apparent[1], but it is clear that the optimal detector is more involved. In fact, while the detector of (4) is appealing and is generally an improvement over the matched filter, there is usually a considerable gap between it and the optimal detector.

3.2. Case II: Clutter not related to signal, known signal with random phase

The hypothesis test is

$$\begin{aligned} H: \quad & \vec{y}_1 = \vec{\nu}_1 + \vec{b}e^{j\theta_1} \\ & \vec{y}_2 = \vec{\nu}_2 + \vec{b}e^{j\theta_2} \\ K: \quad & \vec{y}_1 = \vec{s}_1 e^{j\phi_1} + \vec{\nu}_1 + \vec{b}e^{j\theta_1} \\ & \vec{y}_2 = \vec{s}_2 e^{j\phi_2} + \vec{\nu}_2 + \vec{b}e^{j\theta_2} \end{aligned} \tag{8}$$

We have

$$\begin{aligned} & f(\vec{y}_1, \vec{y}_2 | K, \theta_1, \theta_2) \\ & = \int_{\phi_1}\int_{\phi_2} f(\vec{y}_1, \vec{y}_2 | K, \phi_1, \phi_2, \theta_1, \theta_2) d\phi_2 d\phi_1 \end{aligned} \tag{9}$$

[1] A reasonable approximation is that the logarithm of the Bessel function is quadratic for small arguments and linear for large.

where

$$f(\vec{y}_1,\vec{y}_2|K,\phi_1,\phi_2,\theta_1,\theta_2) = \frac{1}{(2\pi)^{2L}\sigma^{2L}|\sigma^2 I + 2R_b|}\times \exp\left\{-\frac{1}{2\sigma^2}\left[(\vec{y}_1-\vec{s}_1e^{j\phi_1})^H(I-\Sigma^{-1})(\vec{y}_1-\vec{s}_1e^{j\phi_1}) +(\vec{y}_2-\vec{s}_2e^{j\phi_2})^H(I-\Sigma^{-1})(\vec{y}_2-\vec{s}_2e^{j\phi_2}) -2\Re\left((\vec{y}_1-\vec{s}_1e^{j\phi_1})^H\Sigma^{-1}(\vec{y}_2-\vec{s}_2e^{j\phi_2})e^{j\alpha}\right)\right]\right\} \quad (10)$$

and Σ and α are defined in (7). Unfortunately, we cannot get explicit expressions for the two integrations in equation (9), and we are forced to use numerical approximation; fortunately, from simulation, we find that relatively coarse quantization of ϕ_1 and ϕ_2 (see (15) and (16)) yields good performance.

3.3. Case III: Clutter related to signal, known signal

The hypothesis test is

$$\begin{aligned} H: \quad & \vec{y}_1 = \vec{\nu}_1 + (\vec{b}\otimes\vec{s}_1)e^{j\theta_1} \\ & \vec{y}_2 = \vec{\nu}_2 + (\vec{b}\otimes\vec{s}_2)e^{j\theta_2} \\ K: \quad & \vec{y}_1 = \vec{s}_1 + \vec{\nu}_1 + (\vec{b}\otimes\vec{s}_1)e^{j\theta_1} \\ & \vec{y}_2 = \vec{s}_2 + \vec{\nu}_2 + (\vec{b}\otimes\vec{s}_2)e^{j\theta_2} \end{aligned} \quad (11)$$

in which "$\otimes$" denotes element-by-element multiplication (i.e., "b.*s" in Matlab). Note that in (1) the clutter is defined by a convolution rather than by the element-by-element multiplication of (11); however, if the discrete Fourier transform (DFT) of a sampled version of (1) is taken, (11) results.

The joint distribution of $\vec{y}_1$ and $\vec{y}_2$ under K is

$$f(\vec{y}_1,\vec{y}_2|K,\theta_1,\theta_2) = \frac{1}{(2\pi)^{2L}\sigma^{2L}|\sigma^2 I + \Lambda_1^H\Lambda_1 R_b + \Lambda_2^H\Lambda_2 R_b|} \times\exp\left\{-\frac{1}{2\sigma^2}\left[(\vec{y}_1-\vec{s}_1)^H(I-\Lambda_1\Sigma^{-1}\Lambda_1^H)(\vec{y}_1-\vec{s}_1) +(\vec{y}_2-\vec{s}_2)^H(I-\Lambda_2\Sigma^{-1}\Lambda_2^H)(\vec{y}_2-\vec{s}_2) -2\Re\left((\vec{y}_1-\vec{s}_1)^H\Lambda_1\Sigma^{-1}\Lambda_2^H(\vec{y}_2-\vec{s}_2)e^{j\alpha}\right)\right]\right\} \quad (12)$$

(under H the signal terms are missing) where the definition of α is in equation (7), and in which

$$\Sigma = \sigma^2 R_b^{-1} + \Lambda_1^H\Lambda_1 + \Lambda_2^H\Lambda_2 \quad (13)$$

for $\Lambda_i = diag(\vec{s}_i)$, and the definition of α is in equation (7). The explicit optimal test statistic is

$$T_{opt\ III} = \frac{1}{\sigma^2}\Re\left\{\vec{y}_1^H(I-\Lambda_1\Sigma^{-1}\Lambda_1^H)\vec{s}_1 +\vec{y}_2^H(I-\Lambda_2\Sigma^{-1}\Lambda_2^H)\vec{s}_2\right\} +\log I_0\left(\frac{1}{\sigma^2}\left|(\vec{y}_1-\vec{s}_1)^H\Lambda_1\Sigma^{-1}\Lambda_2^H(\vec{y}_2-\vec{s}_2)\right|\right) -\log I_0\left(\frac{1}{\sigma^2}\left|\vec{y}_1^H\Lambda_1\Sigma^{-1}\Lambda_2^H\vec{y}_2\right|\right)$$

3.4. Case IV: Clutter related to signal, known signal with random phase

The hypothesis test is

$$\begin{aligned} H: \quad & \vec{y}_1 = \vec{\nu}_1 + (\vec{b}\otimes\vec{s}_1)e^{j\theta_1} \\ & \vec{y}_2 = \vec{\nu}_2 + (\vec{b}\otimes\vec{s}_2)e^{j\theta_2} \\ K: \quad & \vec{y}_1 = \vec{s}_1e^{j\phi_1} + \vec{\nu}_1 + (\vec{b}\otimes\vec{s}_1)e^{j\theta_1} \\ & \vec{y}_2 = \vec{s}_2e^{j\phi_2} + \vec{\nu}_2 + (\vec{b}\otimes\vec{s}_2)e^{j\theta_2} \end{aligned} \quad (14)$$

As in case II, we cannot get an explicit expression for the two integrations in $f(\vec{y}_1,\vec{y}_2|K)$, and a similar numerical approximation is used. The pdf $f(\vec{y}_1,\vec{y}_2|K,\phi_1,\phi_2,\theta_1,\theta_2)$ is as in (12), with the exception that $\vec{s}_i$ is multiplied by $e^{j\phi_i}$. The optimal statistic is approximated as

$$T_{opt\ IV} \approx \sum_i\sum_j e^{B(\phi_1^i,\phi_2^j)} \quad (15)$$

where

$$B(\phi_1^i,\phi_2^j) = \frac{1}{\sigma^2}\Re\left\{\vec{y}_1^H(I-\Lambda_1\Sigma^{-1}\Lambda_1^H)\vec{s}_1e^{j\phi_1^i} +\vec{y}_2^H(I-\Lambda_2\Sigma^{-1}\Lambda_2^H)\vec{s}_2e^{j\phi_2^j}\right\} +\log I_0\left(\frac{1}{\sigma^2}\left|(\vec{y}_1-\vec{s}_1e^{j\phi_1^i})^H\Lambda_1\Sigma^{-1} \times\Lambda_2^H(\vec{y}_2-\vec{s}_2e^{j\phi_2^j})\right|\right) -\log I_0\left(\frac{1}{\sigma^2}\left|\vec{y}_1^H\Lambda_1\Sigma^{-1}\Lambda_2^H\vec{y}_2\right|\right) \quad (16)$$

4. SIMULATION RESULTS

Here we compare the performance of the optimal or approximated optimal detectors derived in Section 3 to the matched filter. Only the scalar case is simulated, several representative values are selected for s_1 and s_2. The matched filter used for cases I and III is

$$\Re(\vec{y}_1^H\vec{s}_1 + \vec{y}_2^H\vec{s}_2) \quad (17)$$

while that for cases II and IV is

$$|\vec{y}_1^H\vec{s}_1|^2 + |\vec{y}_2^H\vec{s}_2|^2 \quad (18)$$

since in those cases the return signal has a random phase.

Table 1 compares the optimal detector and matched filter, with $\sigma_b = 1$ and $\sigma = 0.1$ (i.e., a high CNR) and $P_{fa} = 0.1\%$. In cases II and IV, in which the optimal detector is only approximated, 10 uniform levels of quantization for ϕ_1 and ϕ_2 are used. The performance difference between the detector based on a Gaussian assumption (i.e., the matched filter) and the optimal detector is startling. The

P_d		$s_1 = 2$, $s_2 = 2$	$s_1 = 2 + 1j$, $s_2 = 2 + 2j$
Case I	MF	0.1467	0.4171
	SD	0.1813	0.2678
	OD	0.8265	0.9465
Case II	MF	0.0342	0.1192
	SD	0.1494	0.2165
	OD	0.6322	0.7806
Case III	MF	0.0178	0.0171
	SD	0.2387	0.0729
	OD	0.7962	0.8313
Case IV	MF	0.0043	0.0047
	SD	0.2228	0.0694
	OD	0.7198	0.7613

Table 1. Probability of detection of the optimal detector and matched filter under four cases of hypothesis test and different transmitted signals. $P_{fa} = 0.1\%$, $\sigma_b = 1$, $\sigma = 0.1$, and there are 10 levels uniform quantization for ϕ_1 and ϕ_2 in both case II and IV. "MF" represents the (appropriate) matched filter, "SD" the detector defined in equation (4) and "OD" the optimal detector.

detector defined in equation (4) generally performs better than the matched filter, while it is always worse than the optimal detector.

Figure 2 is of the probability of detection versus the number of quantization of ϕ_1 and ϕ_2 (quantization of ϕ_1 and ϕ_2 are same) for the approximated optimal detectors in case II and IV, respectively, with $s_1 = 2$, $s_2 = 2$, $P_{fa} = 0.1\%$, $\sigma_b = 1$ and $\sigma = 0.1$. We can see that relatively low level quantization of ϕ_1 and ϕ_2 can yields close-to-optimal detection performance.

5. CONCLUSIONS

Motivated by the detection of random-phase sonar signals in signal-generated clutter, we explore a particular case of two-look (fused) signal detection in noise that is dependent across channels. The noise is complex Gaussian and is the same in each channel except for a random phase. These two noise processes are uncorrelated, and a reasonable but naïve assumption of Gaussianity would lead to a conclusion that these noise processes are independent of each other. They are not independent, and indeed, although their univariate distributions are Gaussian, their joint pdf is strongly non-Gaussian.

Here we derive the optimal detectors for four cases with a known signal: with/without random phase; and clutter that is/is not signal-dependent. The last and most involved case is a proxy for the detection of sonar signals in clutter, at least after DFT processing. When the signal is known precisely

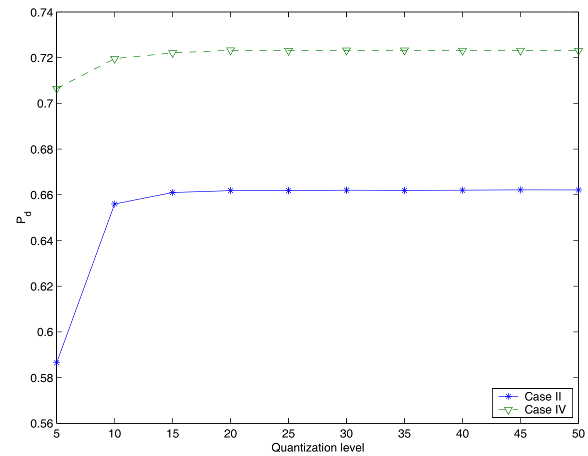

Fig. 2. Probability of detection versus quantization of ϕ_1 and ϕ_2 (quantization level of ϕ_1 and ϕ_2 are same) for the approximated optimal detectors in case II and IV, respectively. $s_1 = 2$, $s_2 = 2$, $P_{fa} = 0.1\%$, $\sigma_b = 1$ and $\sigma = 0.1$.

the optimal detector is explicit, and involves logarithms of Bessel functions of quadratic forms; with a random phase the optimal detector requires an integral, but there is an easy approximation given. In all cases, the performance improvement referred to the appropriate matched filter detector can be enormous when the CNR is large.

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